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Visualisation en mécanique des fluides : illustration

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CTRL-L switch

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Généralités

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0001640	9.020746e-01	9.093018e-01	9.060076e-01	9.020746e-01	0001640	9.020746e-01	9.093018e-01	9.060076e-01	9.020746e-01	0001640	9.020746e-01	9.093018e-01	9.060076e-01	9.002426e-01
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- **Visualization - why ?**

The world is full of information. Simulations, experiments and data collections comprise an enormous and permanently increasing accumulation of data. Therefore new ways have to be found to reveal the information hidden in sometimes huge data sets.

Visualization offers a way. It aims at presenting complex information in a comprehensible way - exploiting the sensory apparatus and the highly developed perceptual capabilities of humans.

- **Terminology**

Visualization. Scientific visualization (science, engineering). Data Visualization (wider understanding). Information visualization (visualize abstract information)

En mécanique des fluides

Production des données par :

- **Expériences** : LDA, PIV, DPIV...
- **Calculs** : simulations numériques.

Simulation Numérique Directe, codes de calcul Navier-Stokes moyennés, modèles...

- Données structurées (différences finies, méthodes spectrales), non structurées (volumes finis, éléments finis)

Exploitation \Rightarrow Comprendre la physique !

Aéronautique (portance, traînée) ; structures (interaction fluides/structures) ;
phénoménologie de la turbulence ; Environnement (atmosphère, océans)

Vision Eulérienne \neq vision Lagrangienne.

Maillages : structuré/non structuré

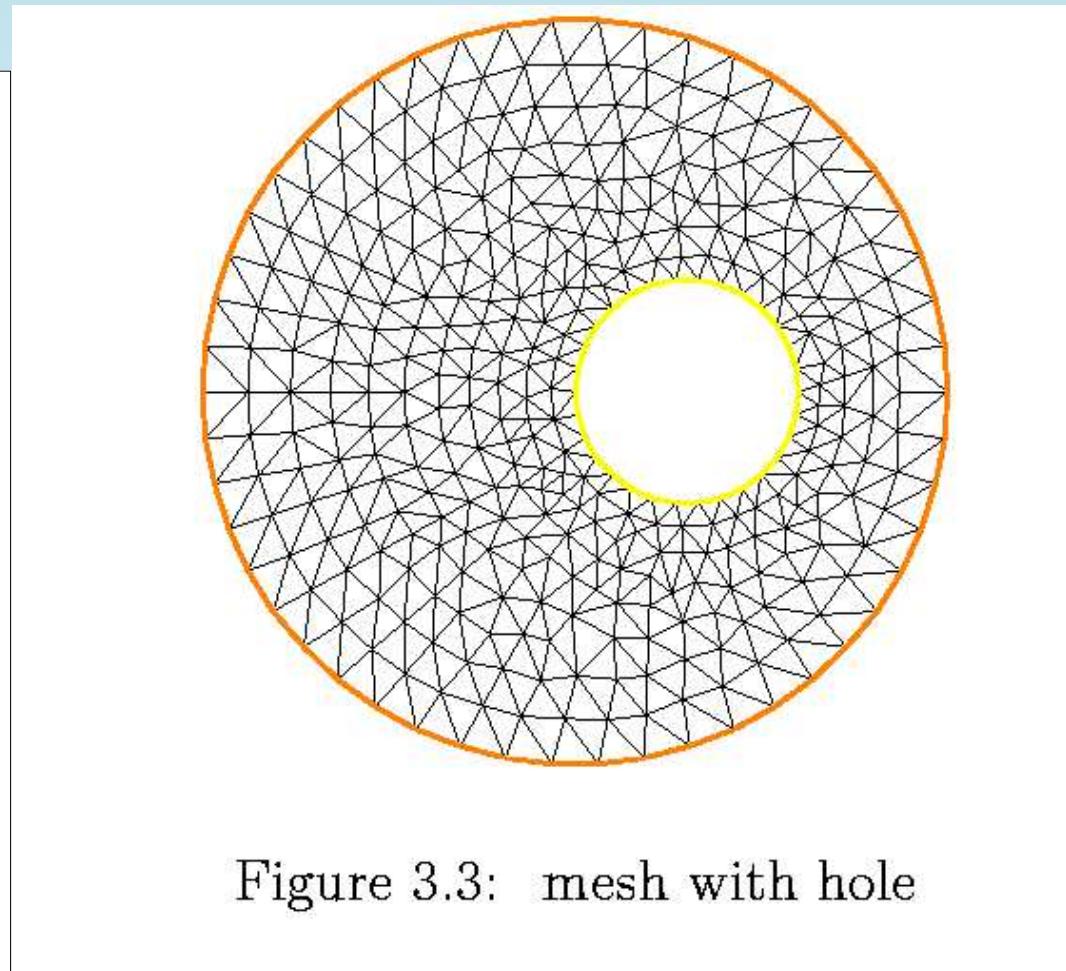
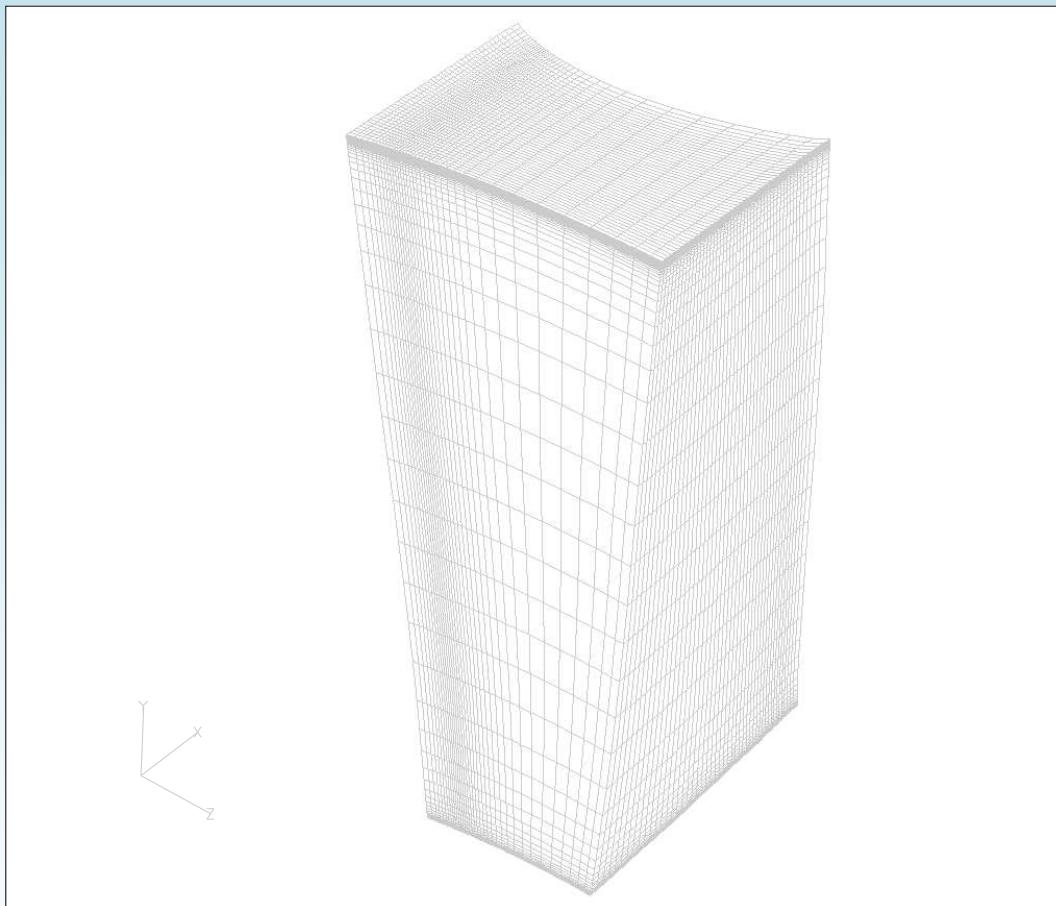


