Discontinuous Galerkin Methods

Part 2: Application to turbulent flows

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Cenaero
Excellence in Simulation Technologies for Aeronautics

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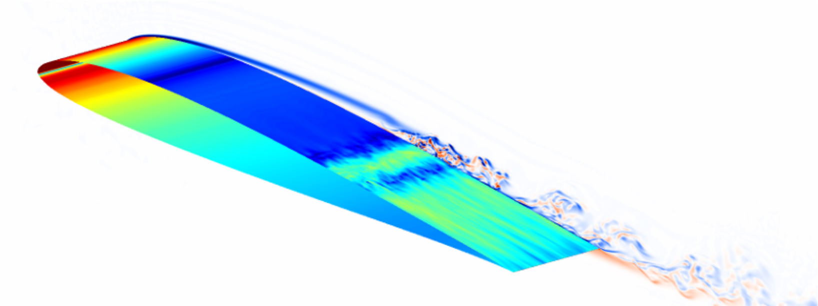
Outline

HOM for industrial turbulence

Modeling approaches : large scale turbulence and transition - SD7003, *Re* = 80.000

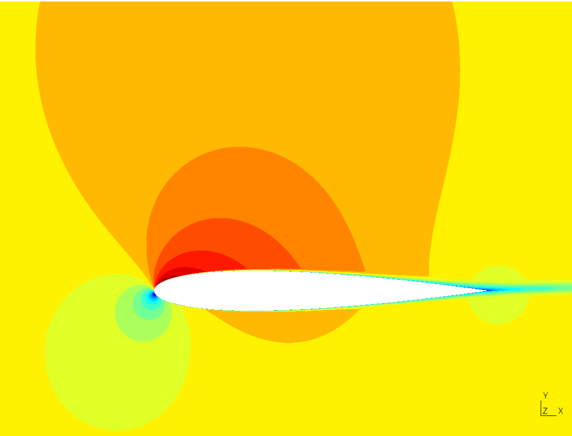


Direct Numerical Simulation

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{v} \otimes \mathbf{v} + \nabla p = \nabla \cdot \mu \nabla \mathbf{v}$$

HOM for industrial turbulence

Modeling approaches : fully turbulent BL - NACA0012 airfoil, $Re = 1.000,000$

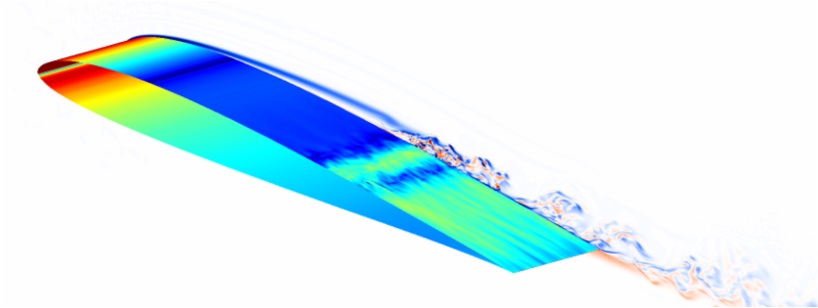


Reynolds Averaged Navier Stokes (RANS) : decompose $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$, $p = \overline{p} + p'$ and average

$$\frac{\partial \overline{\mathbf{v}}}{\partial t} + \nabla \cdot \overline{\mathbf{v}} \otimes \overline{\mathbf{v}} + \nabla \cdot \overline{\mathbf{v}' \otimes \mathbf{v}'} + \nabla \overline{p} = \nabla \cdot \mu \nabla \overline{\mathbf{v}}$$

HOM for industrial turbulence

Modeling approaches : large scale turbulence and transition - SD7003 $Re = 80.000$



Large Eddy Simulation : solve for low-pass filtered solution $\tilde{\mathbf{v}}$

$$\frac{\partial \tilde{\mathbf{v}}}{\partial t} + \nabla \cdot \tilde{\mathbf{v}} \otimes \tilde{\mathbf{v}} + \nabla \cdot \underbrace{\left(\widetilde{\mathbf{v} \otimes \mathbf{v}} - \tilde{\mathbf{v}} \otimes \tilde{\mathbf{v}} \right)}_{\tau_{SGS}} + \nabla \tilde{p} = \nabla \cdot \mu \nabla \tilde{\mathbf{v}}$$

HOM for industrial turbulence
Modeling approaches : cost of turbulence modeling approaches

Equilibrium TBL

$$y^+ = \frac{y\sqrt{u_\tau}}{\nu} \ , \ u_\tau = \sqrt{\frac{\tau}{\rho}}$$

- $\delta_{y+=5} \sim Re^{-3/4}$
- $\delta \sim Re^{-0.2}$

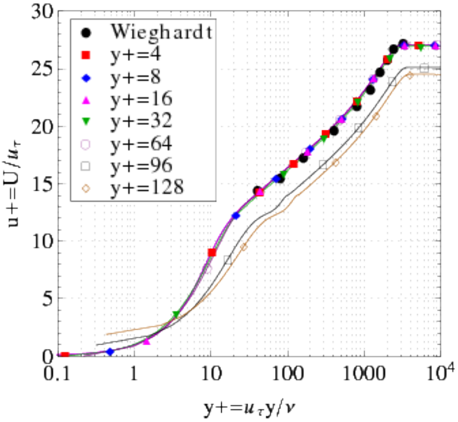
RANS $\sim Re^{3/4}$, steady

Resolved turbulence

- DNS $\sim Re^3$
- LES $\sim Re^{2.8}$
- hybrid RANS-LES $\sim Re^{0.8}$
- wall-modeled LES $\sim Re^{0.8}$

+ statistical convergence to be reached

Drosson & Hillewaert 2012



HOM for industrial turbulence

Modeling approaches : comparison

RANS

- − need for a clear scale separation between geometry and turbulence (higher Re, aligned flow)
- − model is much more global and hence needs tuning/calibration to specific flow situations
- + solve for average flow
- + low resolution since solving for smooth structures
- + industrial standard

LES

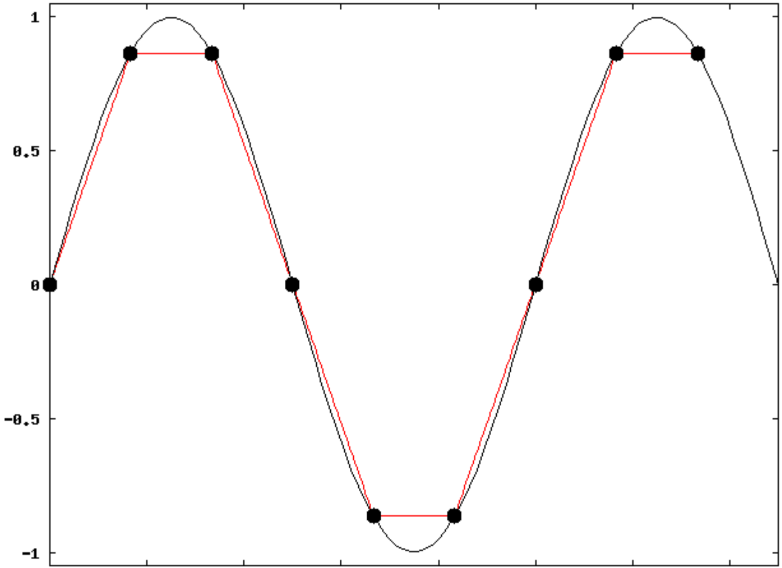
- + resolve part of the turbulence, hence much smaller need for clear scale separation between geometry and turbulence
- + small SGS, hence model should have a more universal, scale independent behaviour
- + SGS time scales are small with respect to geometrically relevant timescales, hence the model should probably only be correct in the statistical average (?)
- − need for unsteady computation and statistical convergence
- − high resolution and accuracy required

DNS :

- + most universally applicable
- − need for unsteady computation and statistical convergence
- − extremely high resolution required ($Re < 200.000$)

