



# Uncertainty Quantification in Simulations of Reactive Flows

## Part 2: Applications

**Gianluca Iaccarino**

ME & iCME

Stanford University

---

CEMRACS Summer School

July 2012

CIRM, Marseille, France

# UQ in Reacting Flows

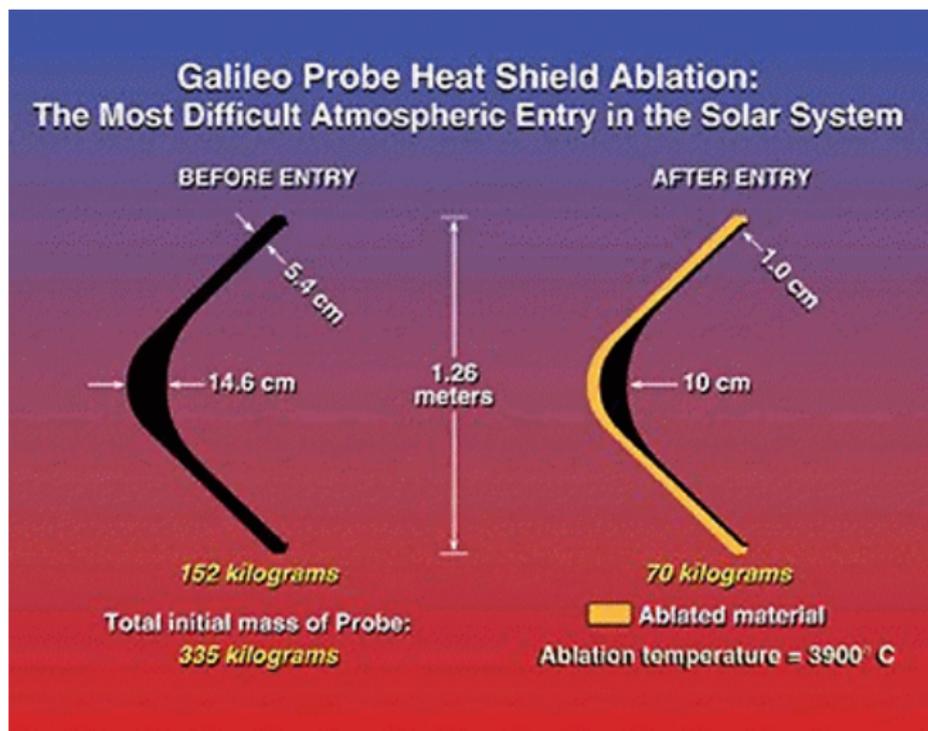
# Planetary Entry Simulations

## High-Temperature Reactive Flow

- ▶ During descent in the atmosphere vehicles experience **extreme** heating loads
- ▶ The design of the thermal protection system (TPS) is the most critical component of every planetary entry mission
- ▶ TPS design is fundamentally **computation-based** because no ground-test can reproduce all the aspects of flight
- ▶ Safety (and reliability) requires **rigorous** evaluation of the uncertainties present



# Jupiter Entry Probe - Galileo



Source: NASA

# Titan Entry Simulations

## High-Temperature Reactive Flow

Predictions of TPS heating loads re-entry are **challenging**

- ▶ Physics Components
  - Chemistry
  - Radiation
  - Turbulence
  - etc.
- ▶ Computational issues
  - Strong shocks
  - Thin boundary layers
  - Flow separation
  - etc.

We focus on the uncertainties in the chemical kinetics, and their impact on the heat transfer at the stagnation point...

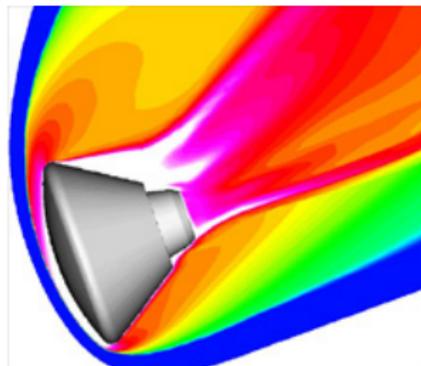
# Titan Entry Simulations

## Aero-thermodynamic model

We consider *nominal* conditions for the Titan entry:

Table: Freestream conditions

$N_2$	$CH_4$	$\rho_\infty$ (kg/m <sup>3</sup> )	$V_\infty$ (km/s)	$T_\infty$ (K)
95%	5%	$1.49 \times 10^{-4}$	5.76	152.7



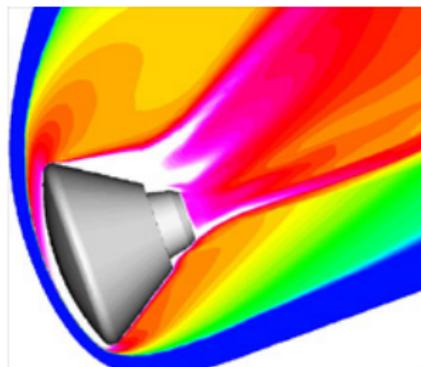
# Titan Entry Simulations

## Aero-thermodynamic model

We consider *nominal* conditions for the Titan entry:

Table: Freestream conditions

$N_2$	$CH_4$	$\rho_\infty$ (kg/m <sup>3</sup> )	$V_\infty$ (km/s)	$T_\infty$ (K)
95%	5%	$1.49 \times 10^{-4}$	5.76	152.7



The strong bow shock induced strong non-equilibrium effects and dissociation/ionization

- ▶ We assumed a 13-species mechanism:
- ▶  $C$ ,  $H$ ,  $N$ ,  $C_2$ ,  $CH_4$ ,  $CH_3$ ,  $CH_2$ ,  $CH$ ,  $CN$ ,  $H_2$ ,  $HCN$ ,  $N_2$ ,  $NH$
- ▶ 26 reactions: 12 dissociation & 14 exchange