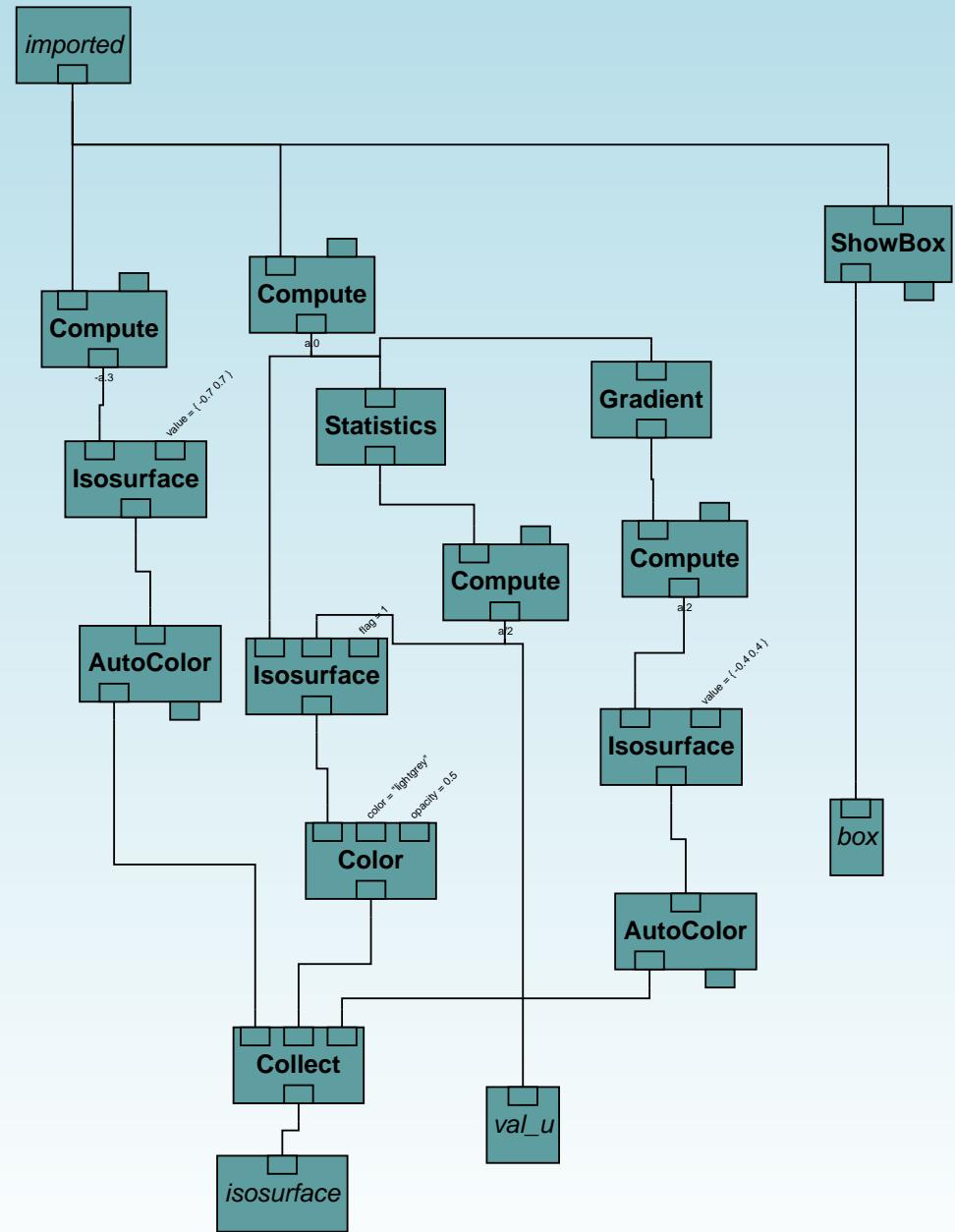
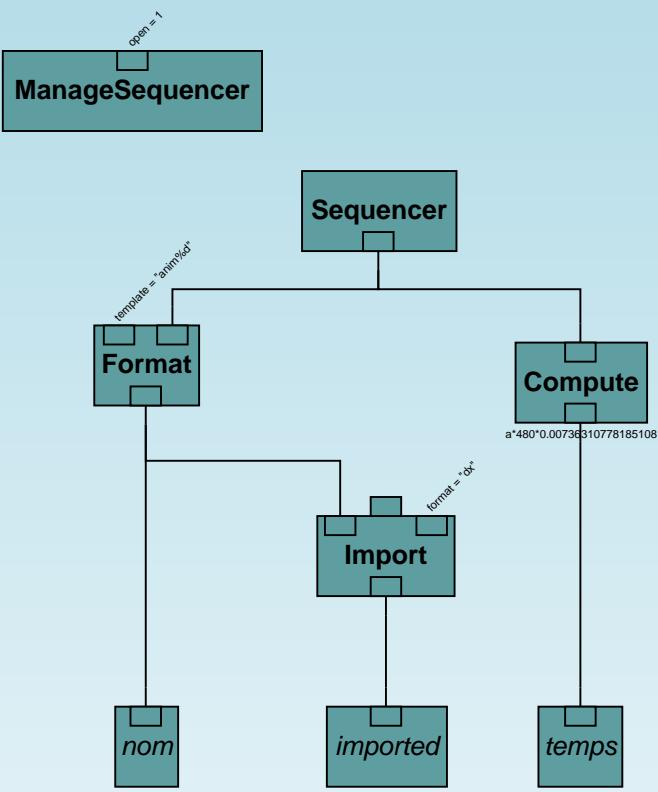
Figure 3.3: mesh with hole