HOM for industrial turbulence

Modeling approaches : accuracy vs resolution



HOM for industrial turbulence

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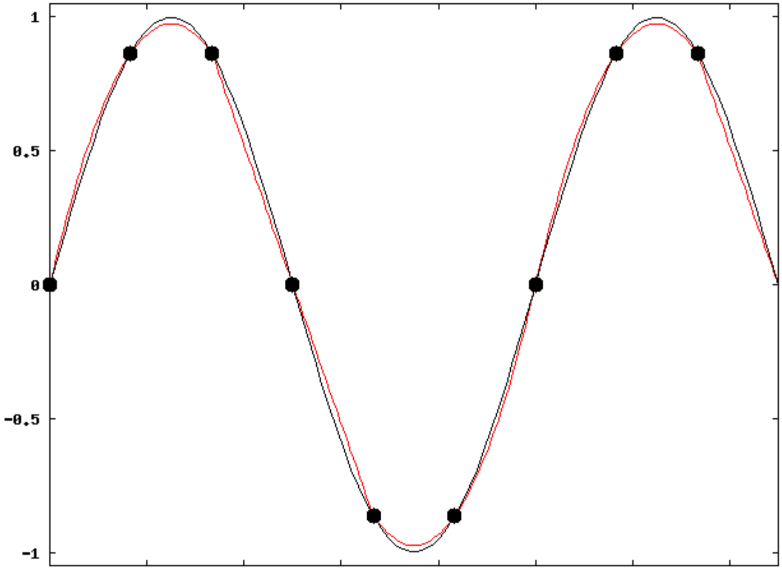
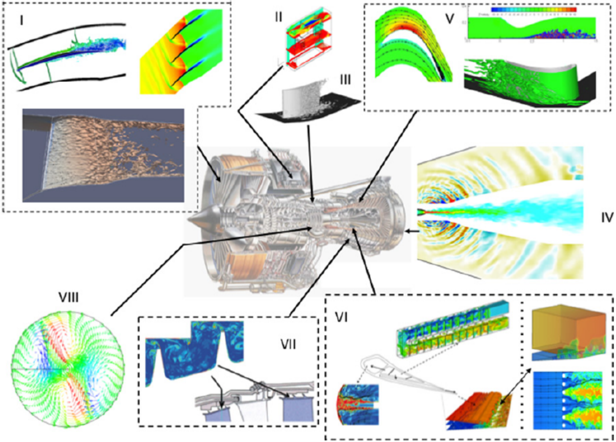


HOM for industrial turbulence

Towards industrial LES : turbulence in turbomachines

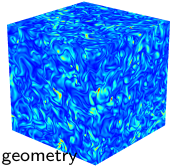


Tucker 2011 [Tuc11a, Tuc11b]

- higher Re
- no clear scale separation for many flow regimes
- however (U)RANS is still used due to cost issues
- advances in modeling required for truely predictive CFD

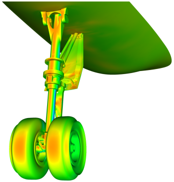
HOM for industrial turbulence
Towards industrial LES : State of the art

Academic codes (PSP, FD, ...) are tuned to canonical testcases in simple geometry



+ high order of accuracy
+ low computational cost
+ optimized dissipation (and dispersion)
- no / small geometric flexibility allowed
- some models are tuned to case

Industrial codes (FVM, stabilised FEM) are tuned for robustness and complex geometry



+ geometric flexibility
+ high scalability and efficiency
+ robustness
- (formally) 2^{nd} order of accuracy
- high dispersion error
- standard methods provide high dissipation (RANS/shocks)
- kinetic energy preserving methods degrade stability

Main effort so far has been on modeling and fundamental turbulence. However, the discretisation is important. We need

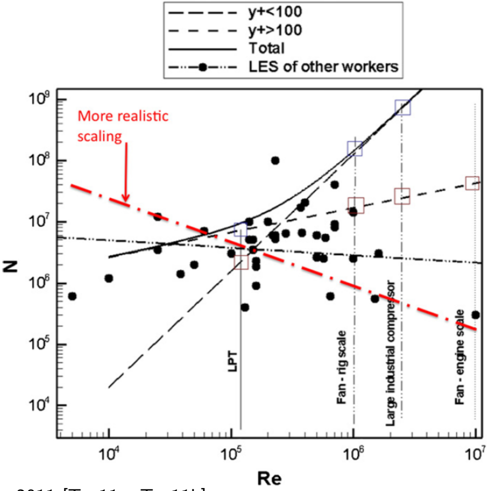
+ high resolution → HPC
+ no dissipation
+ low dispersion
- unstructured meshes complex geometry

DGM seems a good candidate

HOM for industrial turbulence

Towards industrial LES : state of the art

"Although LES is, obviously, much less model dependent than RANS, grids currently used for more practical simulations are clearly insufficiently fine for the LES model and numerical schemes not to be playing an excessively strong role."

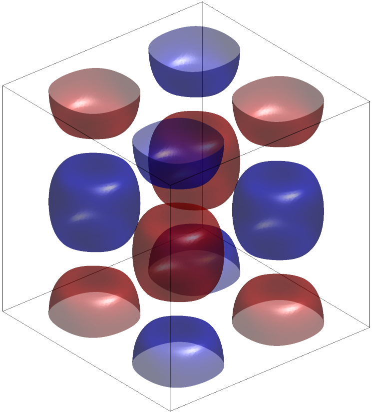


Tucker, Progress in Aerospace Sciences 2011 [Tuc11a, Tuc11b]

Outline

Validation

DNS Taylor-Green : testcase description



Taylor-Green vortex $Re = 1600$

- analytical initial solution
- spectrally resolved for $L/\Delta x \approx 256$ (128^{th} harmonic)
- reference pseudo-spectral computation up to 256^{th} harmonic
- order ($p = 2 \ldots 5$) and grid convergence ($p = 3$, $N = 144, 192, 288, 384$) study for DGM
- testcase C3.5 for the 1^{st} *Intl. Workshop for High Order Methods in CFD (AIAA ASM 2012) - paper submitted to IJNMF*
- *Carton & Hillewaert – in preparation for JCP*

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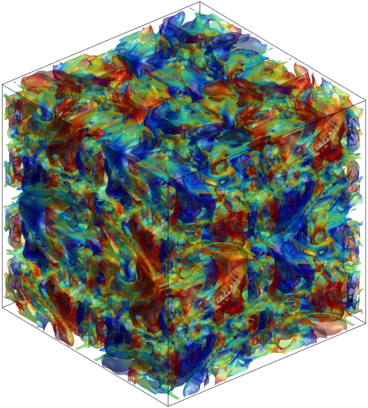

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Validation

DNS Taylor-Green : error criteria

From the conservation of momentum, we find the following equation for the kinetic energy E_k for (nearly) incompressible flow

$$-\frac{\partial}{\partial t}\int_V \rho E_k \, dV = 2\mu \int_V \mathbf{S}:\mathbf{S} \, dV - \underbrace{\int_V p\nabla\cdot\mathbf{v} \, dV}_{\approx 0} + \mu_v \underbrace{\int_V (\nabla\cdot\mathbf{v})^2}_{\approx 0} \, dV$$

$$\approx 2\mu \int_V \mathbf{S}:\mathbf{S} \, dV = 2\mu\mathcal{E}$$

with $\mathbf{S}$ the deviatoric part of the strain rate tensor

$$\mathbf{S} = \frac{\nabla\mathbf{v} + \nabla\mathbf{v}^T}{2}$$

Three errors can be defined

- $\Delta\epsilon_1$ = error on theoretical dissipation based on the enstrophy integral $\mathcal{E}$;
- $\Delta\epsilon_2$ = error on measured dissipation rate ;
- $\Delta\epsilon_3$ = difference between theoretical and measured dissipation rate

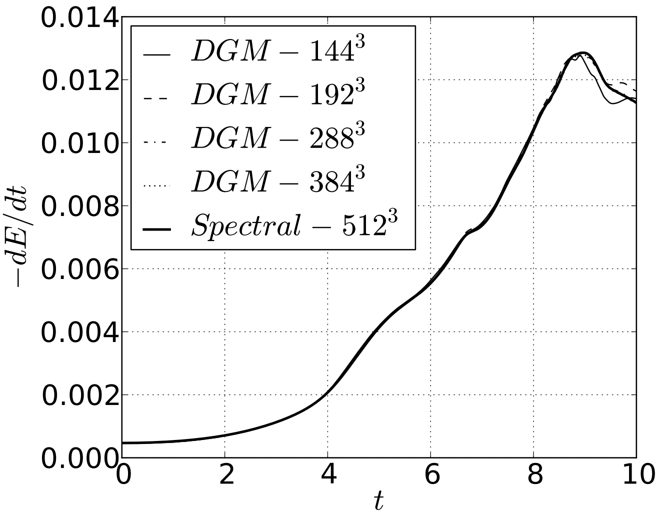
Resolution criteria in function of DGM as $n = Np$

- counts # dofs
- Nyquist criterion refers to number of points (cfr. zeros)