# Probabilistic approach

## Reaction rates in ionization/dissociation models

- Uncertainty in the reactions rates, gathered from

- theory
- experiments
- engineering judgment

- Uncertainty in the reaction rates is described using independent u.r.v.s

	Dissociation reactions $k_f = A_f T^b \exp(-C_f/T)$	$A_f$ (cc/mol-s)	$b_f$	$C_f$ (K)	95% conf. limit [Ref.]
1	$N_2 + M \rightleftharpoons 2N + M$ M=N,C,H	$7.00 \times 10^{21}$ $3.00 \times 10^{22}$	-1.60 -1.60	113200 113200	See Table 2
2	$CH_4 + M \rightleftharpoons CH_3 + H + M$	$4.70 \times 10^{17}$	-8.20	59200	$\pm 0.30$ [22]
3	$CH_3 + M \rightleftharpoons CH_2 + H + M$	$1.02 \times 10^{16}$	0.00	45600	$\pm 0.35$ [22]
4	$CH_2 + M \rightleftharpoons CH + H_2 + M$	$5.00 \times 10^{15}$	0.00	42800	$\pm 0.30$ [23]
5	$CH_2 + M \rightleftharpoons CH + H + M$	$4.00 \times 10^{15}$	0.00	41800	$\pm 0.30$ [23]
6	$CH_2 + M \rightleftharpoons C + H_2 + M$	$1.30 \times 10^{14}$	0.00	29700	$\pm 0.30$ [23]
7	$CH + M \rightleftharpoons C + H + M$	$1.90 \times 10^{14}$	0.00	33700	$\pm 0.30$ [23]
8	$C_2 + M \rightleftharpoons 2C + M$	$1.50 \times 10^{16}$	0.00	71600	$\pm 0.30$ [24]
9	$H_2 + M \rightleftharpoons 2H + M$	$2.23 \times 10^{14}$	0.00	48350	$\pm 0.30$ [22,25]
10	$CN + M \rightleftharpoons C + N + M$	$2.53 \times 10^{14}$	0.00	71000	$\pm 0.30$ [26,27]
11	$NH + M \rightleftharpoons N + H + M$	$1.80 \times 10^{14}$	0.00	37600	$\pm 0.30$ [28]
12	$HCN + M \rightleftharpoons CN + H + M$	$3.57 \times 10^{20}$	-2.60	62845	$\pm 0.30$ [29]
Exchange reactions					
13	$CH_3 + H \rightleftharpoons CH_2 + H_2$	$6.03 \times 10^{13}$	0.00	7600	$\pm 1.00$ [25]
14	$CH_3 + N_2 \rightleftharpoons HCN + NH$	$4.82 \times 10^{12}$	0.00	18000	$\pm 1.00$ [28]
15	$CH_2 + N \rightleftharpoons HCN + H$	$5.00 \times 10^{13}$	0.00	0	$\pm 1.00$ [30]
16	$CH_2 + H \rightleftharpoons CH + H_2$	$6.03 \times 10^{12}$	0.00	-900	$\pm 0.87$ [25,28]
17	$CH + N_2 \rightleftharpoons HCN + N$	$4.40 \times 10^{12}$	0.00	11060	$\pm 0.35$ [30]
18	$CH + C \rightleftharpoons C_2 + H$	$2.00 \times 10^{14}$	0.00	0	$\pm 1.00$ [23]
19	$C_2 + N_2 \rightleftharpoons 2CN$	$1.50 \times 10^{13}$	0.00	21000	$\pm 0.30$ [31]
20	$CN + H_2 \rightleftharpoons HCN + H$	$2.95 \times 10^5$	0.00	1130	$\pm 0.60$ [32]
21	$CN + C \rightleftharpoons C_2 + N$	$5.00 \times 10^{13}$	0.00	13000	$\pm 0.54$ [18]
22	$N + H_2 \rightleftharpoons NH + H$	$1.60 \times 10^{14}$	0.00	12650	$\pm 0.30$ [33]
23	$C + N_2 \rightleftharpoons CN + N$	$5.24 \times 10^{13}$	0.00	22600	$\pm 0.50$ [T]
24	$C + H_2 \rightleftharpoons CH + H$	$4.00 \times 10^{14}$	0.00	11700	$\pm 0.30$ [34]
25	$H + N_2 \rightleftharpoons NH + N$	$3.00 \times 10^{12}$	0.50	71400	$\pm 0.50$ [T]
26	$CH_4 + H \rightleftharpoons CH_3 + H_2$	$1.32 \times 10^4$	3.00	4045	$\pm 0.30$ [22,25]

# Another uncertainty source

## Radiation modeling

NASA has identified the *heating from shock layer radiation due to the CN radical formed in the  $N_2/CH_4$  atmosphere as a primary uncertainty*

# Another uncertainty source

## Radiation modeling

NASA has identified the *heating from shock layer radiation due to the CN radical formed in the  $N_2/CH_4$  atmosphere as a primary uncertainty*

Can we predict the CN radical?

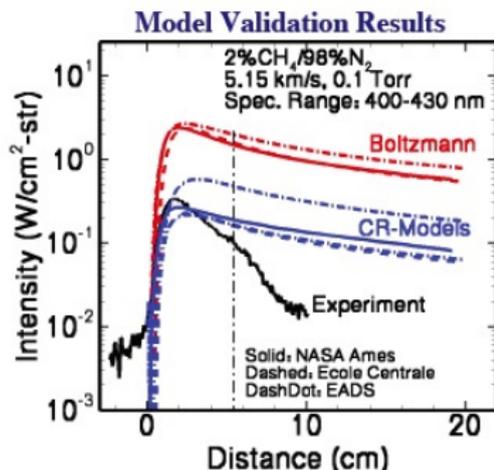
# Another uncertainty source

## Radiation modeling

NASA has identified the *heating from shock layer radiation due to the CN radical formed in the N<sub>2</sub>/CH<sub>4</sub> atmosphere as a primary uncertainty*

Can we predict the CN radical?

- ▶ State-of-the art knowledge during the design of the Huygen's probe was the Boltzmann model
- ▶ This led to **overprediction** of the heating rates - **conservative design**
- ▶ Recent work has lead to **collisional-radiative** (CR) models



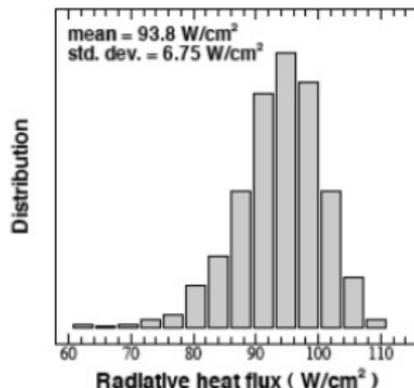
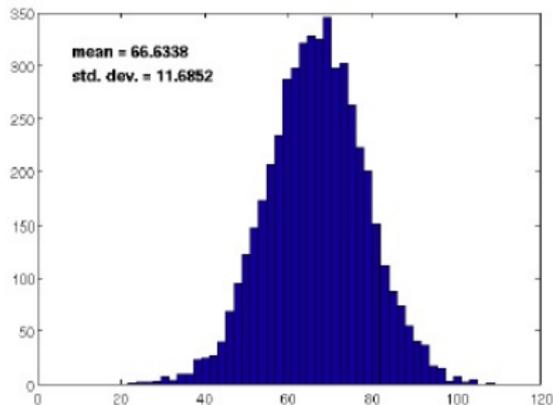
## TPS Heating load uncertainty

- ▶ We used Monte Carlo sampling (10,000 runs) to study the effect of the kinetics uncertainties
- ▶ We employed the **CR** model but compared to NASA earlier work (with Boltzmann model) in an attempt to characterize the **epistemic uncertainty**

# TPS Heating load uncertainty

- ▶ We used Monte Carlo sampling (10,000 runs) to study the effect of the kinetics uncertainties
- ▶ We employed the **CR** model but compared to NASA earlier work (with Boltzmann model) in an attempt to characterize the **epistemic uncertainty**

## Stagnation point heat flux ( $\text{W}/\text{cm}^2$ )



## TPS Heating load uncertainty

- ▶ Both the mean and the variance of the heat loads are affected by the radiation model

## TPS Heating load uncertainty

- ▶ Both the mean and the variance of the heat loads are affected by the radiation model
- ▶ Can we learn something more?

## TPS Heating load uncertainty

- ▶ Both the mean and the variance of the heat loads are affected by the radiation model
- ▶ Can we learn something more?
- ▶ **Correlate** and **rank** the uncertainty sources

**Correlation**: based on cross-plots of output (amount of  $CN$ ) vs. input (uncertainty in the reaction rates)

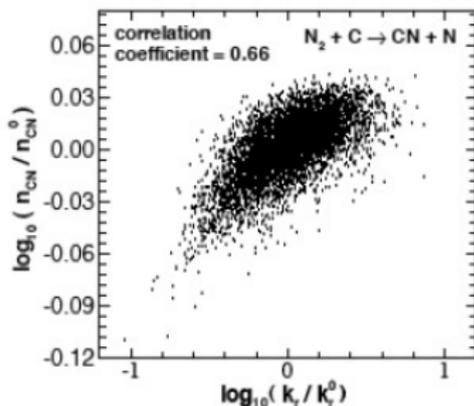
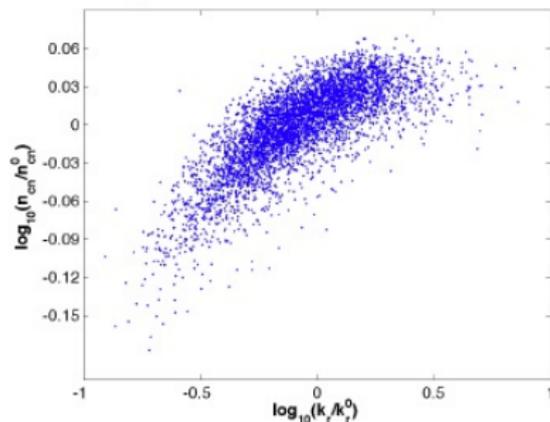
# TPS Heating load uncertainty