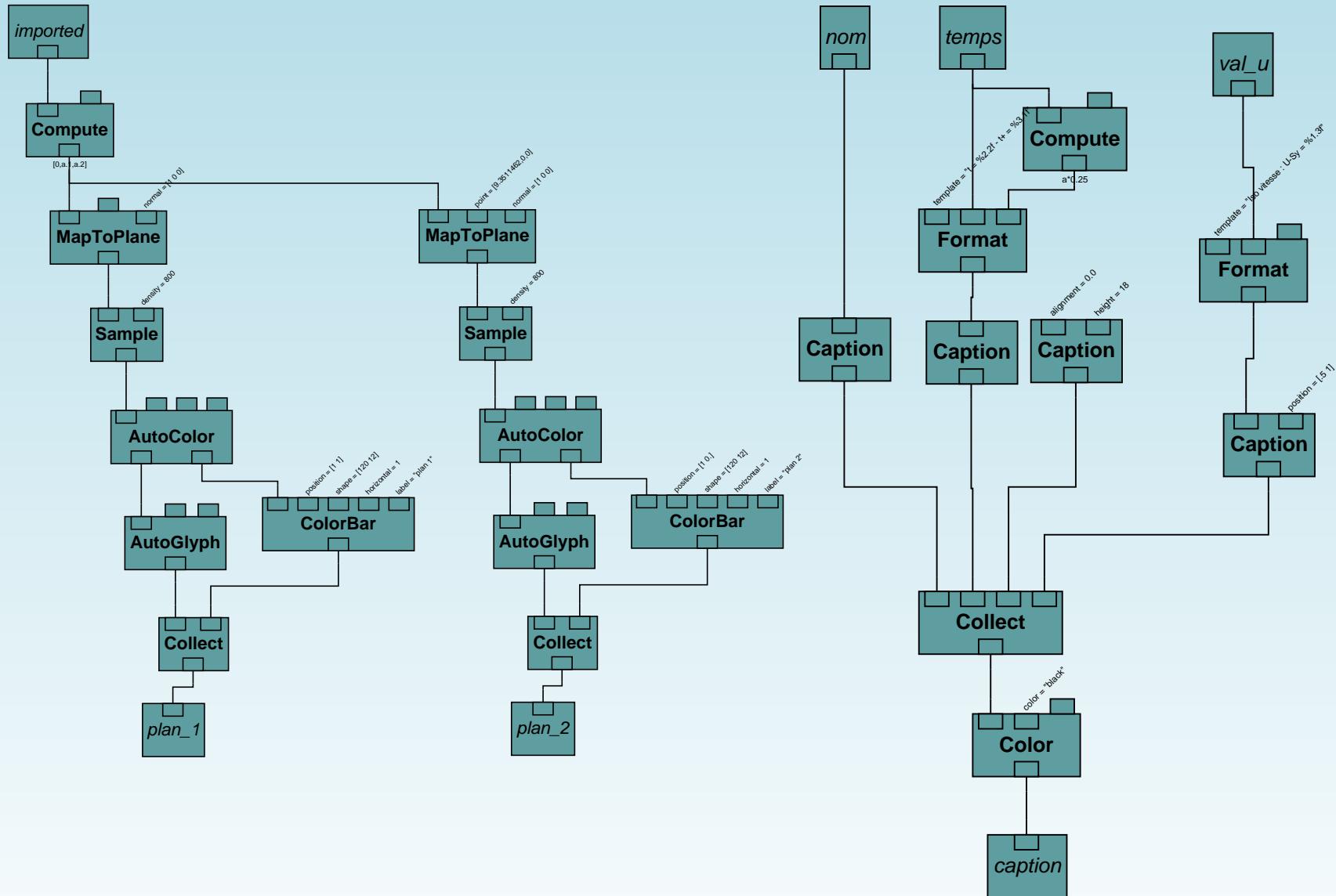
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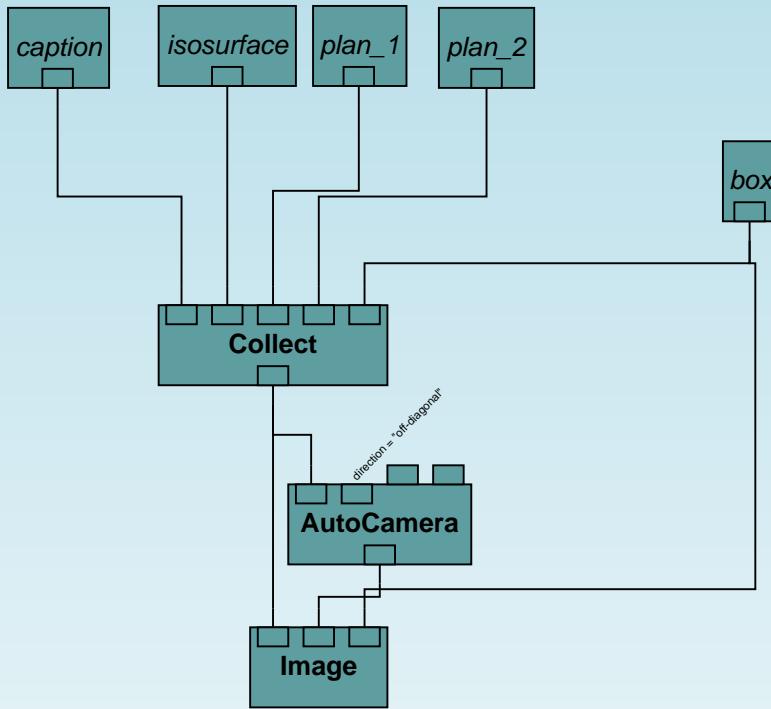
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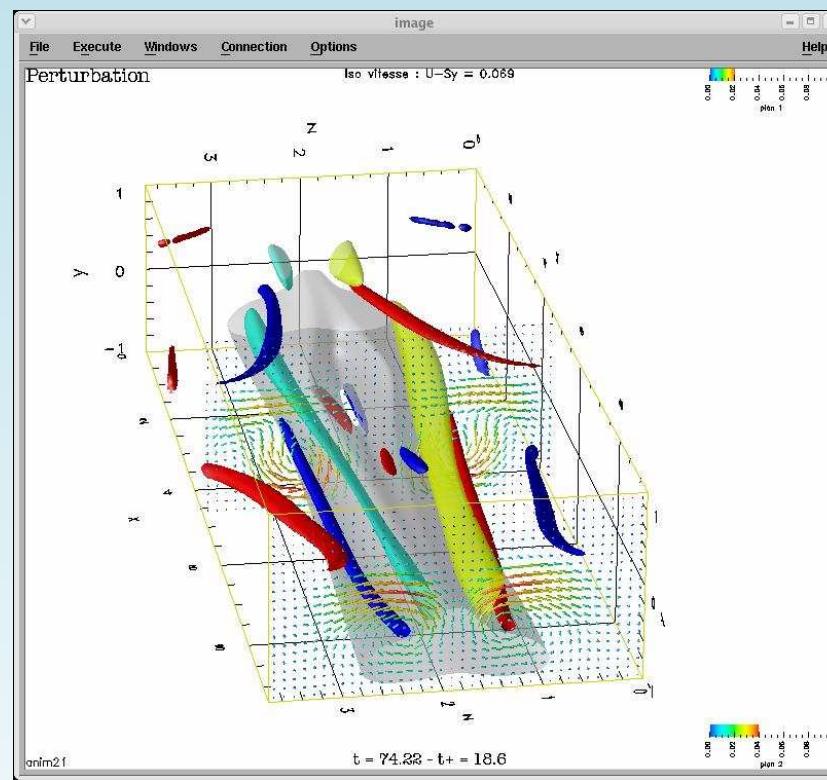
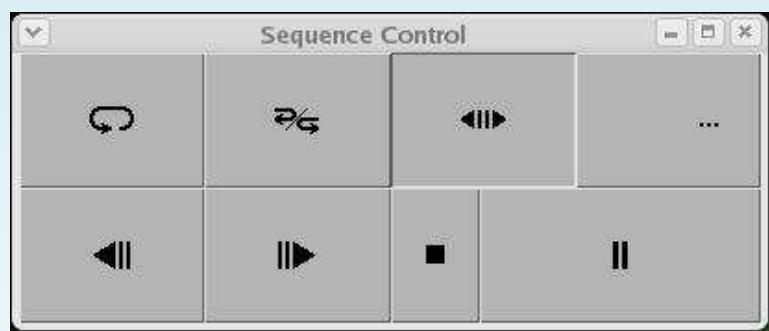
www.opendx.org

- OpenDX is the open source software version of IBM's Visualization Data Explorer Product.
- Visualization for anything from examining simple data sets to analyzing complex, time-dependent data from disparate source ; **sophisticated data model**
- Build visualization applications for your end users ; **functional modules**
- Advanced Execution Environment : distributed processing
- Client/Server architecture