Comparison of DGM(3) vs FD4

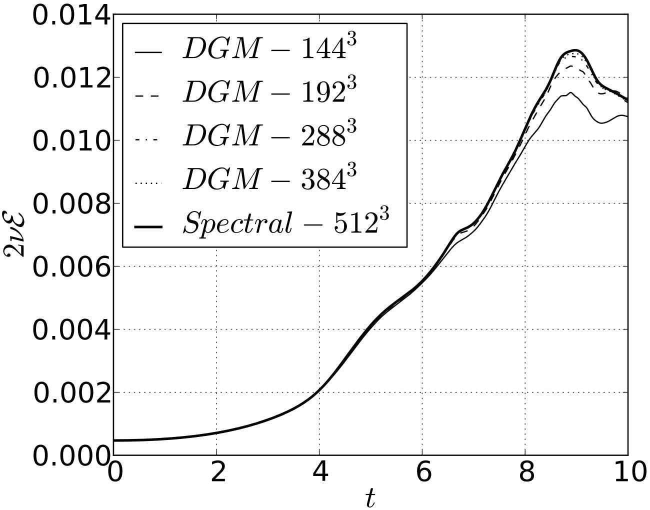
Validation

DNS Taylor-Green : energy dissipation rate



Validation

DNS Taylor-Green : energy dissipation rate



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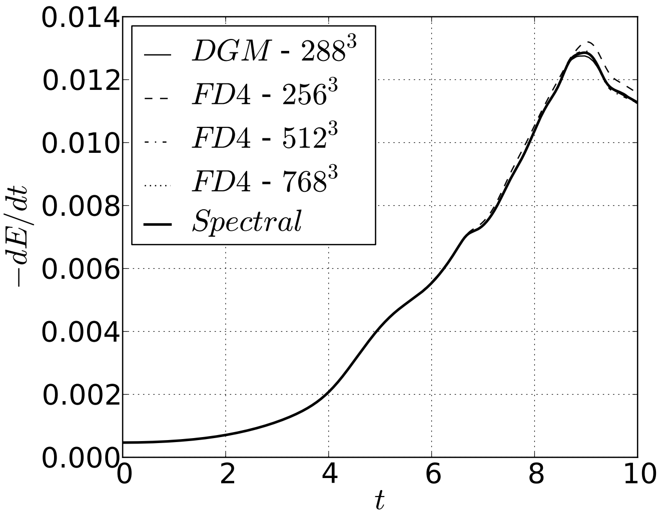
DNS Taylor-Green : energy dissipation rate



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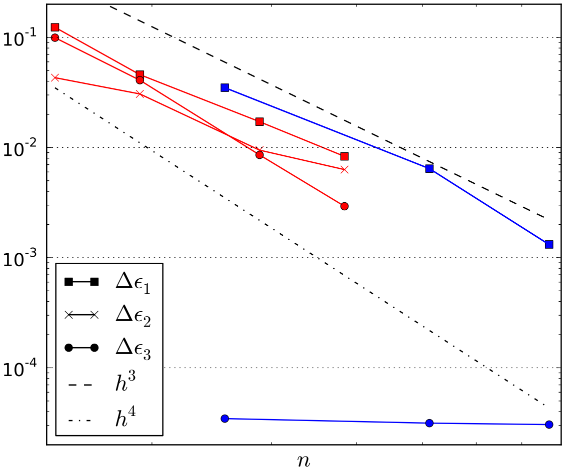
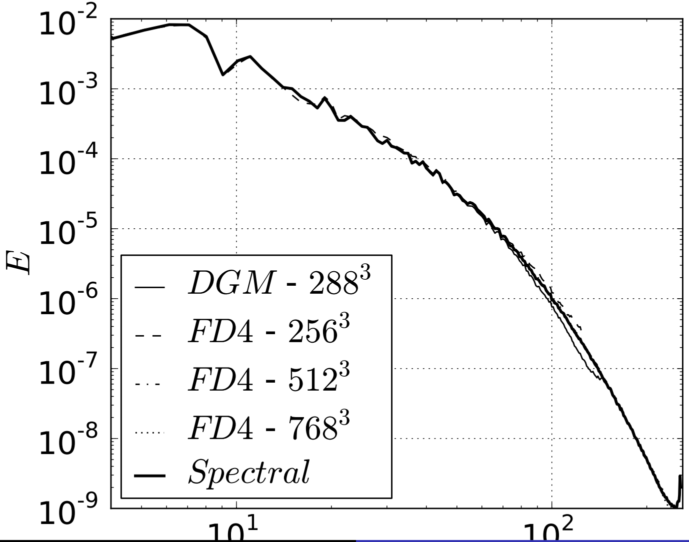


Validation

DNS Taylor-Green : spectral distribution of kinetic energy



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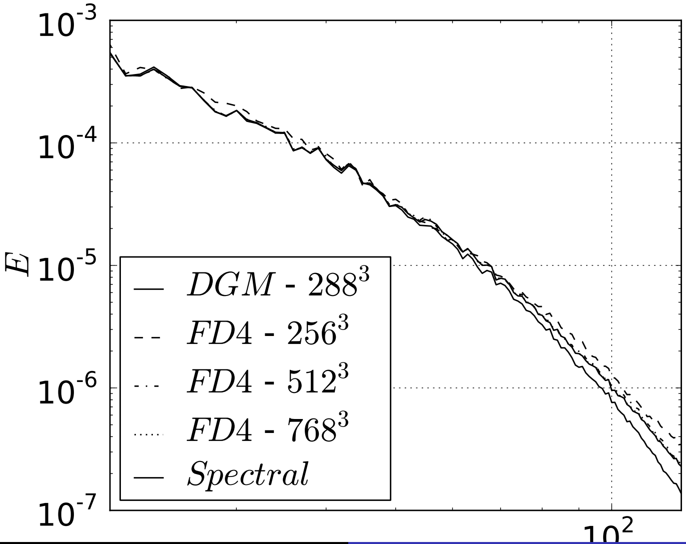
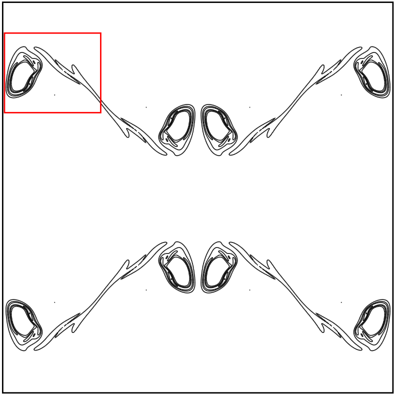
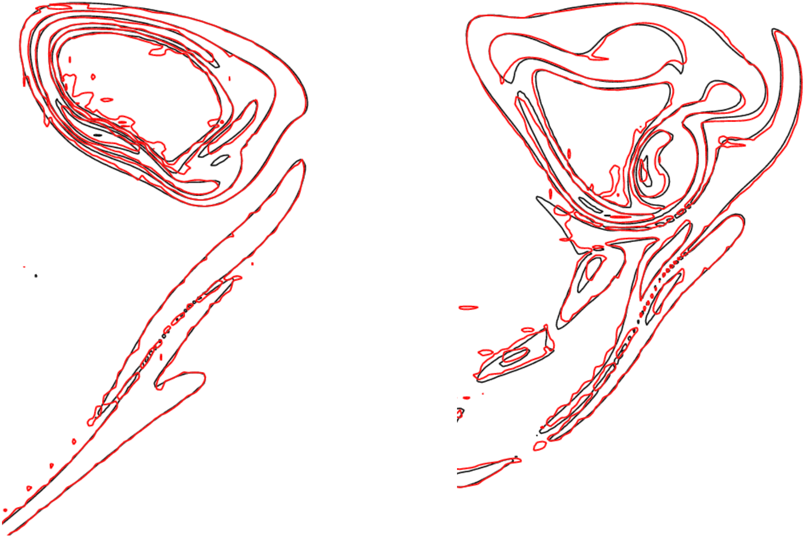