- ▶ Both the mean and the variance of the heat loads are affected by the radiation model
- ▶ Can we learn something more?
- ▶ **Correlate** and **rank** the uncertainty sources

**Correlation:** based on cross-plots of output (amount of  $CN$ ) vs. input (uncertainty in the reaction rates)

## Correlation plot for $N_2 + C \rightarrow CN + N_2$

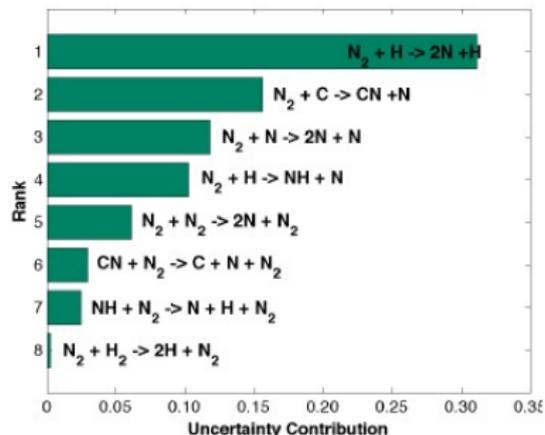
$N_2 + C \rightarrow CN + N$ , correlation coeff. = 0.76415



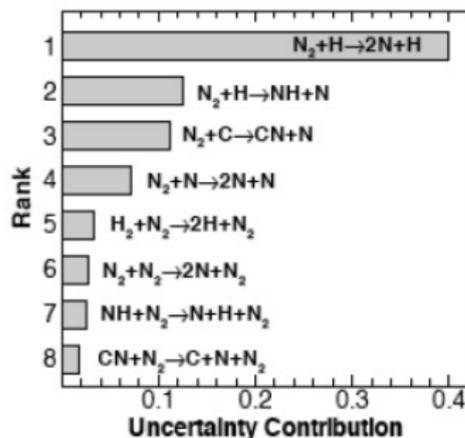
# TPS Heating load uncertainty

**ANOVA** ANalysis Of Variance: **separate** the variance in factors contributed by each input uncertainty

## 8 major contributors to uncertainty



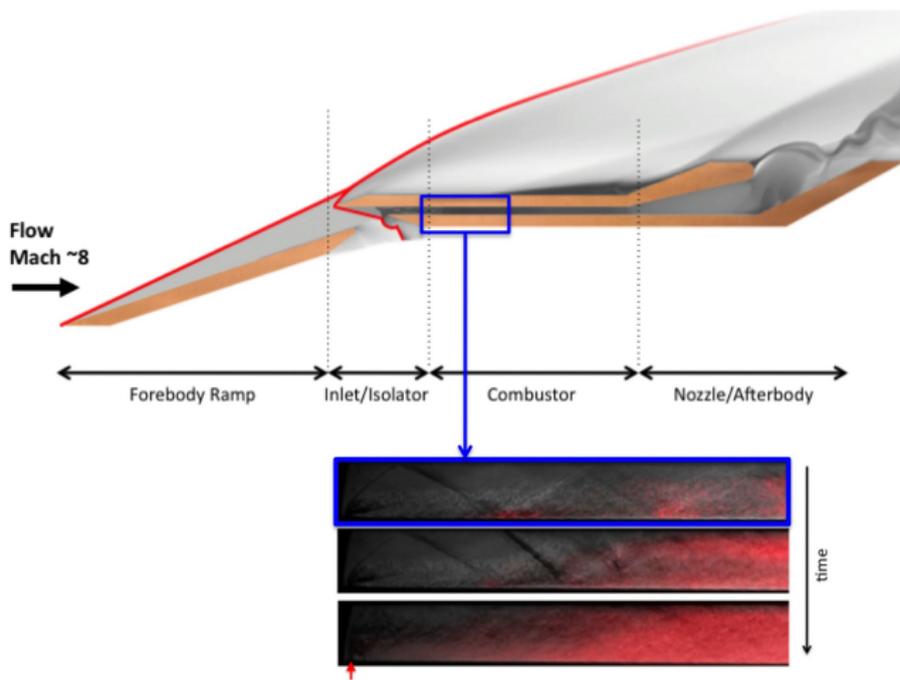
CR Model: Ghaffari, Iaccarino, Magin, 2009



Boltzmann model: Bose & Wright, 2004

# Uncertainty in Ignition Delay Time

Determination of Ignition Delay Time is an important design consideration, for example in air-breathing hypersonic propulsion systems



# Uncertainty in Ignition Delay Time

In scramjet there are two competing mechanism causing sudden ignition of mixture:

- ▶ Mixing-induced accumulation of radicals starts chain reaction
- ▶ Shock-induced radical farming

# Uncertainty in Ignition Delay Time

In scramjet there are two competing mechanism causing sudden ignition of mixture:

- ▶ Mixing-induced accumulation of radicals starts chain reaction
- ▶ Shock-induced radical farming
- ▶ What is the effect of uncertainties in reaction rates?

# Uncertainty in Ignition Delay Time

In scramjet there are two competing mechanism causing sudden ignition of mixture:

- ▶ Mixing-induced accumulation of radicals starts chain reaction
- ▶ Shock-induced radical farming
- ▶ **What is the effect of uncertainties in reaction rates?**
- ▶ Simplified Problem:
  - ▶ Integrate evolution of reacting mixture in homogeneous isochoric (constant volume) reactor
  - ▶ Hydrogen chemistry (9 species, 25 elementary reactions)

# Uncertainty in Ignition Delay Time

In scramjet there are two competing mechanism causing sudden ignition of mixture:

- ▶ Mixing-induced accumulation of radicals starts chain reaction
- ▶ Shock-induced radical farming
- ▶ **What is the effect of uncertainties in reaction rates?**
- ▶ Simplified Problem:
  - ▶ Integrate evolution of reacting mixture in homogeneous isochoric (constant volume) reactor
  - ▶ Hydrogen chemistry (9 species, 25 elementary reactions)
  - ▶ **What is the uncertainty?**

# Hydrogen Chemistry

## Reaction Rate Uncertainties

- ▶ Rate (and their uncertainties) are available in the literature
- ▶ Modified Arrhenius form  
 $k = AT^n \exp(-E/RT)$
- ▶ The uncertainty factor  $UF$  is such that  
 $[k/UF : k \times UF]$  provide *probable* bounds!
- ▶ Assume that the reaction rate are **independent, lognormally distributed** r.v.