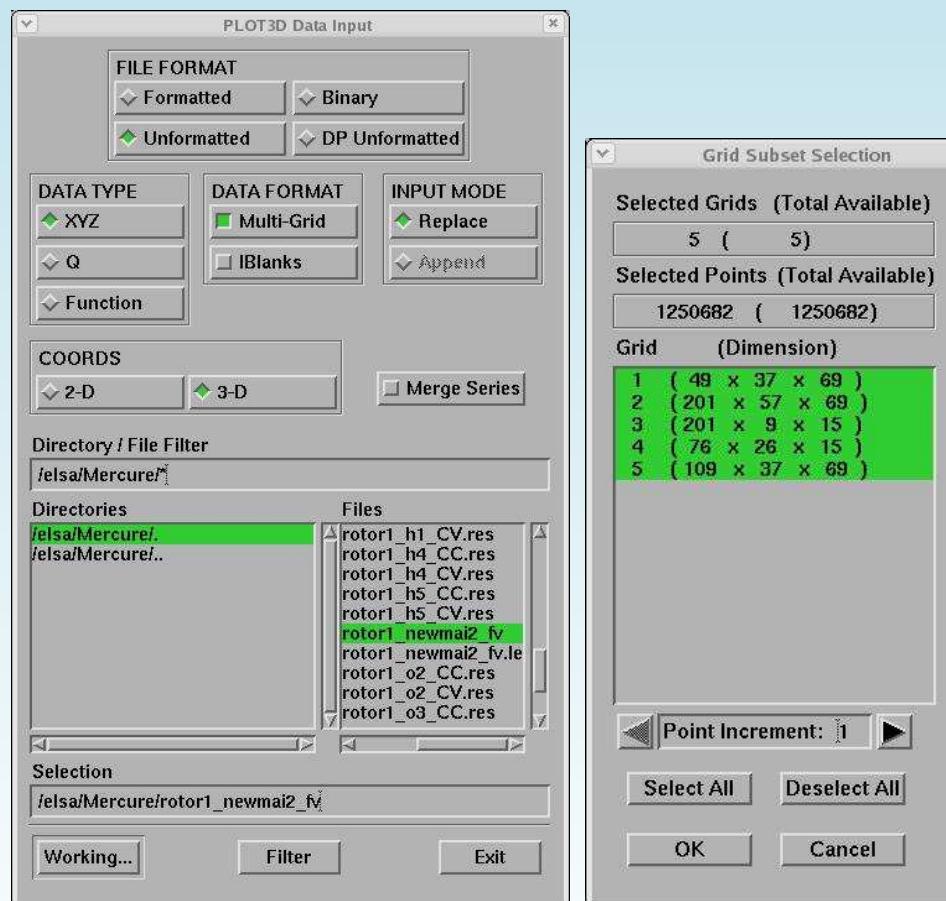


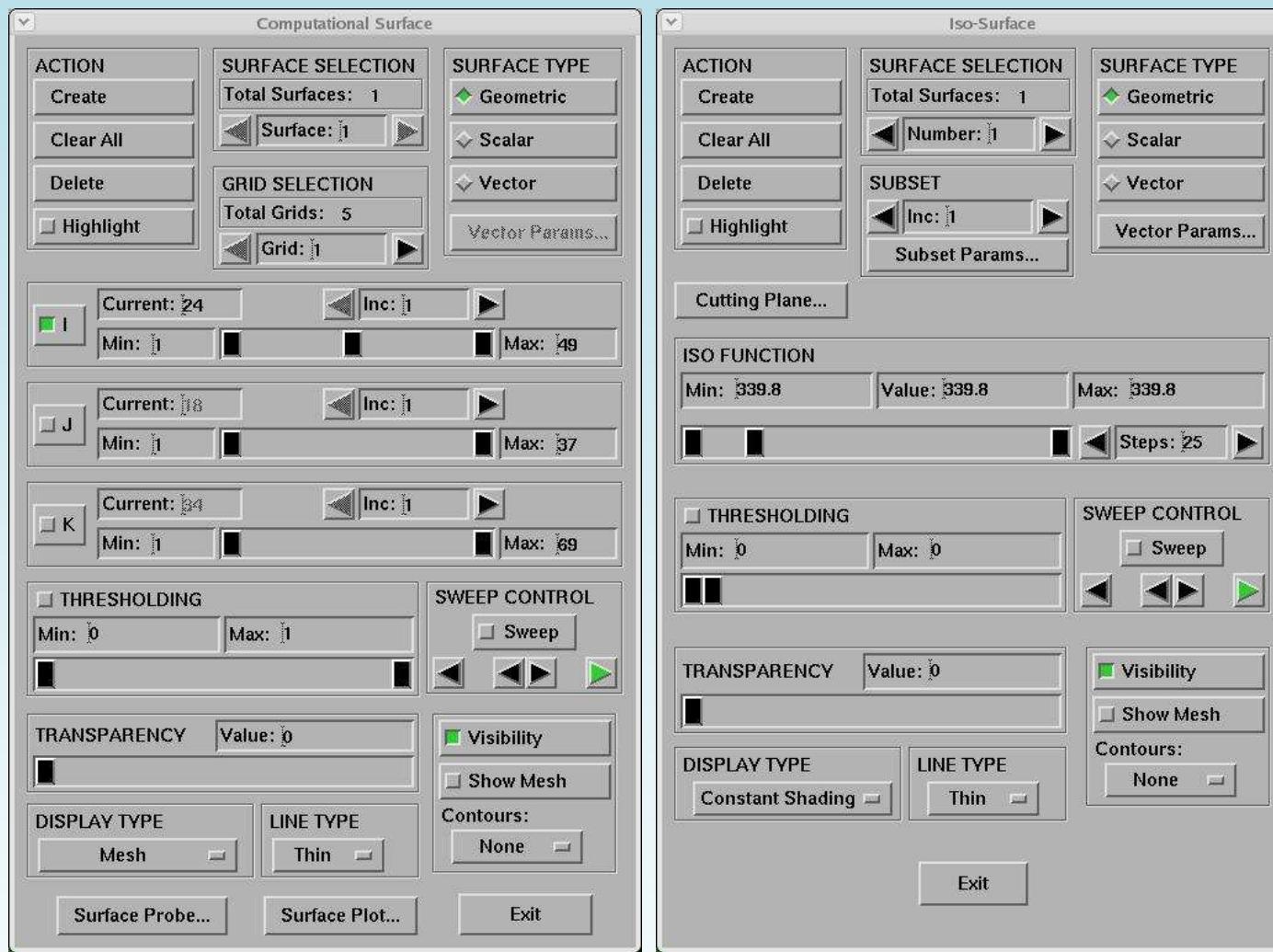


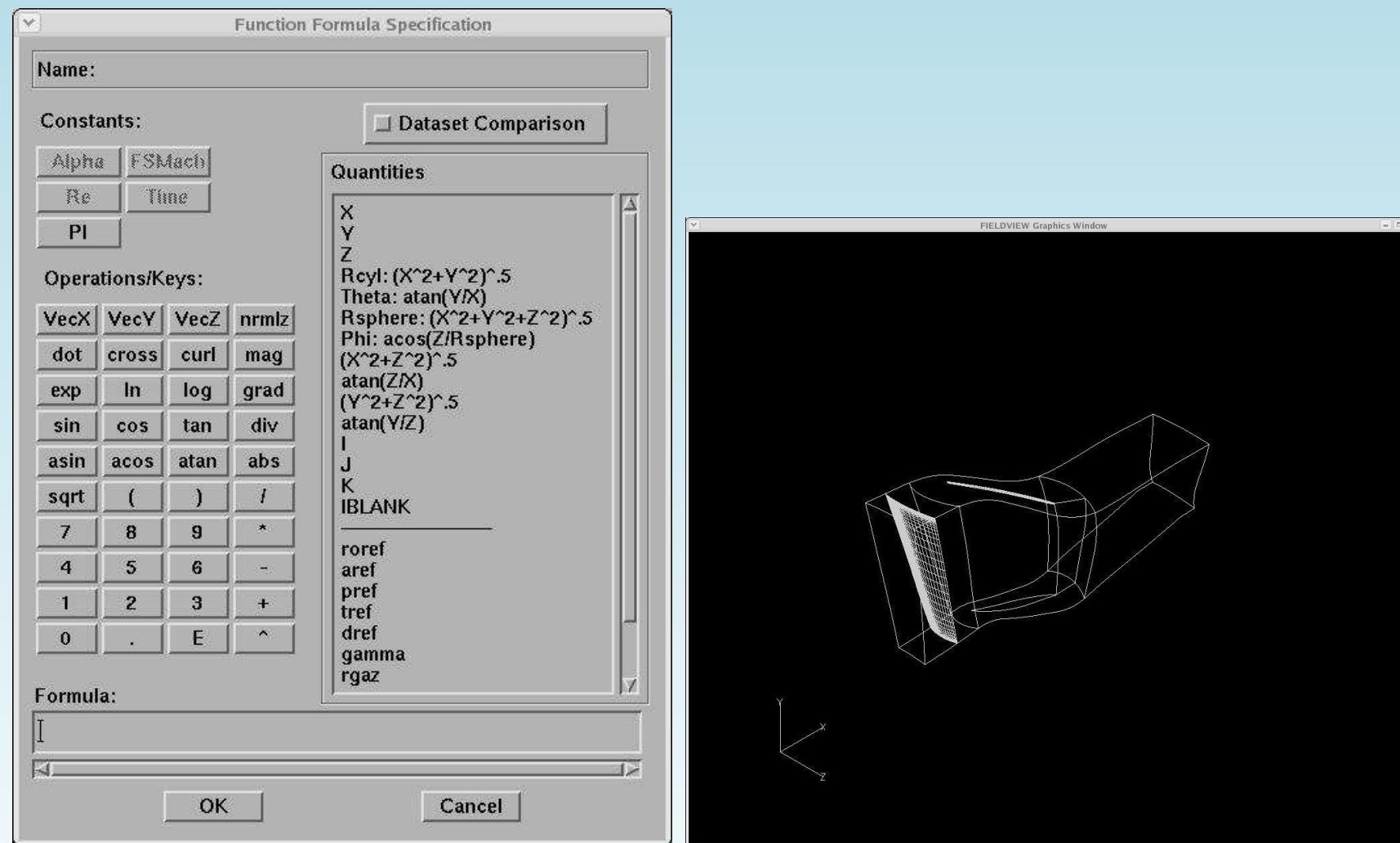


Fieldview

www.ilight.com







Streamlines, particle paths, Core detection, ...

Extrait de la documentation Fieldview

“Automatic Feature Detection algorithms process CFD results data to locate flow features such as vortex cores, separation and re-attachment lines and shock surfaces. Until now, these features would be located using indirect methods that relied heavily on the fluid mechanics knowledge of the user.”

Autres logiciels utilisés

- Amira
- Tecplot
- Fluent

Critères de visualisation en mécanique des fluides

⇒ On voit ce que l'on veut voir !

- Visualisation directe des champs de vitesse, vorticité, pression...
- La visualisation de tourbillons passe par la définition même de **ce qu'est un tourbillon** ou une structure cohérente et ce problème est encore ouvert à ce jour.

Visualisation des tourbillons : divers critères

Il sont basés sur le comportement de quantités physiques au centre et autour des tourbillons.

- Diverses observations ont mené à ces critères. Par exemple, le critère de basse pression au centre des tourbillons se justifie uniquement dans le cas d'écoulement plans stationnaires.
- Une définition intuitive du tourbillon comme un tube dont la surface est composée de lignes de vorticité conduit à calculer la vorticité et à rechercher ses extrema. Cette technique est très efficace dans la limite d'écoulements relativement homogènes car la détection des tourbillons est tributaire du seuillage du niveau de vorticité choisi.
- Proposé pour les écoulements 2D, le critère de Weiss, qui consiste à rechercher le minimum du laplacien de pression, a démontré son efficacité mais n'est pas applicable aux résultats tridimensionnels.

Reste donc à trouver un moyen robuste de repérer des tourbillons tridimensionnels dans un écoulement fortement inhomogène.

- Arguments dynamiques : critères de pression
- Arguments topologiques : critères Q , R , discriminant, λ_2
- Arguments cinématiques : critère NAM

Critère de pression

Les particules fluides au sein d'un tourbillon sont en équilibre si

$$\partial_t \mathbf{u} + \boldsymbol{\omega} \times \mathbf{u} = -\frac{1}{\rho} \nabla P$$

La pression dynamique doit décroître dans le tourbillon pour contrebalancer les effets centrifuges.

Le cœur d'un tourbillon correspond ainsi à un **minimum local de pression**.

Les surfaces iso-pression peuvent donc être utilisées.

Critères Q , R , discriminant, λ_2