Validation
DNS Taylor-Green : structures



Validation

DNS Taylor-Green : structures DGM - 288^3

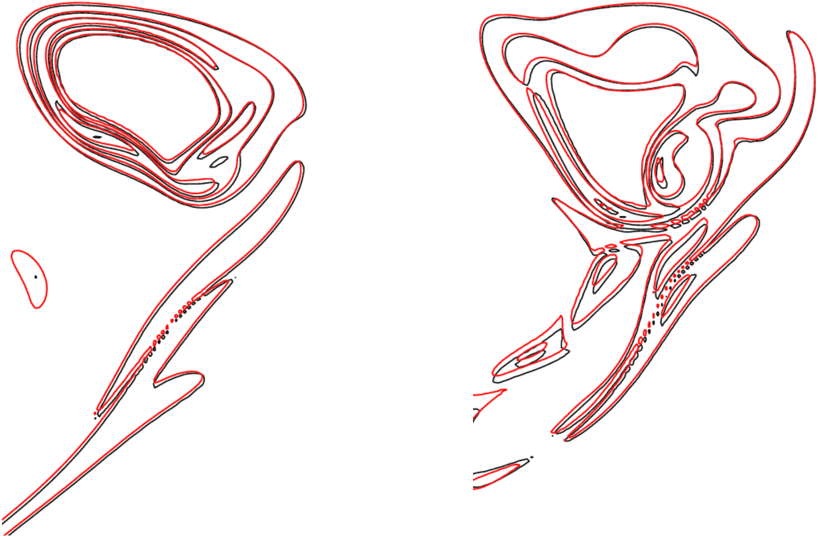


Validation

DNS Taylor-Green : structures FD4 - 256^3

Validation

DNS Taylor-Green : structures FD4 - 512^3

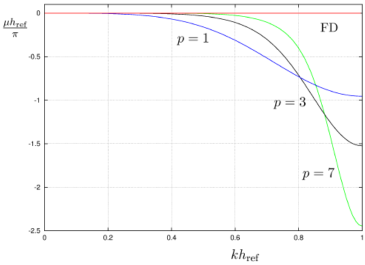


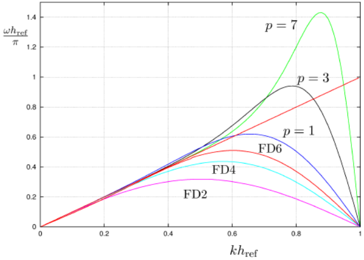
Validation

DNS Taylor-Green : structures DGM - 384^3

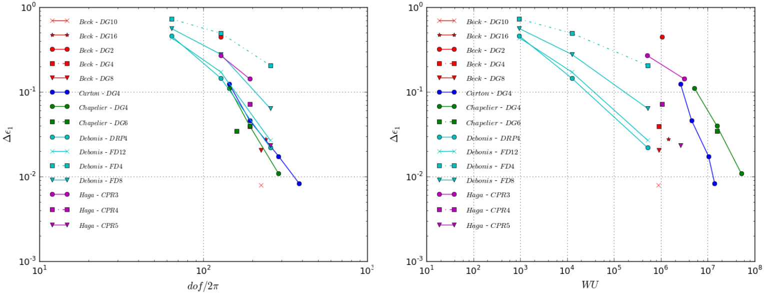
Validation

DNS Taylor-Green : error properties of DGM



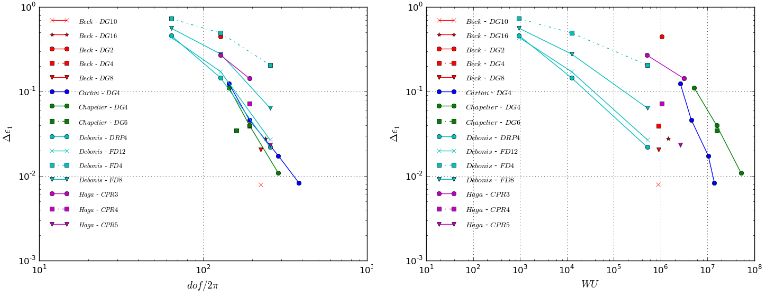


DNS Taylor-Green : high order workshop results



- unstructured methods competitive with finite differences in terms of resolution
- importance of the dispersion error
- unstructured methods are superior in terms of cpu if target error low enough

DNS Taylor-Green : high order workshop results



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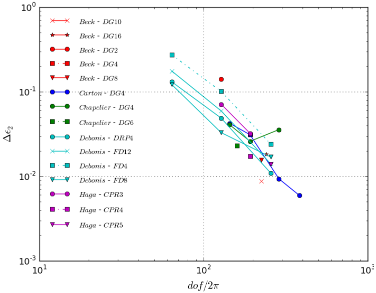

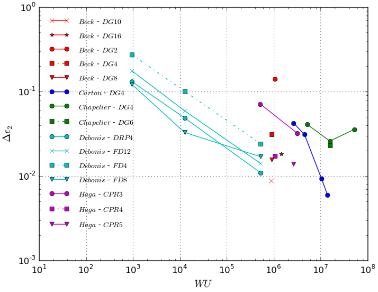

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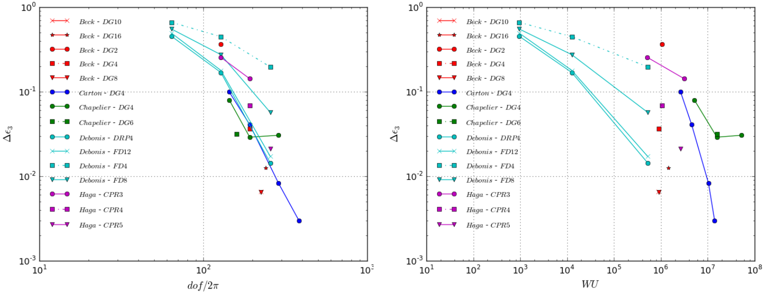

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Validation

Homogeneous isotropic turbulence : homogeneous isotropic turbulence (HIT)

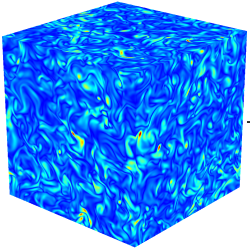
Kolmogorov theory

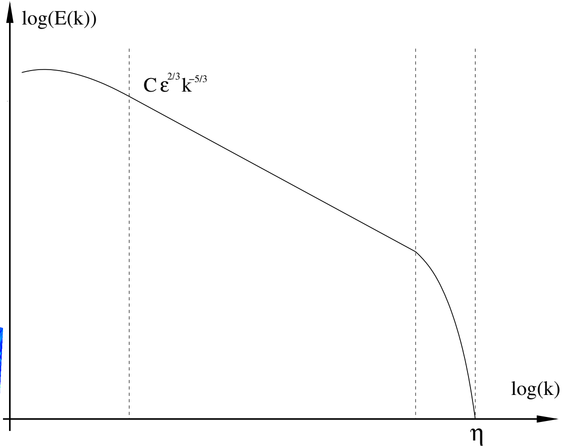
$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}}$$

DNS resolution

$$\Delta x \sim \eta \sim Re^{-3/4}$$

$$\mathrm{CPU} \sim Re^3$$





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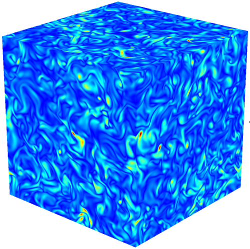
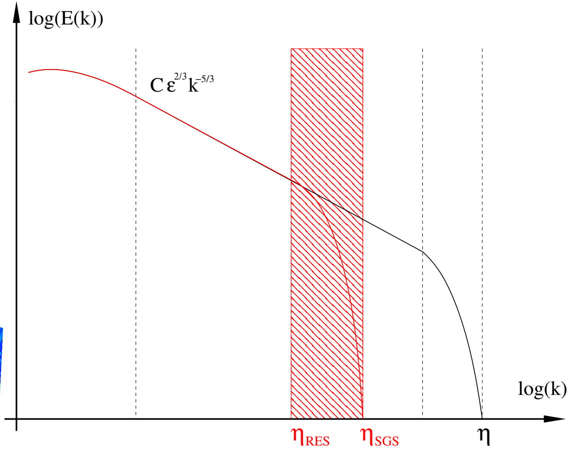




Homogeneous isotropic turbulence : LES models

Local explicit eddy viscosity models

$$\tau = \tau + \tau_{SGS}$$
$$\tau_{SGS} = \mu_{SGS}(u)\mathbf{S}$$
$$S_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}$$

- Smagorinsky - freestream turbulence

$$\nu_T = (C_s\Delta)^2|S|$$

- WALE - wall-bounded flows

$$\nu_T = (C_w\Delta)^2\frac{|S^d|^3}{|S|^5 + |S^d|^{5/2}}$$
$$S^d_{ij} = \frac{1}{2}\left(\left(\frac{\partial v_i}{\partial x_j}\right)^2 + \left(\frac{\partial v_j}{\partial x_i}\right)^2\right) - \frac{1}{3}\frac{\partial v_k}{\partial x_k}^2\delta_{ij}$$

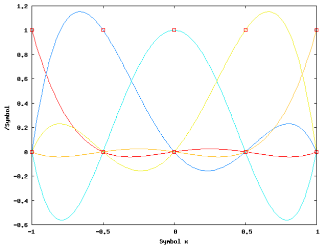
- Variational multiscale model are based on high-pass filtered solutions

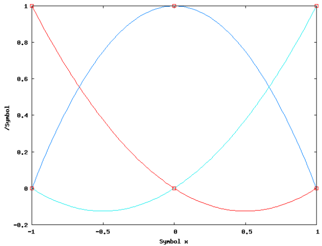
$$\tau = \tau + \tau_{SGS}$$
$$\tau_{SGS} = \mu^*_{SGS}\mathbf{S}^{**}$$

Implicit LES models (ILES, MILES, ...) rely on the discretisation

Validation

Homogeneous isotropic turbulence : variational multiscale DG





Low pass filtered solution ~ low order polynomial content

$$u_m = \sum_i \mathbf{u}_{im} \phi_i^p = u_m^* + \sum_i \mathbf{u}_{im}^q \phi_i^q$$

Use Galerkin projection to find u^q

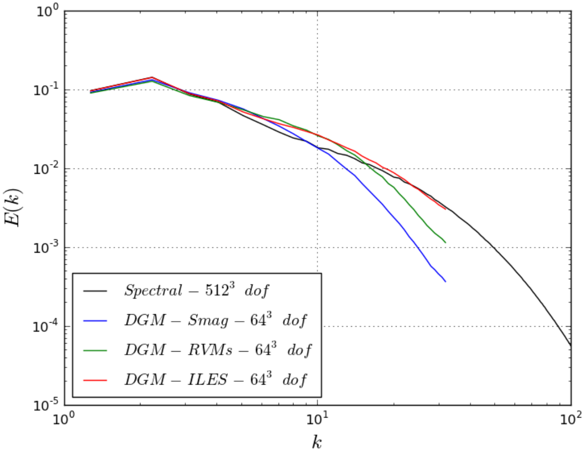
$$\sum_i \mathbf{u}_{jm}^q \int_V \phi_i^q \phi_j^q = \sum_i \mathbf{u}_{km} \int_V \phi_k^p \phi_i^q \Rightarrow \mathbf{u}^q = \left(\mathbf{M}^q\right)^{-1} \mathbf{M}^{qp}$$