Reaction	$A$	$n$	$E$	$UF$
$H + O_2 \leftrightarrow O + OH$	2.64e16	-0.67	71.30	1.5
$O + H_2 \leftrightarrow H + OH$	4.59e4	2.70	26.19	1.3
$OH + H_2 \leftrightarrow H + H_2O$	1.73e8	1.51	14.35	2.0
$OH + OH \leftrightarrow O + H_2O$	3.97e4	2.40	-8.83	1.5
$H + H + M \leftrightarrow H_2 + M$	1.78e18	-1.00	0.00	2.0
$H + H + H_2 \leftrightarrow H_2 + H_2$	9.00e16	-0.60	0.00	2.5
$H + H + H_2O \leftrightarrow H_2 + H_2O$	5.62e19	-1.25	0.00	2.0
$H + OH + M \leftrightarrow H_2O + M$	4.40e22	-2.00	0.00	2.0
$H + O + M \leftrightarrow OH + M$	9.43e18	-1.00	0.00	3.0
$O + O + M \leftrightarrow O_2 + M$	1.20e17	-1.00	0.00	2.0
$H + O_2 + M \leftrightarrow HO_2 + M$	6.33e19	-1.40	0.00	1.2
$H_2 + O_2 \leftrightarrow HO_2 + H$	5.92e5	2.43	223.85	2.0
$OH + OH + M \leftrightarrow H_2O_2 + M$	2.01e17	-0.58	-9.59	2.5
$HO_2 + H \leftrightarrow O + H_2O$	3.97e12	0.00	2.81	3.0
$HO_2 + H \leftrightarrow OH + OH$	7.49e13	0.00	2.66	2.0
$HO_2 + O \leftrightarrow OH + O_2$	4.00e13	0.00	0.00	1.2
$HO_2 + OH \leftrightarrow H_2O + O_2$	2.38e13	0.00	-2.09	3.0
	1.00e16	0.00	72.51	3.0
$HO_2 + HO_2 \leftrightarrow O_2 + H_2O_2$	1.30e11	0.00	-6.82	1.4
	3.66e14	0.00	50.21	2.5
$H_2O_2 + H \leftrightarrow HO_2 + H_2$	6.05e6	2.00	21.76	3.0
$H_2O_2 + H \leftrightarrow H_2O + OH$	2.41e13	0.00	16.61	2.0
$H_2O_2 + O \leftrightarrow HO_2 + OH$	9.63e6	2.00	16.61	3.0
$H_2O_2 + OH \leftrightarrow HO_2 + H_2O$	2.00e12	0.00	1.79	2.0
	2.67e41	-7.00	157.32	2.0

# Ignition Delay Time

## Uncertainty Propagation

- ▶ Conditions: Stoichiometric Hydrogen-Air Mixture (29.6% H<sub>2</sub>; 14.8% O<sub>2</sub>); Temperature: 1000 K; Pressure: 1 atm;
- ▶ Non-intrusive LHS Sampling (25 uncertain variables)

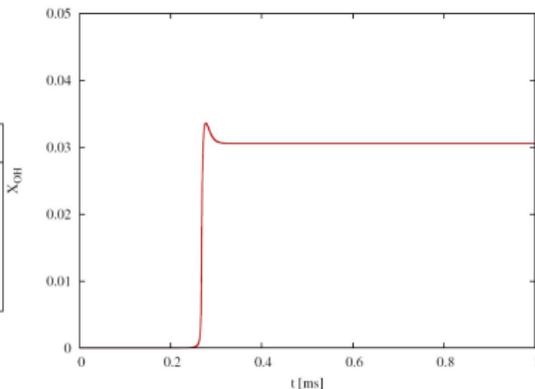
# Ignition Delay Time

## Uncertainty Propagation

- ▶ Conditions: Stoichiometric Hydrogen-Air Mixture (29.6% H<sub>2</sub>; 14.8% O<sub>2</sub>); Temperature: 1000 K; Pressure: 1 atm;
- ▶ Non-intrusive LHS Sampling (25 uncertain variables)

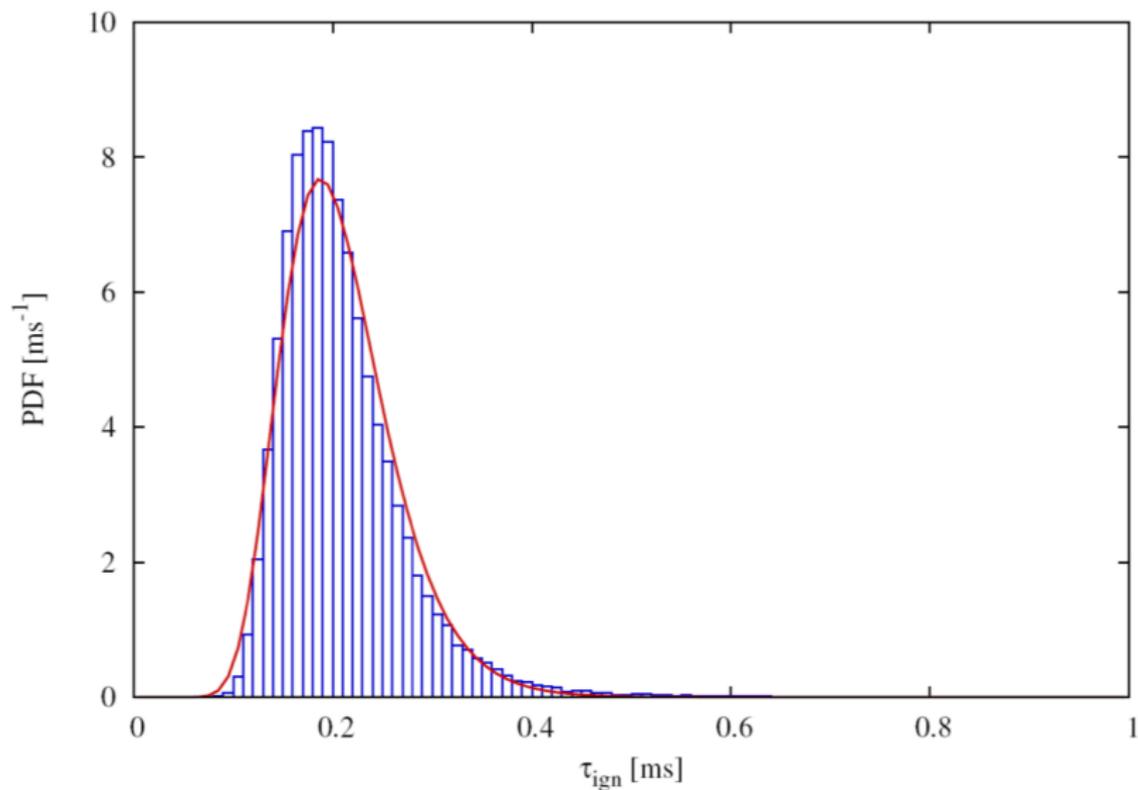
Number of Samples	Mean [ms]	$UF$
100	0.202060	1.7362
1000	0.201304	1.7054
10000	0.201256	1.7039
100000	0.201263	1.7075