(Chong et Perry, 1990 ; Dubief et Delcayre, 2000) En supposant que le champ de vitesse se comporte “bien” partout, on peut développer localement en chaque point de l’écoulement le champs de vitesse en série de Taylor :

$$u_i = \dot{x}_i = A_i + A_{ij}x_j + A_{ijk}x_jx_k + \dots$$

Les coefficients dépendent du temps si l’écoulement est instationnaire.

En re-centrant le repère local au point considéré, on peut récrire au premier ordre la caractérisation topologique de l’écoulement sous la forme :

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Les coefficients $a_{ij} = \frac{\partial u_i}{\partial x_j}$ sont donc les éléments du tenseur des déformations, qui se décompose en parties symmétriques et antisymétriques :

$$\frac{\partial u_i}{\partial x_j} = S_{ij} + R_{ij}.$$

On identifie les valeurs propres $\lambda_1, \lambda_2, \lambda_3$ de \mathbf{A} et les vecteurs propres, et l'on étudie la topologie du point considéré dans le repère des éléments propres. L'équation caractéristique

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0$$

introduit trois quantités :

$$P_{ij} = -S_{ii} = -\text{tr}(\mathbf{A}) \quad Q = \frac{1}{2} (P^2 - \text{tr}(\mathbf{A}^2)) \quad R = -\det(\mathbf{A})$$

et l'on peut relier la structure locale de l'écoulement aux valeurs de Q et R . (Diagramme dans le plan $Q-R$.)

-
- Critère $Q = \frac{1}{4}(\omega^2 - S_{ij}S_{ij})$ (critère de Weiss) :

$Q < 0$ sont des régions à cisaillement dominant

$Q > 0$ sont les régions à vorticité dominante (tourbillons ?..)

(Lien entre les tourbillons cohérents et les régions convexes à $\nabla^2 P > 0 \Rightarrow$ Critère de M. Larchevêque, bi-dimensionnel.)

- Critère du discriminant $\Delta = \left(\frac{Q}{3}\right)^3 + \frac{R}{2} > 0$

Critère NAM : Moment Angulaire Normé

Méthode d'abord mise au point pour le post-traitement de données expérimentales bruitées, ce qui limitait sérieusement la capacité du critère de la vorticité à repérer les tourbillons (Michard, LMFA, 1997).

Définition du N.A.M.

Outil basé sur des propriétés géométriques simples du tourbillon.

Pour la première fois appliqué à des mesures P.I.V. (“Particle Image Velocimetry”). Le but de l'expérience, menée par Michard et al. est de localiser le centre d'un tourbillon dans une chambre de moteur.

Pour introduire la fonction utilisée, appelée Moment Angulaire Normé (noté N.A.M.), considérons tout d'abord un tourbillon bidimensionnel. L'idée de base est de constater que, dans ce cas, les lignes de courant sont fermées au voisinage du centre du tourbillon si l'on se place dans le repère se déplaçant avec le tourbillon.