Filtering = parametric operation, castable as matrix-matrix multiply

$$\mathbf{u}^* = \left(\mathbf{I} - \left(\mathbf{M}^p\right)^{-1} \mathbf{M}^{pq} \left(\mathbf{M}^q\right)^{-1} \mathbf{M}^{qp}\right) \mathbf{u}$$

Validation

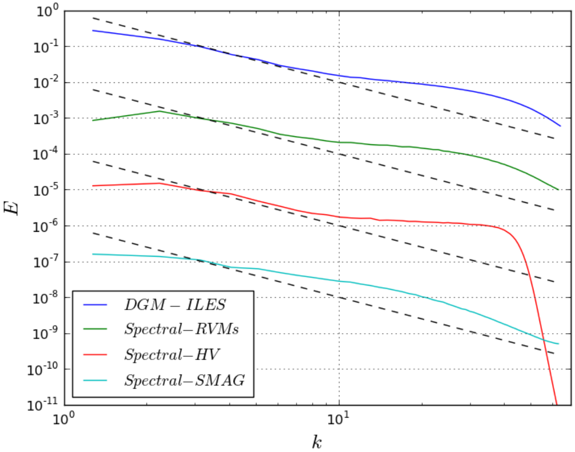
LES of Homogeneous isotropic turbulence : $Re_\lambda = 136$



Carton, Hillewaert et al. ETMM9 [CdWHG+12b]

Validation

LES of Homogeneous isotropic turbulence : Euler



Energy distribution has same quality as spectral code with optimised SGS *Cocle et al. 2009 [CBW09]*

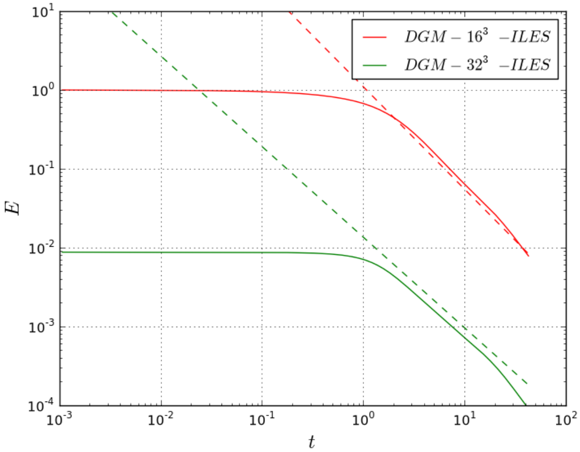
Validation

LES of Homogeneous isotropic turbulence : Euler

DNS literature

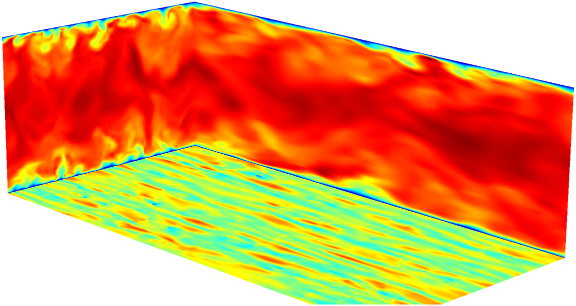
$$\frac{E(t)}{E(0)} \sim t^{-p}$$

$p = 1.30 \pm 0.1$



Validation

ILES of channel flow $Re_\tau = 395$:setup



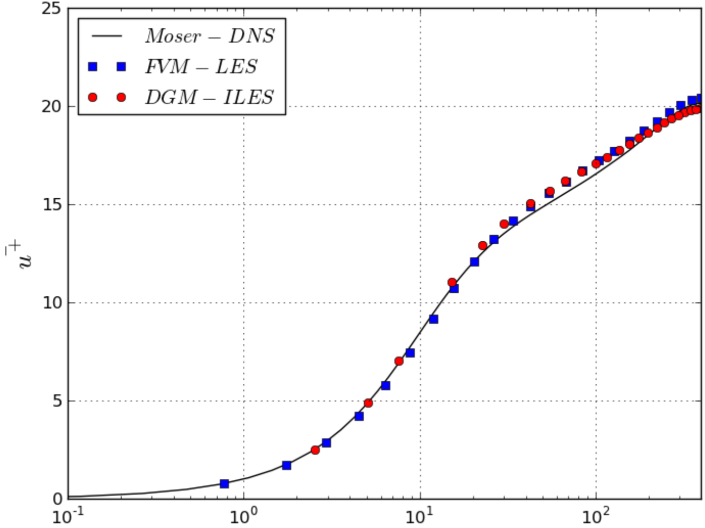
- developed flow in x-direction
- spanwise periodicity
- constant pressure gradient in x
- periodicity of the pressure perturbation p'

$$p = \frac{\partial \bar{p}}{\partial x} x + p'$$

Carton, Hillewaert et al. ETMM9 [CdWHG+12b]

Validation

ILES of channel flow $Re_\tau = 395$: velocity correlations



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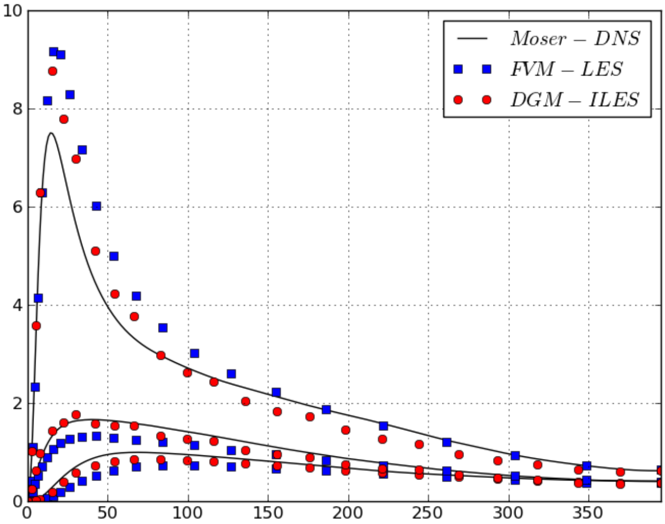
ILES of channel flow $Re_\tau = 395$: velocity correlations



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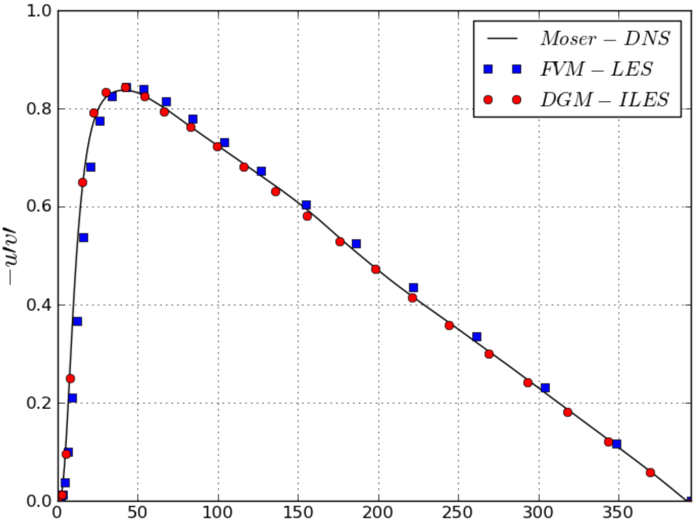
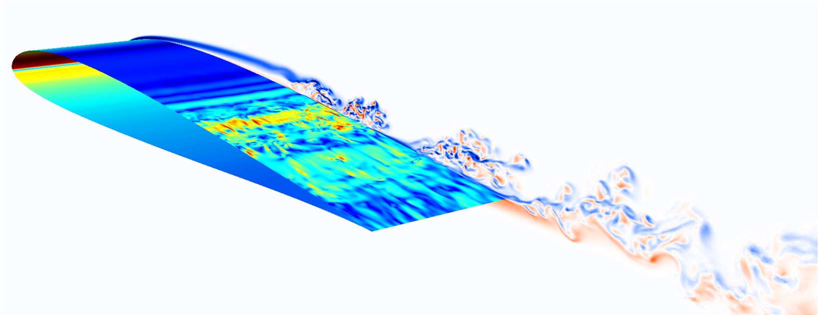


Outline

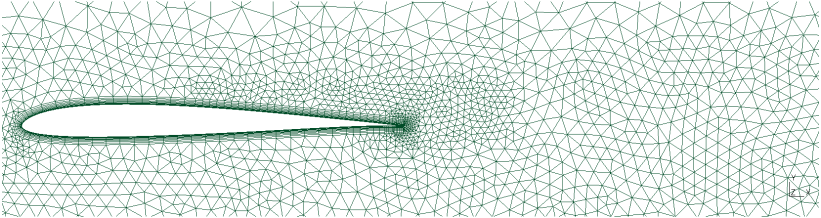
Towards real-life applications
ILES of SD7003 airfoil : Overview