One Sample Showing Ignition at ~0.25 ms



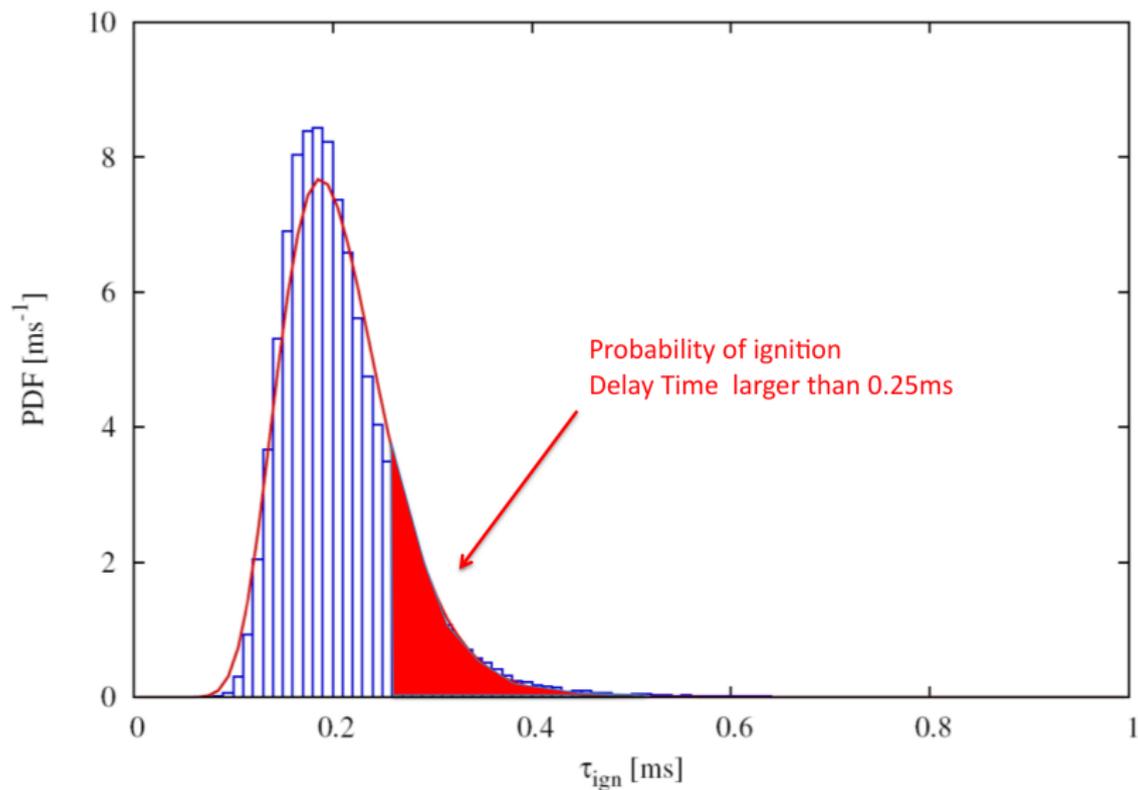
# Ignition Delay Time

## Uncertainty Propagation



# Ignition Delay Time

## Uncertainty Propagation



# Ignition Delay Time

## Uncertainty Propagation

- ▶ UQ provides an effective quantification of the range (and likelihood) of the ignition delay time

# Ignition Delay Time

## Uncertainty Propagation

- ▶ UQ provides an effective quantification of the range (and likelihood) of the ignition delay time
- ▶ Given that hydrogen chemistry is the **simplest** possible chose and we still need  $\approx 5000$  solutions to get an accurate answer (using LHS), two questions remain

# Ignition Delay Time

## Uncertainty Propagation

- ▶ UQ provides an effective quantification of the range (and likelihood) of the ignition delay time
- ▶ Given that hydrogen chemistry is the **simplest** possible chose and we still need  $\approx 5000$  solutions to get an accurate answer (using LHS), two questions remain
  - ▶ Good to know, so what?

# Ignition Delay Time

## Uncertainty Propagation

- ▶ UQ provides an effective quantification of the range (and likelihood) of the ignition delay time
- ▶ Given that hydrogen chemistry is the **simplest** possible chose and we still need  $\approx 5000$  solutions to get an accurate answer (using LHS), two questions remain
  - ▶ Good to know, so what?
  - ▶ Can we do this faster?

# Ignition Delay Time

Pose the UQ quest as an Inverse Problem

- ▶ What uncertainty in the reaction rates can we **tolerate** to ensure that the probability of ignition delay time exceeding 0.25 ms is less than 10%?

# Ignition Delay Time

Pose the UQ quest as an Inverse Problem

- ▶ What uncertainty in the reaction rates can we **tolerate** to ensure that the probability of ignition delay time exceeding 0.25 ms is less than 10%?
- ▶ Can be cast as an **optimization problem under uncertainty**: find the maximum UF such that the  $p(IDT > IDT_{cr}) < 0.1$

# Ignition Delay Time

Pose the UQ quest as an Inverse Problem

- ▶ What uncertainty in the reaction rates can we **tolerate** to ensure that the probability of ignition delay time exceeding 0.25 ms is less than 10%?
- ▶ Can be cast as an **optimization problem under uncertainty**: find the maximum UF such that the  $p(IDT > IDT_{cr}) < 0.1$
- ▶ Problem: too many parameters! Focus only on the branching reaction ( $H + O_2 \leftrightarrow O + OH$ )

# Ignition Delay Time

Pose the UQ quest as an Inverse Problem

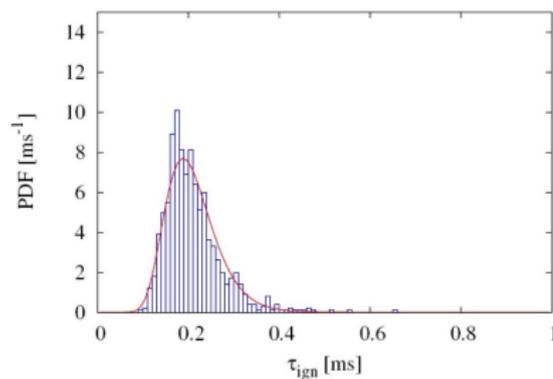
- ▶ What uncertainty in the reaction rates can we **tolerate** to ensure that the probability of ignition delay time exceeding 0.25 ms is less than 10%?
- ▶ Can be cast as an **optimization problem under uncertainty**: find the maximum UF such that the  $p(IDT > IDT_{cr}) < 0.1$
- ▶ Problem: too many parameters! Focus only on the branching reaction ( $H + O_2 \leftrightarrow O + OH$ )

Quantity	Nominal	Optimal
$UF$ Branching Reaction	1.5	1.29
Mean $\tau_{ign}$ [ms]	0.201304	0.198947
$UF$ $\tau_{ign}$	1.7054	1.4023
Probability of Failure	0.191	0.100

# Ignition Delay Time

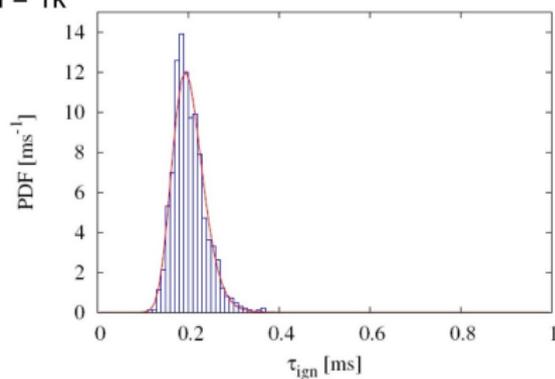
## Uncertainty Propagation

Nomimal



Optimal

N = 1k



- ▶ Overall uncertainty in the IDT is reduced
- ▶ Failure probability below critical requirement

# Ignition Delay Time

## Efficiency of UQ

The results shown so far use **sampling** to compute the statistics of the output of interest

# Ignition Delay Time

## Efficiency of UQ

The results shown so far use **sampling** to compute the statistics of the output of interest

- ▶ The problem is characterized by high-dimensionality of the input ( $H_2$  chemistry has 25 uncertain factors)

# Ignition Delay Time

## Efficiency of UQ

The results shown so far use **sampling** to compute the statistics of the output of interest

- ▶ The problem is characterized by high-dimensionality of the input ( $H_2$  chemistry has 25 uncertain factors)
- ▶ Polynomial chaos methods cannot be applied because of the exponential cost of building tensorial basis functions (recall cardinality  $\mathcal{P} = \frac{(P + d)!}{P! + d!}$ )
- ▶ But the key is that NOT all of the inputs are important!

# Ignition Delay Time

## Efficiency of UQ

The results shown so far use **sampling** to compute the statistics of the output of interest

- ▶ The problem is characterized by high-dimensionality of the input ( $H_2$  chemistry has 25 uncertain factors)
- ▶ Polynomial chaos methods cannot be applied because of the exponential cost of building tensorial basis functions (recall cardinality  $\mathcal{P} = \frac{(P + d)!}{P! + d!}$ )
- ▶ But the key is that NOT all of the inputs are important!