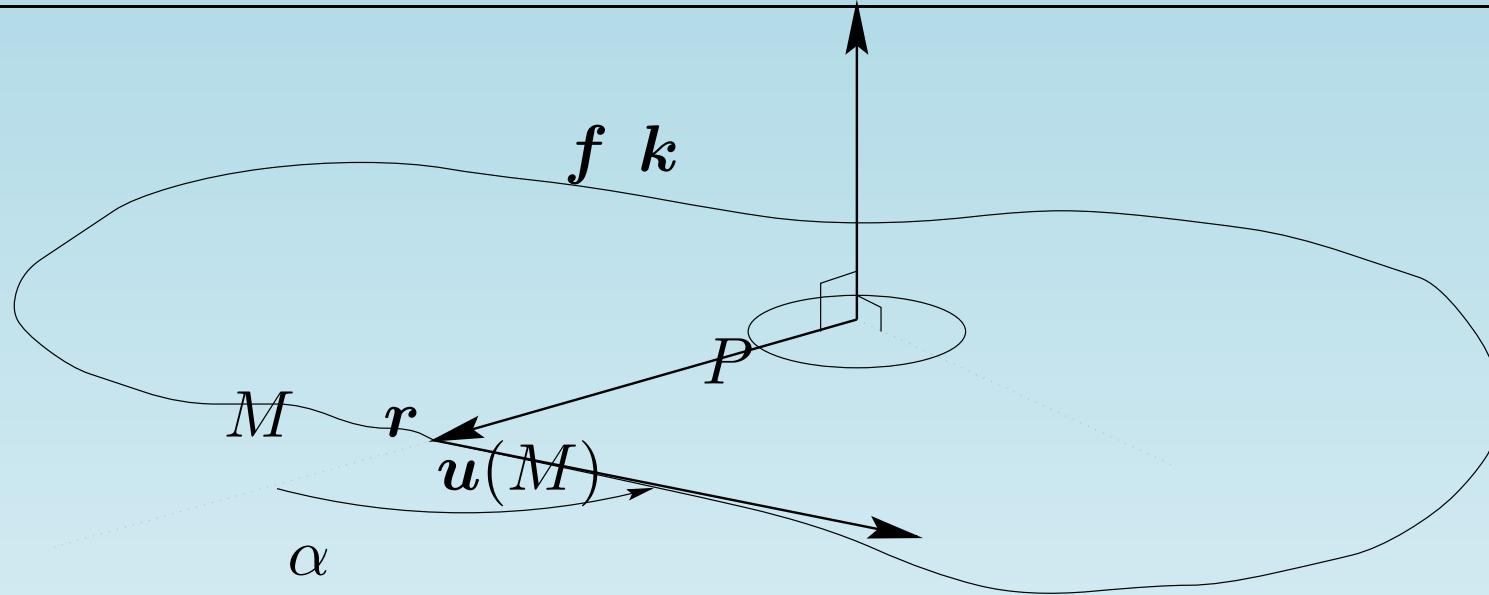


Figure 1: Schéma du calcul du moment angulaire normé.

Sur la figure 1, r et α désignent respectivement le rayon vecteur et l'angle entre r et la vitesse u prise au point M . Soit P un point à l'intérieur du tourbillon. Si M se déplace le long d'une ligne de courant, la fonction $\sin \alpha$, qui est la valeur algébrique du produit vectoriel $r \times u$ normé, garde toujours le même signe. Sa valeur moyenne est soit positive soit négative suivant le sens de rotation du tourbillon mais non nulle. Cette valeur tend vers zéro en dehors du tourbillon.

On généralise cette idée pour un écoulement tridimensionnel en définissant la fonction f comme :

$$f(\mathbf{x}_p) = \frac{1}{V} \int_{\mathbf{x} \in V} \frac{(\mathbf{x} - \mathbf{x}_p) \times \mathbf{u}(\mathbf{x})}{|\mathbf{x} - \mathbf{x}_p| |\mathbf{u}(\mathbf{x})|} d\mathbf{x} \quad (1)$$

Dans les cas d'**écoulements bidimensionnels** et des mesures expérimentales se faisant dans des plans, seule la composante perpendiculaire au plan de l'écoulement ou au plan de mesure est non nulle et est donc utilisée pour localiser les tourbillons. Le signe de cette composante donnera le sens de rotation des tourbillons repérés ; nous noterons cette fonction f_{2D} .

Dans le cas d'**écoulements tridimensionnels**, il y a deux cas de figure. Si l'on connaît a priori l'orientation privilégiée des tourbillons (par exemple pour les "streaks" dans les écoulements en canal) on peut ne calculer que la composante du moment angulaire normé concernée. Si l'on ne sait rien de l'écoulement, en revanche, alors le calcul de la norme du N.A.M. s'impose avec perte d'information sur le sens de rotation du tourbillon. On notera cette fonction f_{3D} .

Dans la pratique, les champs de vitesse sont bien sûr discrêts et l'on n'utilise pas directement l'équation 1. La définition suivante est adoptée :

$$\mathbf{f}(P) = \frac{1}{(2N + 1)^3 - 1} \sum_i \frac{\mathbf{r}_i \times \mathbf{u}(M_i)}{|\mathbf{r}_i| |\mathbf{u}(M_i)|}. \quad (2)$$

Ici, le point P parcourt tout le domaine de calcul mais le N.A.M. est calculé sur un nombre restreint de points autour de ce point. On définit N comme le nombre de "couches" de points constituant le domaine restreint ; il y a donc $(2N + 1)^3 - 1$ points dans ce domaine (on exclut le point P où la fonction présente une singularité). La figure 2 montre un exemple de sous domaine à quatre couches.