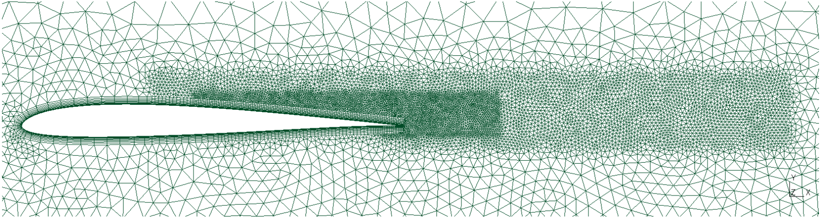


- transitional flow on a low Re airfoil $Re = 80.000, \alpha = 4^{\circ}$
- comparison between ILES and DNS
- scheme : DGM(3), Newton-Krylov-Jacobi, 3PtBDF

Carton & Hillewaert, ICCFD7 [CdWH12]

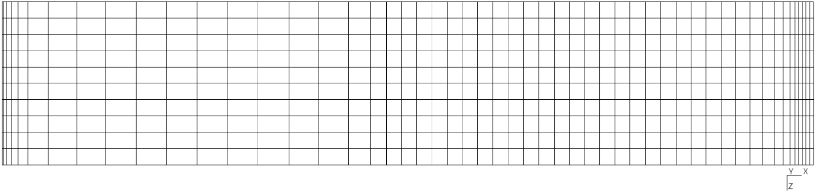
Towards real-life applications
ILES of SD7003 airfoil : mesh resolution

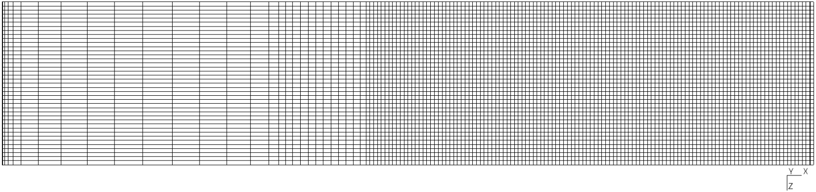




Towards real-life applications

ILES of SD7003 airfoil : mesh resolution



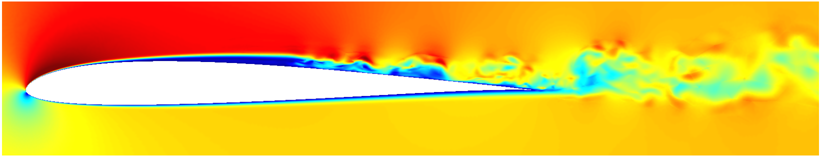


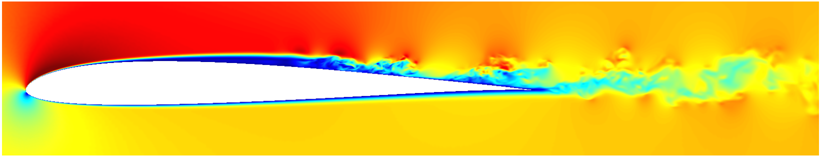
Towards real-life applications

ILES of SD7003 airfoil : mesh characteristics

	DNS	LES
$\Delta y_0/c$ (wall-normal)	$3.33\ 10^{-4}$	$3.33\ 10^{-4}$
$\Delta x/c$ (box 1)	$1.67\ 10^{-3}$	$6.67\ 10^{-3}$
$\Delta x/c$ (box 2)	$3.33\ 10^{-3}$	$1.33\ 10^{-2}$
$\Delta z/c$ (spanwise)	$1.67\ 10^{-3}$	$6.67\ 10^{-3}$
y^+ at $x/c = 0.8$	1.2	1.2
$x^+ = z^+$ at $x/c = 0.8$	6	24
Number of hexahedra (/1000)	84.7	8.7
Number of wedges (/1000)	646.1	47.9
Total number of dof per variable (at continuity) [k]	10934.3	874.5

Towards real-life applications
ILES of SD7003 airfoil : velocity field

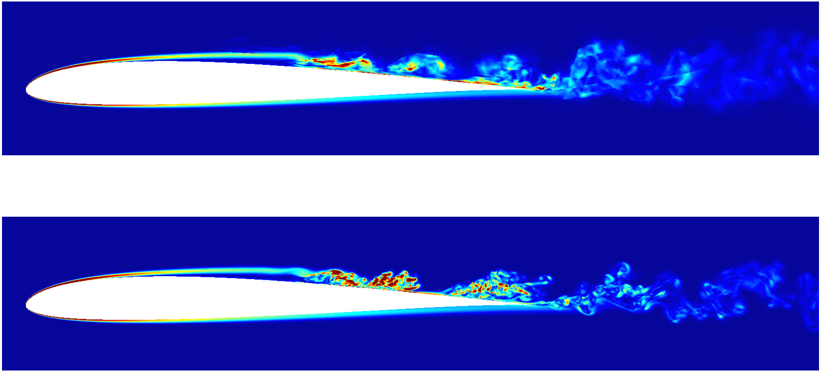




LES (top) vs DNS (bottom)

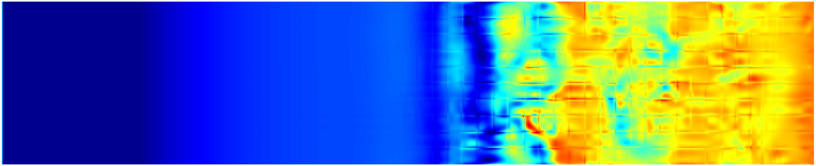
Towards real-life applications

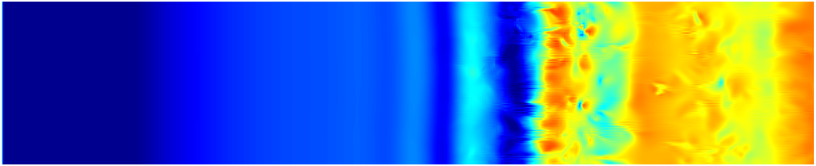
ILES of SD7003 airfoil : vorticity field



LES (top) vs DNS (bottom)

Towards real-life applications
ILES of SD7003 airfoil : wall pressure

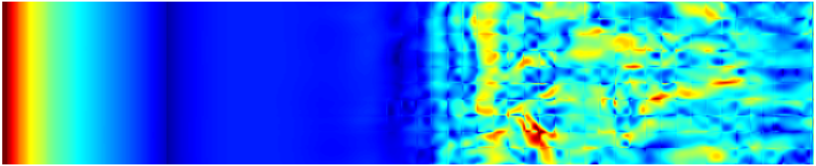


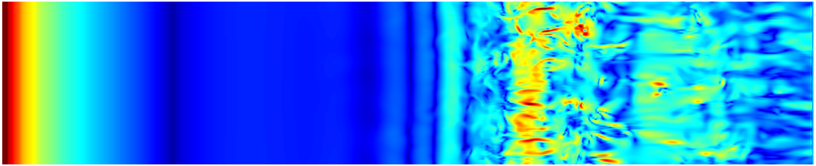


LES (top) vs DNS (bottom)

Towards real-life applications

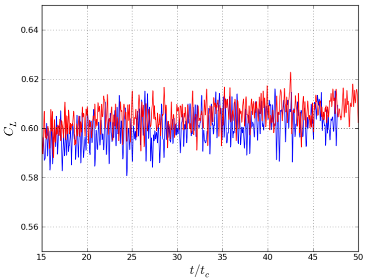
ILES of SD7003 airfoil : wall friction

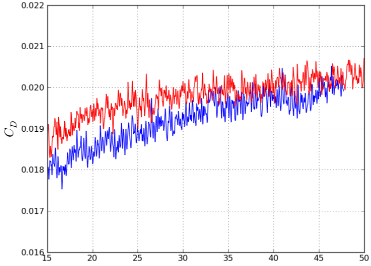




LES (top) vs DNS (bottom)

Towards real-life applications
ILES of SD7003 airfoil : statistical convergence lift and drag

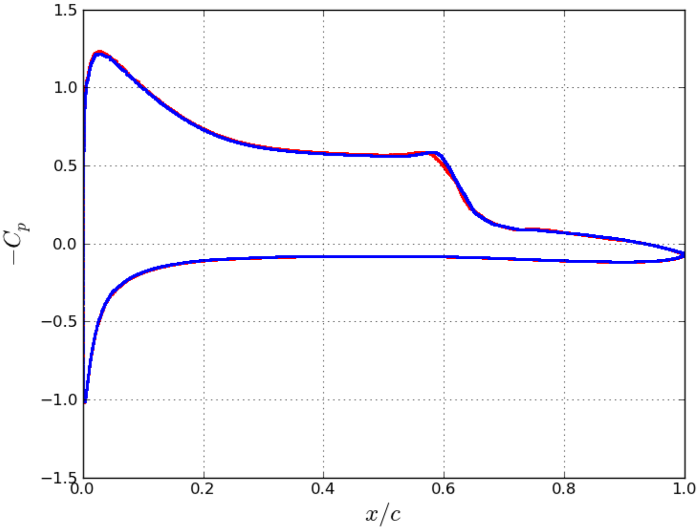




very slow convergence to statistically constant state

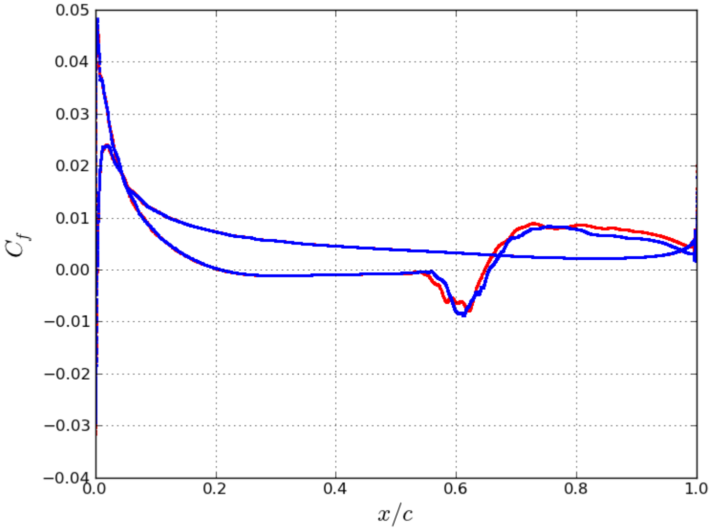
Towards real-life applications

ILES of SD7003 airfoil : comparison DNS vs ILES of pressure coefficient distribution