We need to **use/develop** algorithms that **discover** the true dependency of the solution

# Low-Rank Approximations

## High-Dimensional UQ

Extend the concept of **Separation of variables** to computational methodologies

# Low-Rank Approximations

## High-Dimensional UQ

Extend the concept of **Separation of variables** to computational methodologies

- ▶ Assume  $y_j$  for  $j = 1, \dots, d$  are the input uncertainties
- ▶ Define

$$u(y_1, \dots, y_d) \approx \sum_{k=1}^r u_1^{(k)}(y_1) \times u_2^{(k)}(y_2) \times \dots \times u_d^{(k)}(y_d)$$

# Low-Rank Approximations

## High-Dimensional UQ

Extend the concept of **Separation of variables** to computational methodologies

- ▶ Assume  $y_j$  for  $j = 1, \dots, d$  are the input uncertainties

- ▶ Define

$$u(y_1, \dots, y_d) \approx \sum_{k=1}^r u_1^{(k)}(y_1) \times u_2^{(k)}(y_2) \times \dots \times u_d^{(k)}(y_d)$$

- ▶ Need to **discover** the functions  $u^{(k)}$  and the *rank*  $r$

# Low-Rank Approximations

## High-Dimensional UQ

Extend the concept of **Separation of variables** to computational methodologies

- ▶ Assume  $y_j$  for  $j = 1, \dots, d$  are the input uncertainties

- ▶ Define

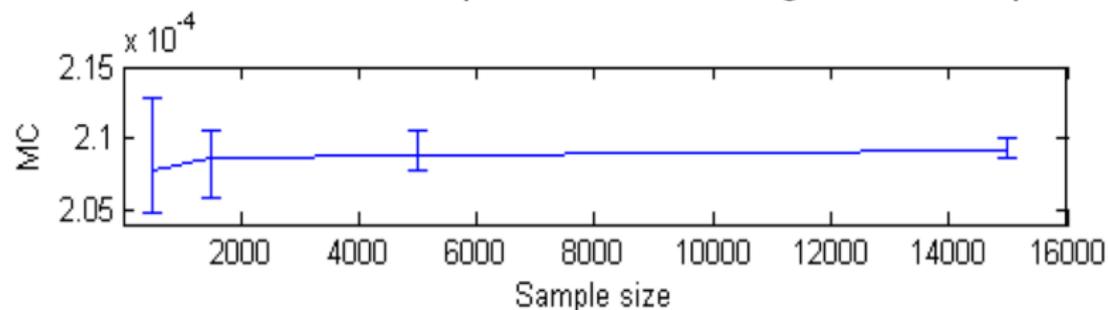
$$u(y_1, \dots, y_d) \approx \sum_{k=1}^r u_1^{(k)}(y_1) \times u_2^{(k)}(y_2) \times \dots \times u_d^{(k)}(y_d)$$

- ▶ Need to **discover** the functions  $u^{(k)}$  and the *rank*  $r$
- ▶ We cast it as an optimization problem: Find the lowest possible  $r$  which approximates a set of given function evaluations with  $u^{(k)}$  being polynomials of fixed maximum order

# Low-Rank Approximations

## Mean Ignition Delay Time

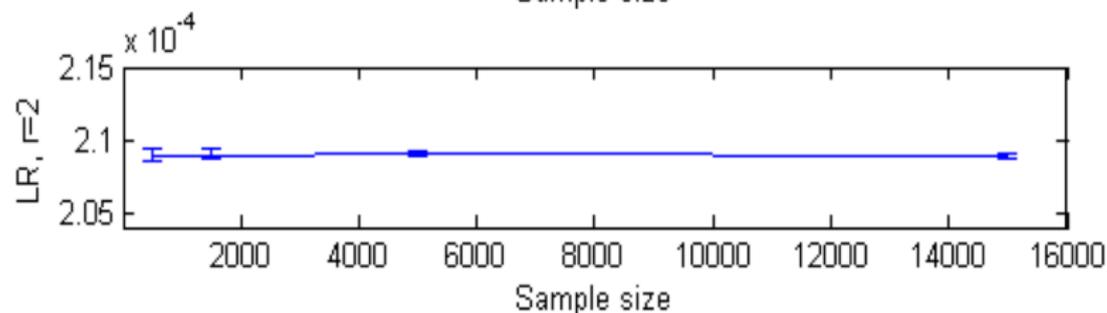
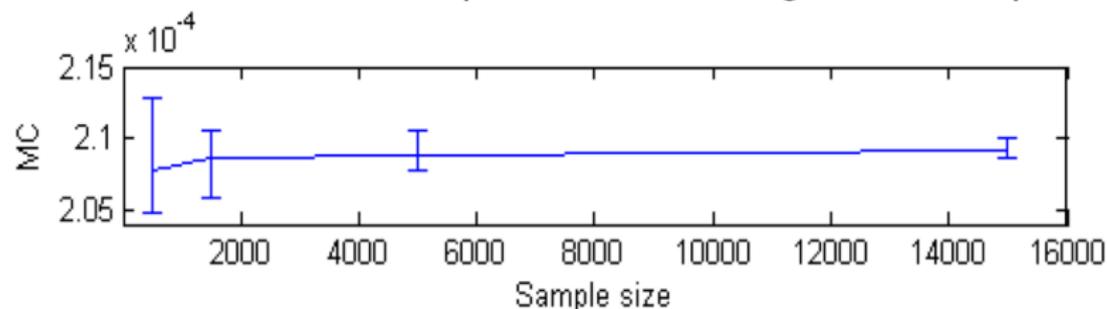
Reuse the solutions computed in the autoignition example



# Low-Rank Approximations

## Mean Ignition Delay Time

Reuse the solutions computed in the autoignition example

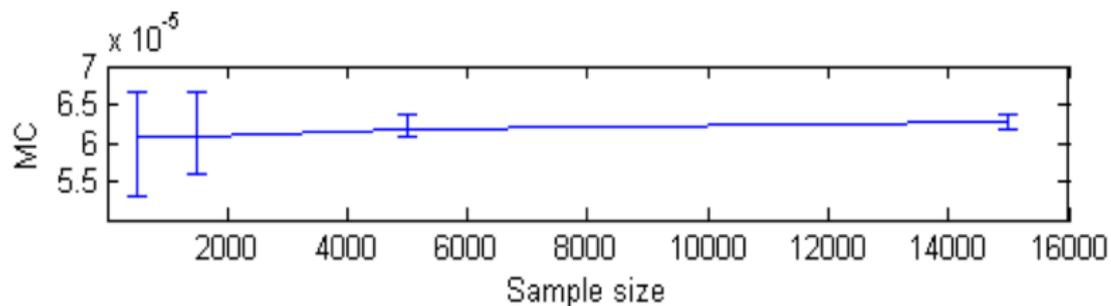


Even with 500 samples the estimate of the mean ignition delay is acceptable (smaller than MC with 14,000)