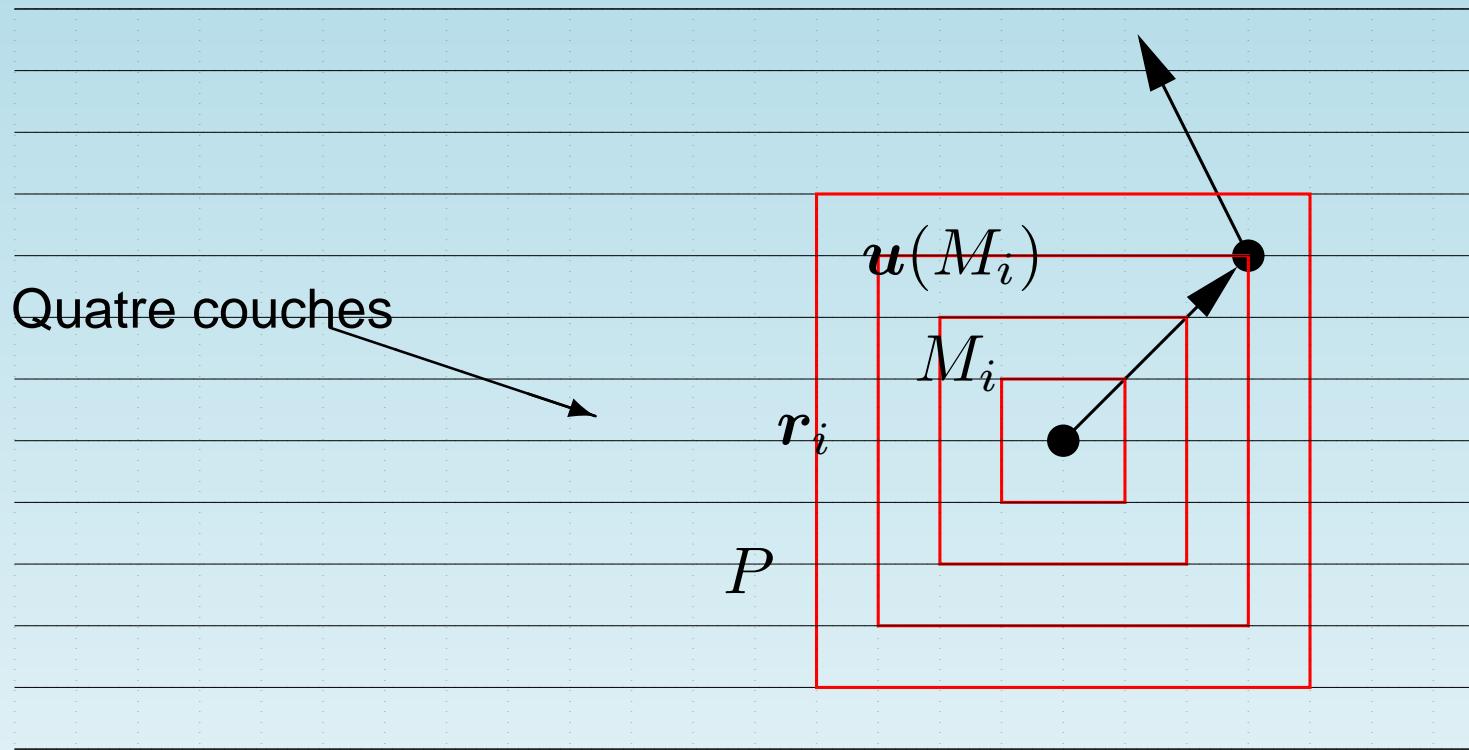


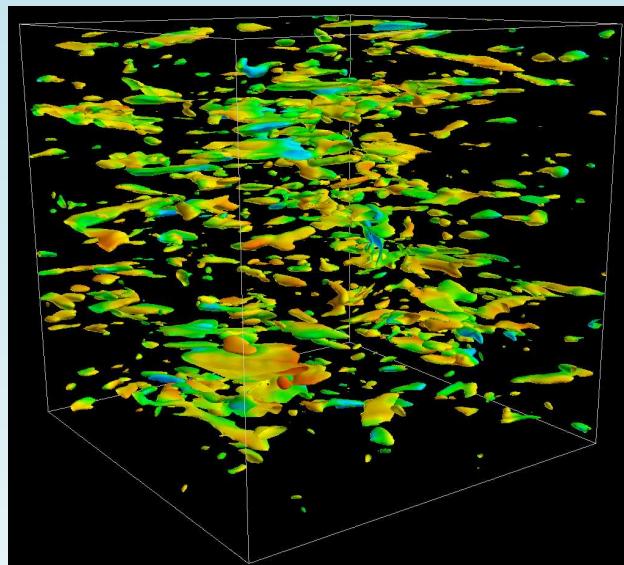
Figure 2: Illustration du domaine de calcul discr  t et r  duit du moment angulaire norm  . Ici, $N = 4$.

Turbulence homogène

Calculs de Lukas Liechtenstein (2005)

Rotating stratified turbulence

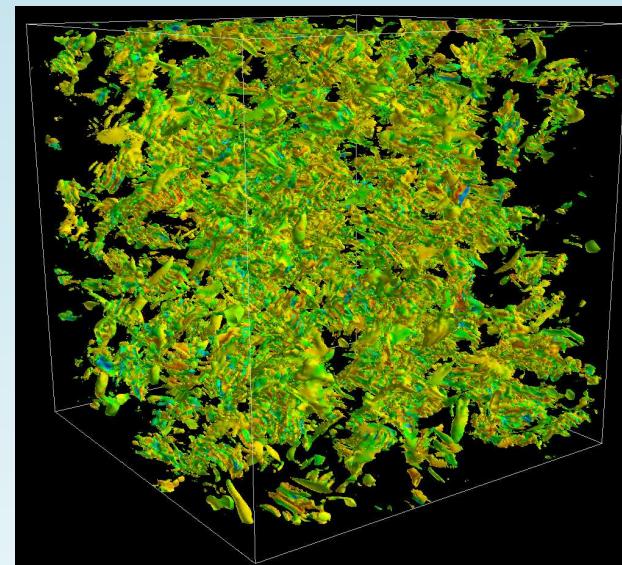
DNS, Res: 256^3 , $Re_\lambda \approx 150$



STRATIFIED

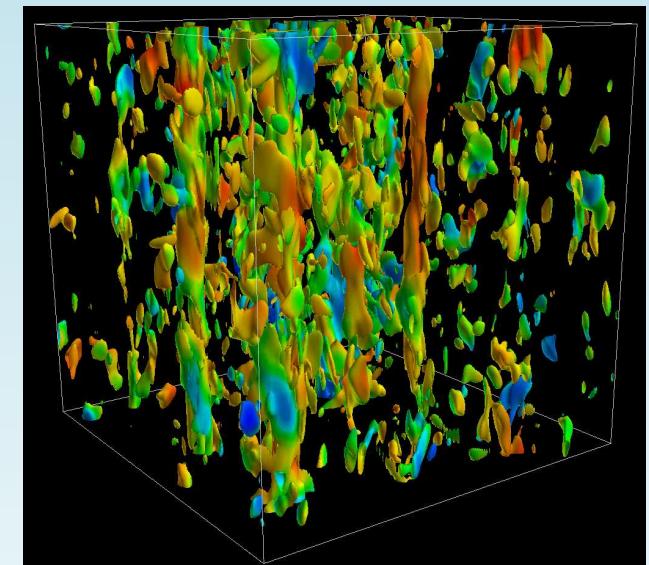
Brunt-Vaisala Frequency N

$$\alpha < 1$$



$2\Omega = N$

$$\alpha = \frac{2\Omega}{N} = 1$$



ROTATING

Vertical System Rotation Ω

$$\alpha > 1$$

Boussinesq approximation

Divergence free Navier-Stokes with body forces

$$\frac{\partial \mathbf{u}}{\partial t} - \frac{1}{Re} \nabla^2 \mathbf{u} = -\nabla(p^* + \frac{1}{2} \mathbf{u}^2) + \mathbf{u} \times \nabla \times \mathbf{u} - 2\Omega \mathbf{n}_3 \times \mathbf{u} - b \mathbf{n}_3$$