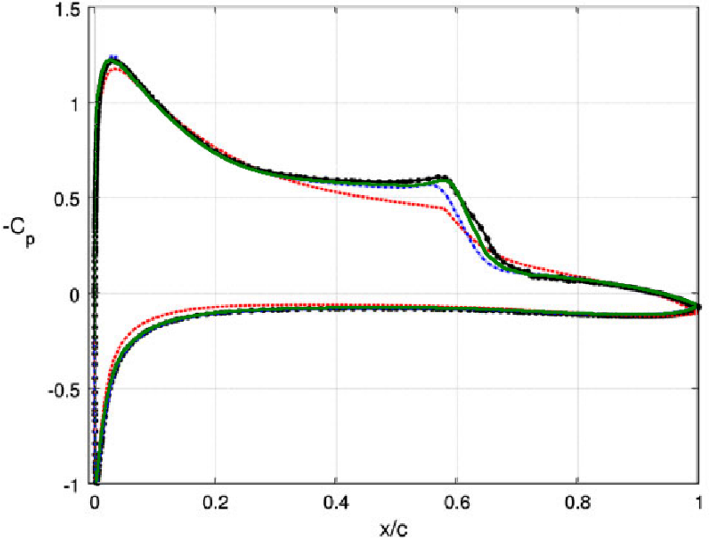
Towards real-life applications

ILES of SD7003 airfoil : comparison DNS vs ILES of skin friction coefficient distribution



Towards real-life applications

ILES of SD7003 airfoil : comparison to literature (Uranga 2009 [UPDP09])



- SDM/ILES - Uranga, Persson and Drela 2009 [UPDP09]

Towards real-life applications

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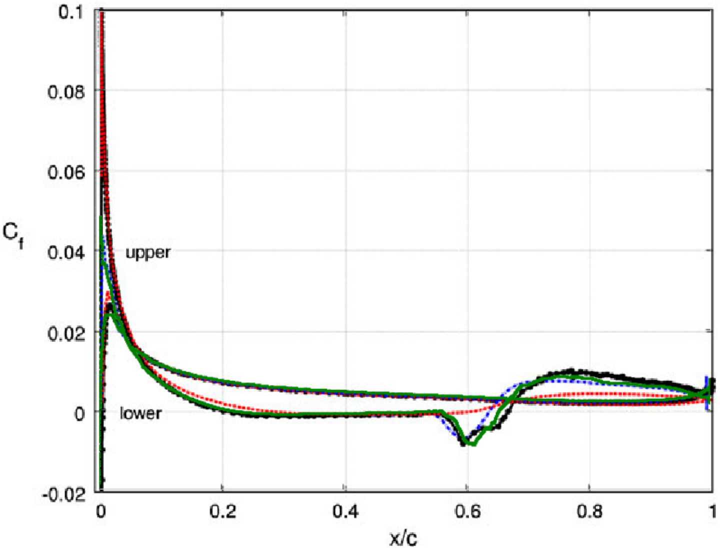


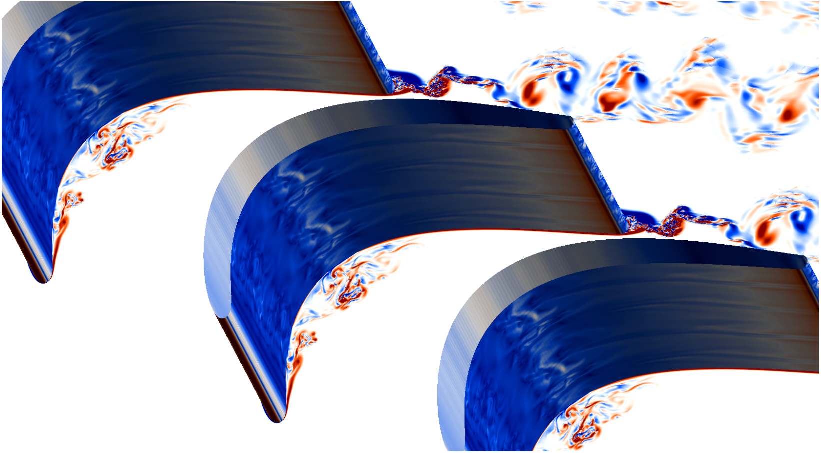
- SDM/ILES - Uranga, Persson and Drela 2009 [UPDP09]

Towards real-life applications

ILES of SD7003 airfoil : comparison to literature

	DNS/DGM	ILES/DGM	Uranga [UPDP09]	Visbal [GV08]
$\overline{C_L}$	0.196	0.201	0.22	-
$\overline{C_D}$	0.602	0.607	0.603	-
Separation	0.209	0.207	0.21	0.23
Reattachment	0.654	0.647	0.65	0.67
Cost to compute one t_c [CPUh]	11001	415	-	-

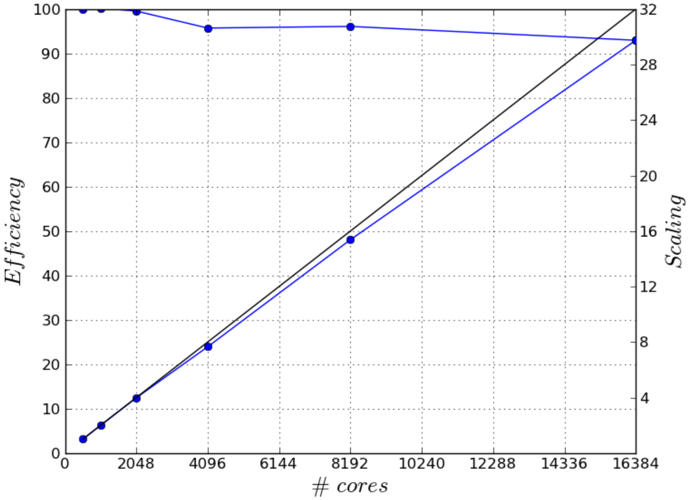
Towards real-life applications
LP turbine blade



PRACE industrial pilot project *noFUDGE*

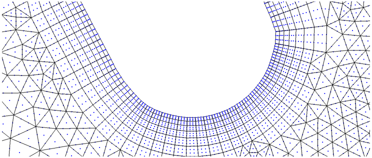
- 2mio core hours on BlueGene/P
- code optimisation and weak scaling up to 16384 cores
- DNS of a LP turbine blade, $Re = 85.000$, $M = 0.6$ (engine conditions)
- DGM(3), Newton-GMRES-Jacobi, 3Pt BDF
- comparison to previous LES computations using FVM

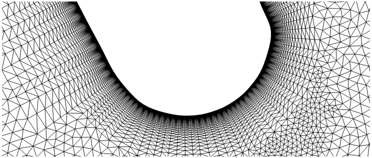
Towards real-life applications
LP turbine blade : weak scaling



Towards real-life applications

LP turbine blade : computational cost





	FVM		DGM		
	$x^+ = z^+$	y^+	x^+	z^+	y^+
Leading edge	35	0.5	15	30	3
Pressure side	5	0.1	3	3	0.3
Suction side	50	0.3	15	15	1.5
Trailing edge edge	10	0.3	2	10	1