# Low-Rank Approximations

STD of Ignition Delay Time

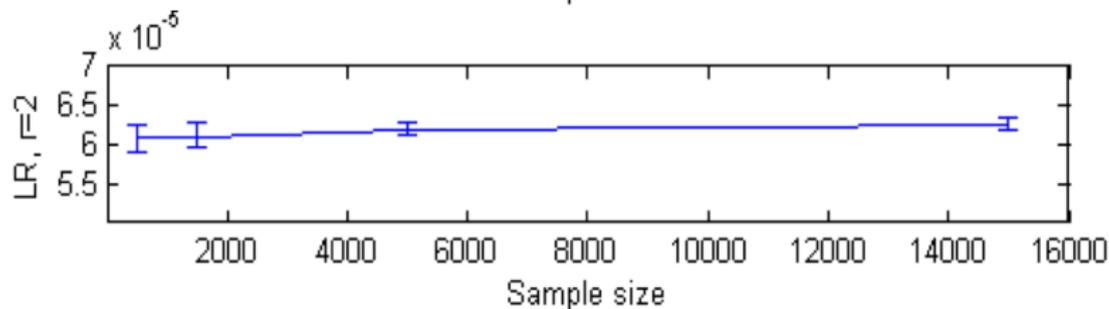
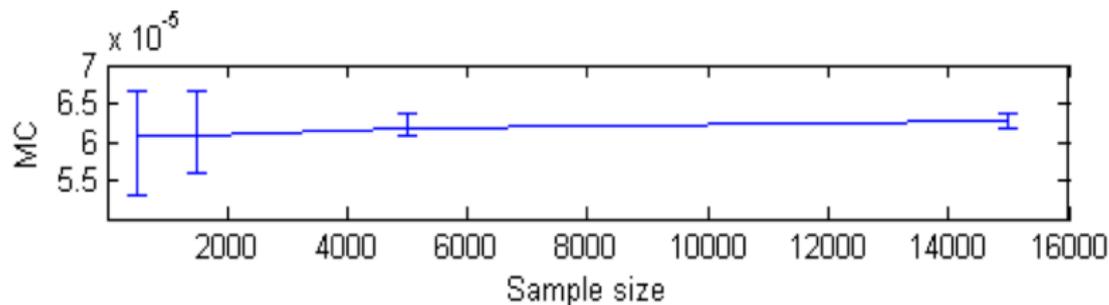
Similar results for the variance



# Low-Rank Approximations

## STD of Ignition Delay Time

Similar results for the variance



# Summary of the examples

...so far

- ▶ Illustrated Uncertainty Propagation: effect of the input uncertainties on the output

# Summary of the examples

...so far

- ▶ Illustrated Uncertainty Propagation: effect of the input uncertainties on the output
- ▶ Demonstrated the concept of **ranking** of the uncertainties

# Summary of the examples

...so far

- ▶ Illustrated Uncertainty Propagation: effect of the input uncertainties on the output
- ▶ Demonstrated the concept of **ranking** of the uncertainties
- ▶ Showed an example of Backward Uncertainty Propagation: from tolerable outputs to acceptable input uncertainties

# Summary of the examples

...so far

- ▶ Illustrated Uncertainty Propagation: effect of the input uncertainties on the output
- ▶ Demonstrated the concept of **ranking** of the uncertainties
- ▶ Showed an example of Backward Uncertainty Propagation: from tolerable outputs to acceptable input uncertainties
- ▶ Given one example of efficiency gains with modern UQ algorithms

# Summary of the examples

...so far

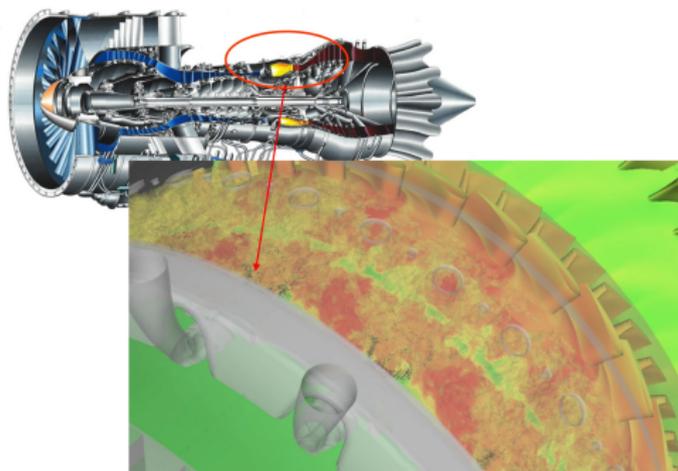
- ▶ Illustrated Uncertainty Propagation: effect of the input uncertainties on the output
- ▶ Demonstrated the concept of **ranking** of the uncertainties
- ▶ Showed an example of Backward Uncertainty Propagation: from tolerable outputs to acceptable input uncertainties
- ▶ Given one example of efficiency gains with modern UQ algorithms

To finish, I want to give you an example of UQ combined with **realistic** flow simulations...

# Turbulent reacting flows

## Challenges

- ▶ **Turbulent** flow simulations in **realistic geometries** with **detailed kinetic mechanisms** are still beyond the reach of computational engineering



PW6000 simulations enabled by the **flamelet modeling** approach, but still requiring 1000 CPUs

# Turbulent reacting flows

## UQ Challenges

- ▶ Uncertainties in kinetic mechanisms might still dominate, especially for the **prediction of pollutants**
- ▶ It is **impossible** to perform more than a handful of simulation ( $\mathcal{O}(5)$ )
- ▶ Non-intrusive approach (even our fancy LR method) have no hope of success
- ▶ Need to be **intrusive** and connect the physics/mathematics/UQ

# Turbulent reacting flows

Non-intrusive framework



- ▶ Perform MANY simulations and sample the outputs

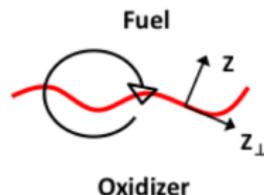
# Reacting Flow Modeling - Flamelet

## ▶ Basic Premise

- ▶ Since scales of chemical reaction are much smaller than the smallest scales of turbulent, a turbulent flame is simply an ensemble of laminar "flamelets" embedded in a turbulent flow field
- ▶ Solve for flame structure independently from the flow field

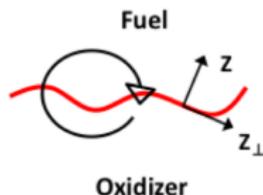
## ▶ Coordinate Transformation

- ▶ Transform to coordinate system attached to the flame  
 $(x, y, z) \rightarrow (Z, Z_{\perp})$
- ▶ Neglect all gradients in tangential directions



# Reacting Flow Modeling - Flamelet

- ▶ Basic Premise
  - ▶ Since scales of chemical reaction are much smaller than the smallest scales of turbulent, a turbulent flame is simply an ensemble of laminar "flamelets" embedded in a turbulent flow field
  - ▶ Solve for flame structure independently from the flow field
- ▶ Coordinate Transformation
  - ▶ Transform to coordinate system attached to the flame  
 $(x, y, z) \rightarrow (Z, Z_{\perp})$
  - ▶ Neglect all gradients in tangential directions
  - ▶ Resulting equations are a one-dimensional set of reaction-diffusion equations parameterized by the mixture fraction  $Z$
  - ▶ Solved in advance and tabulated for a given fuel



# Reacting Flow Modeling - Governing Equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ 2\mu \left( S_{ij} - \frac{1}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right]$$

$$\frac{\partial Z}{\partial t} + \frac{\partial \rho Z}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \rho D \frac{\partial Z}{\partial x_j} \right]$$

$$T = T(\rho, p, Z) \quad \rightarrow \quad \text{tabulated}$$

# Reacting Flow Modeling - Governing Equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ 2\mu \left( S_{ij} - \frac{1}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right]$$

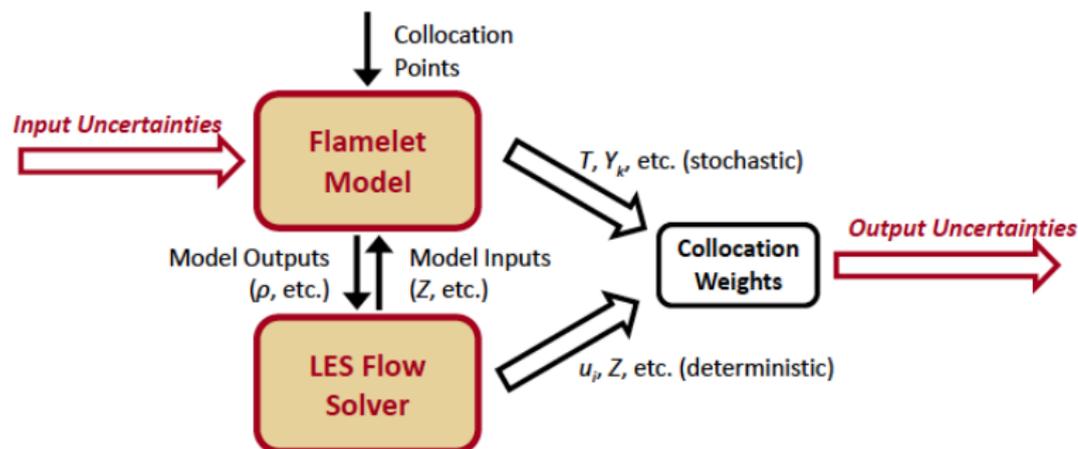
$$\frac{\partial Z}{\partial t} + \frac{\partial \rho Z}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \rho D \frac{\partial Z}{\partial x_j} \right]$$

$$T = T(\rho, p, Z) \quad \rightarrow \quad \text{tabulated}$$

- ▶ Uncertainty in the kinetic rates appears indirectly through the density
- ▶ Use the flamelet equations to condition the high-dimensional uncertainty
- ▶ Can use efficient UQ methods requiring few full system simulations

# Turbulent reacting flows

## Intrusive framework



- ▶ Split the flamelet-generation part from the actual flow simulations
- ▶ Propagate the uncertainty through the flamelet
- ▶ Inject uncertainties in the link between flamelets and flow equations (via mixture fraction, density, etc.)

# Turbulent reacting flows

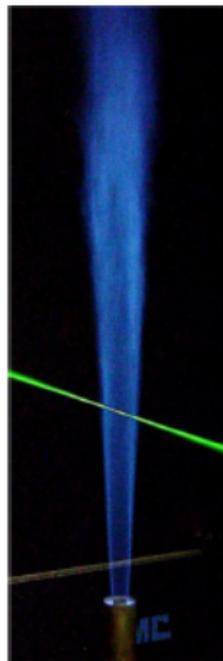
## Sandia Flame D

### Piloted partially premixed methane/air flame

- ▶ Used NGA (low Mach, structured grid)
- ▶ GRI 3.0 mechanism
- ▶ Uncertainties in rates from Sheen *et al.* 2009

### Simulations

1. Used LHS sampling for flamelets (10,000 solutions)
2. Compiled tables with mean and variances of density (other uncertainties, e.g. viscosity ignored for now)
3. Performed 7 LES simulations sampling on the density distribution

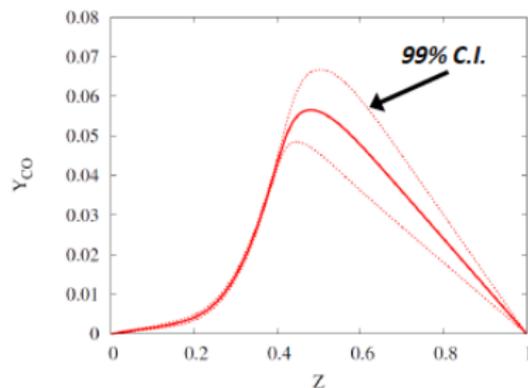
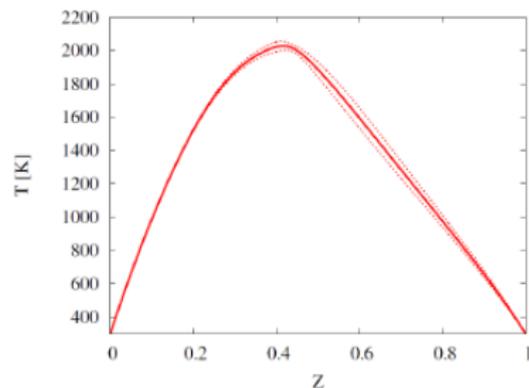


# Turbulent reacting flows

## Sandia Flame D

### Step 1

From kinetic rate uncertainties to flamelet output uncertainties



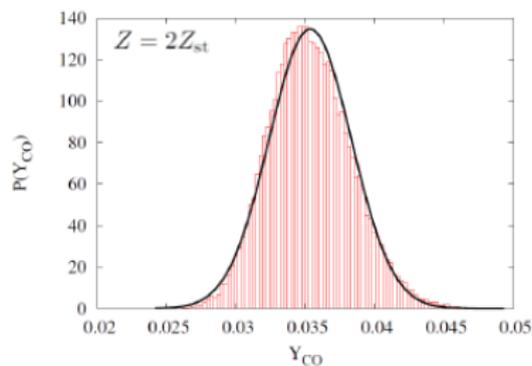
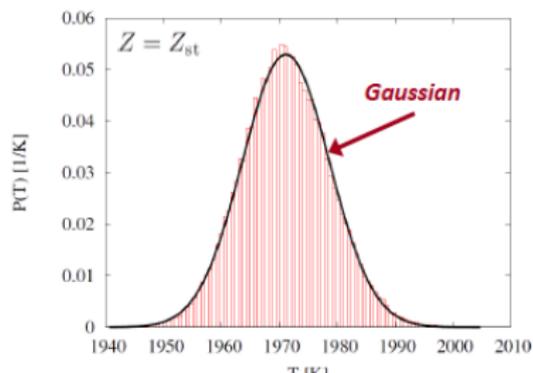
Rich part of the flame more *uncertain*

# Turbulent reacting flows

Sandia Flame D

## Step 2

Create a **stochastic** flamelet table



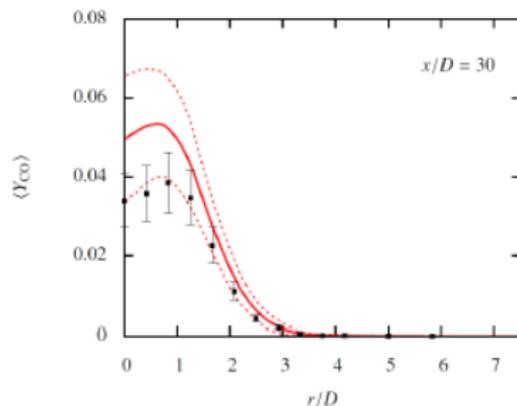
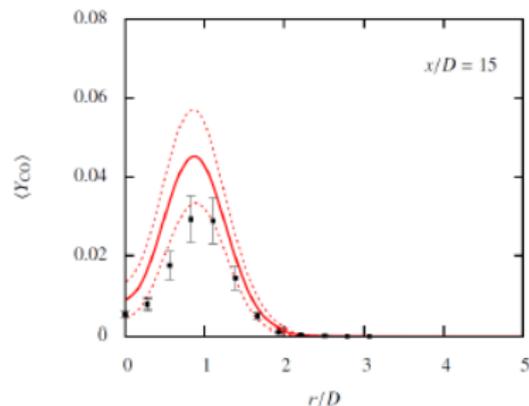
Distributions are Gaussian-like

# Turbulent reacting flows

## Sandia Flame D

### Step 3

LES with **stochastic** flamelet table



Emission uncertainty is quite high, and might be comparable with other uncertainties (steady flamelet assumption, kinetic mechanism, etc.)



**CEMRACS Summer School**  
**July 2012**  
**CIRM, Marseille, France**