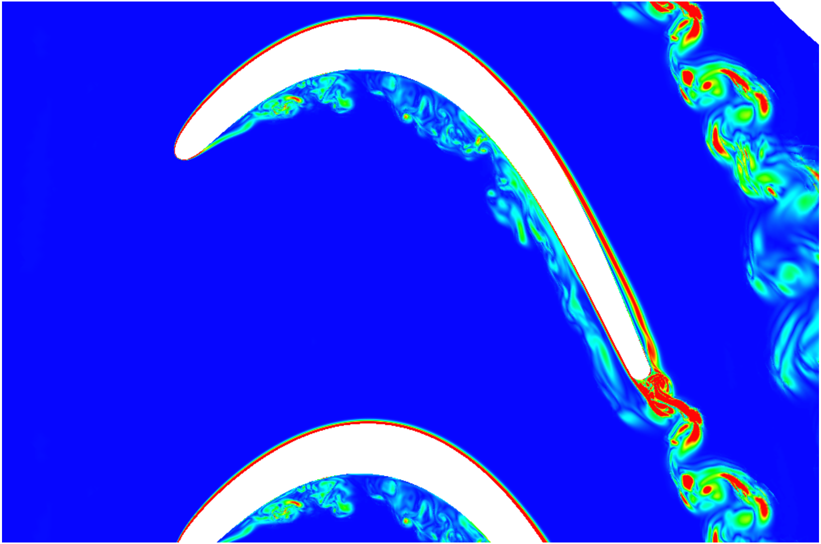
	FVM	DGM
Computer	Intel Cluster	BlueGene/P
Order of accuracy	2	4
Mesh (M nodes)	8.5	15
CPU time for one t_c (kCPUh)	11	112
Memory per core (MB)	700	500
Number of CPU	256	4096
CPU time / mesh size	0.0013	0.0019
Memory / mesh size	0.02	0.036

NB : BG/P 3-4 times slower than intel cluster → DGM is about 2 times more expensive but for far higher (almost DNS) accuracy

Towards real-life applications

LP turbine blade : vorticity snapshot

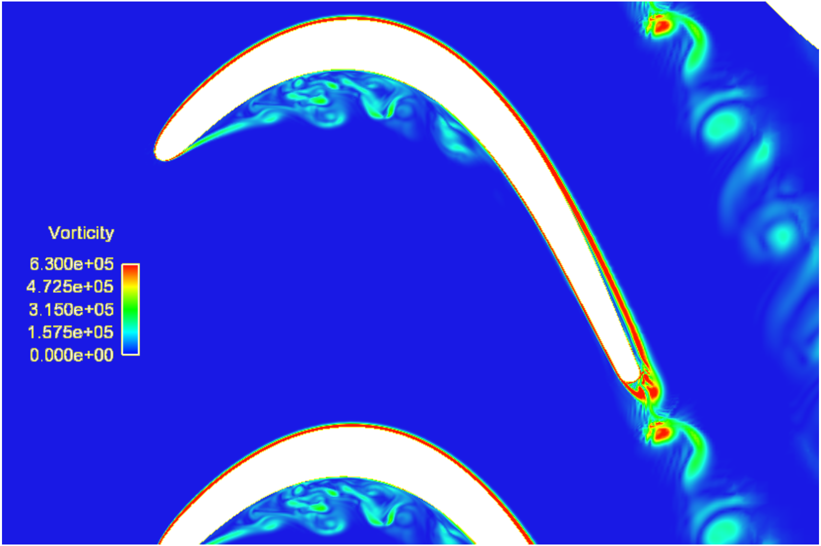
DGM



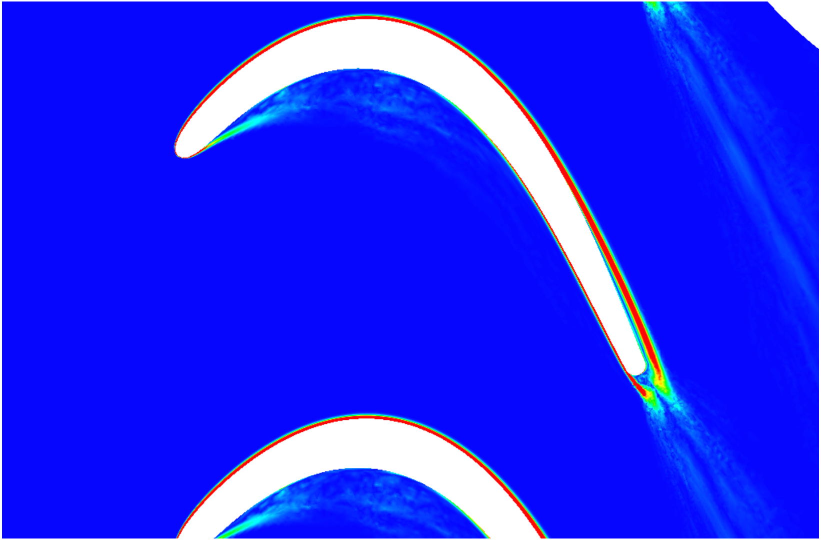
Towards real-life applications

LP turbine blade : vorticity snapshot

FVM

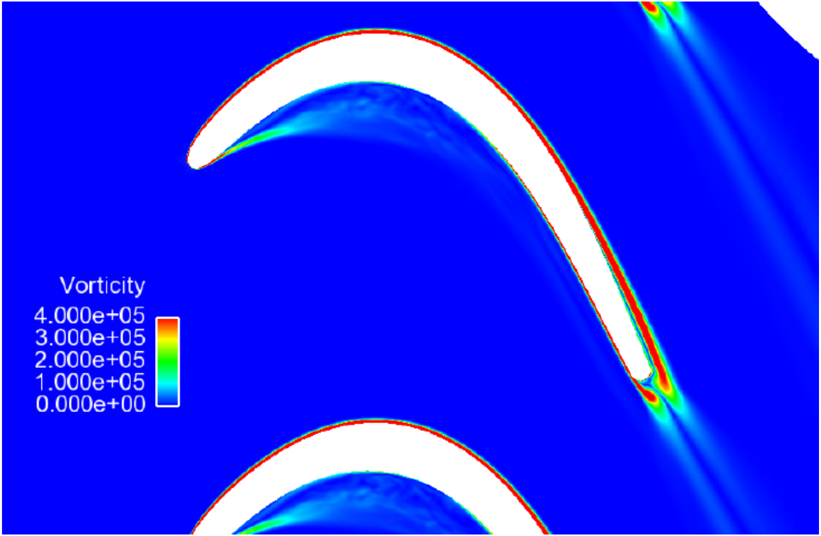


Towards real-life applications
LP turbine blade : mean vorticity DGM



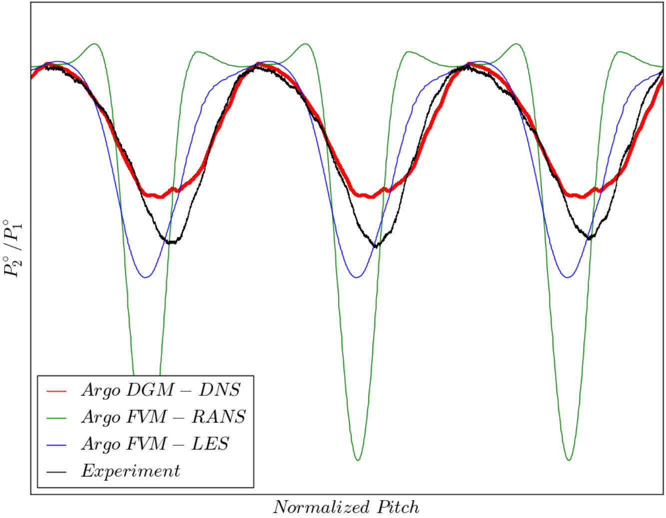
Towards real-life applications

LP turbine blade : mean vorticity FVM



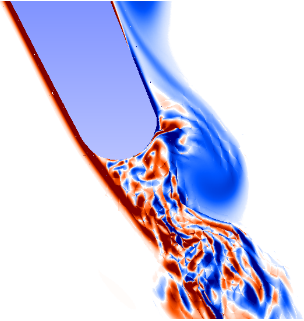
Towards real-life applications

LP turbine blade : comparison of loss distribution



Towards real-life applications

LP turbine blade : further improving the results



- resolution at trailing edge → need for p-adaptation
- no inlet turbulence
- acoustic perturbations (periodicity, inlet and outlet) → absorbing boundary conditions for vortices and pressure waves
- loss computed based on averaged entropy
- statistical convergence reached (very different time-scales)

Outline

Concluding remarks

Main conclusion : DGM is a good candidate for industrial resolved turbulence

- high order obtained
- implicit way of checking for resolution
- ILES would remove need for model tuning
- efficient use of large scale HPC resources

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A lot of work should still be done on the solver side ...

- further assessment of LES approaches - channel $Re_\tau = 590, 1000, 2000$
- absorbing boundaries and synthetic turbulence
- development of hp-adaptive strategy
- shock capturing and interaction with LES modeling
- multigrid algorithms
- rotor-stator interaction
- speeding up transients

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... but also on the peripheral technology → Gmsh

- creation of the curvilinear mesh (splines on pyramids anyone ?)
- visualisation of large high-order data sets

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Contributors and colleagues

- Philippe Geuzaine
- Corentin Carton de Wiart - turbulence
- François Pochet - free surface flows and viscoplasticity
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- Pierre Schrooyen (UCL) - interaction turbulence and ablation

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- Jean-Franois Remacle (UCL) - Gmsh and DGM
- Grgoire Winckelmancs (UCL) - fundamental turbulence
- Laurent Bricteux (UMons) - fundamental turbulence
- Christophe Geuzaine (ULg) - Gmsh

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- FP7 project IDIHOM

Outline

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