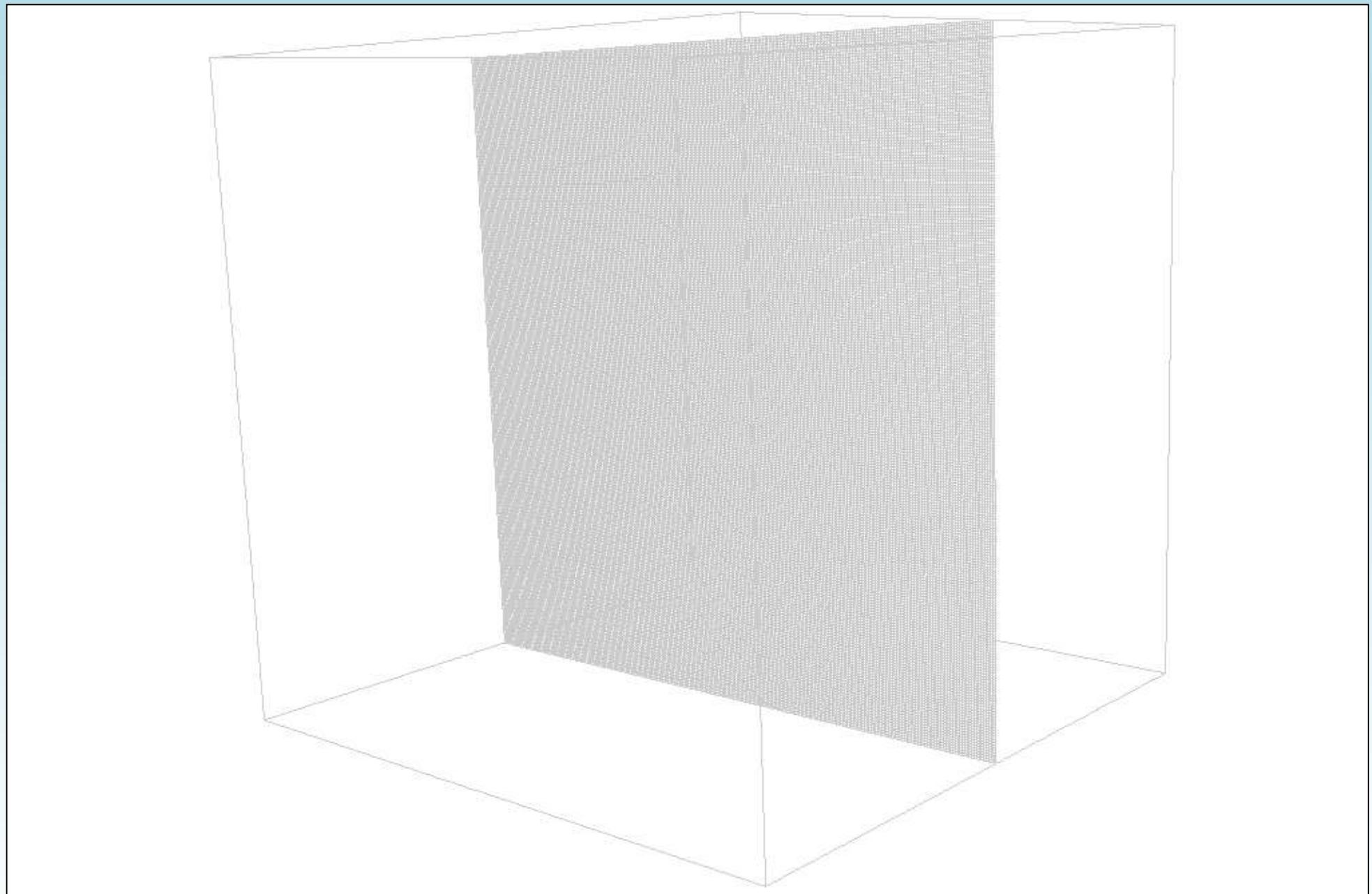
$$\nabla \cdot \mathbf{u} = 0$$

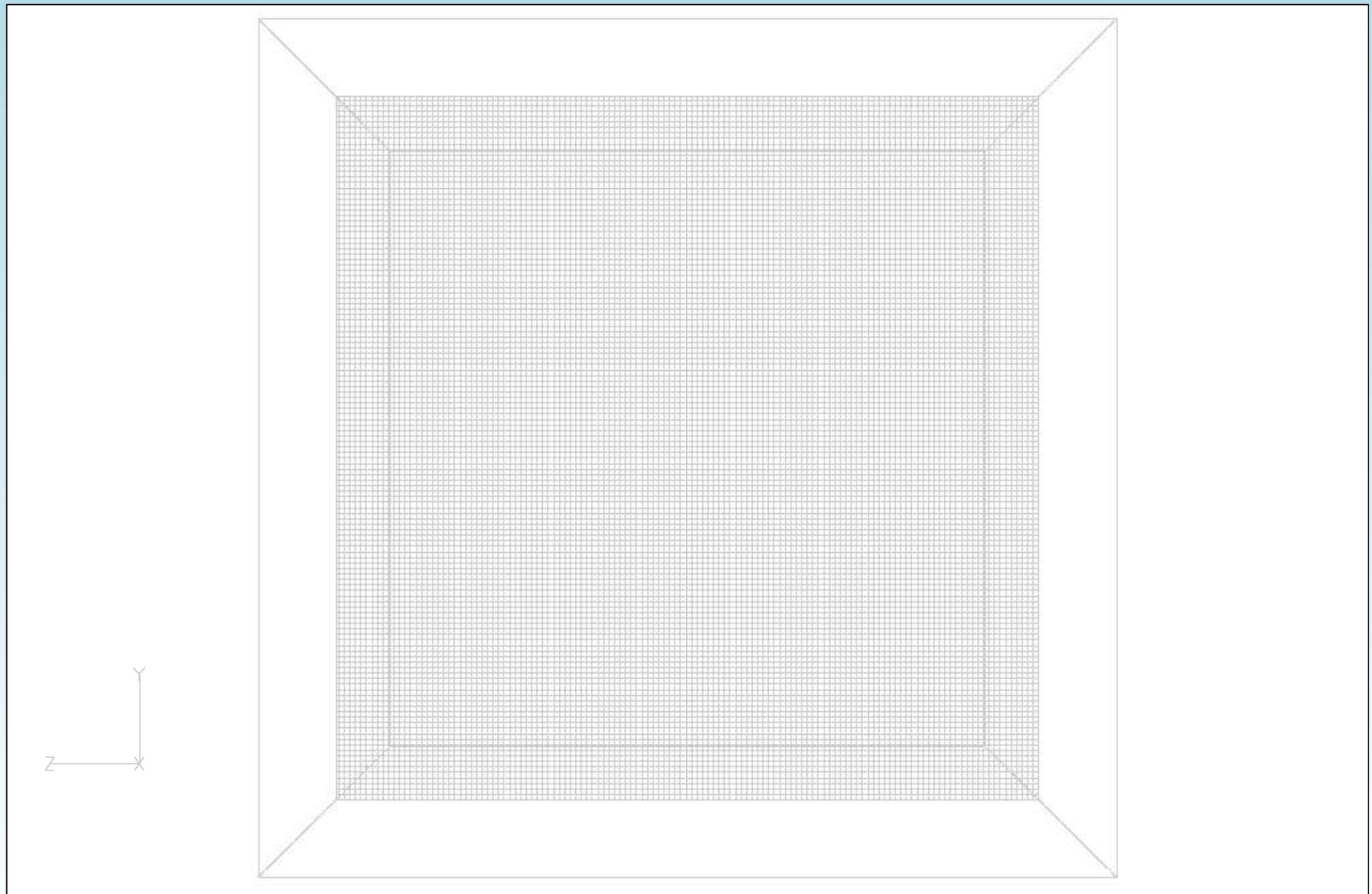
Equation for density fluctuations

$$\frac{\partial b}{\partial t} - \kappa \nabla^2 b = -(\mathbf{u} \times \nabla) \cdot b + N^2 (\mathbf{n}_3 \cdot \mathbf{u})$$

⇒ Méthode de projection sur une base de polynômes de Fourier (pseudo-spectrale, DNS). Résolution courante $512^3 = 134$ millions de points ! ($\times N$ champs)

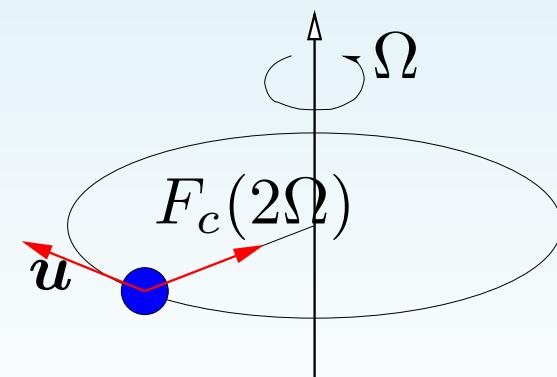
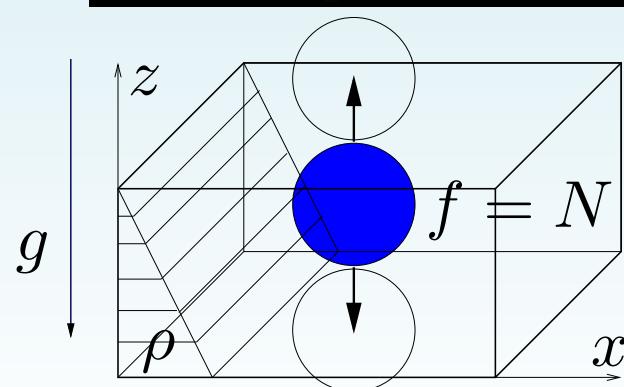
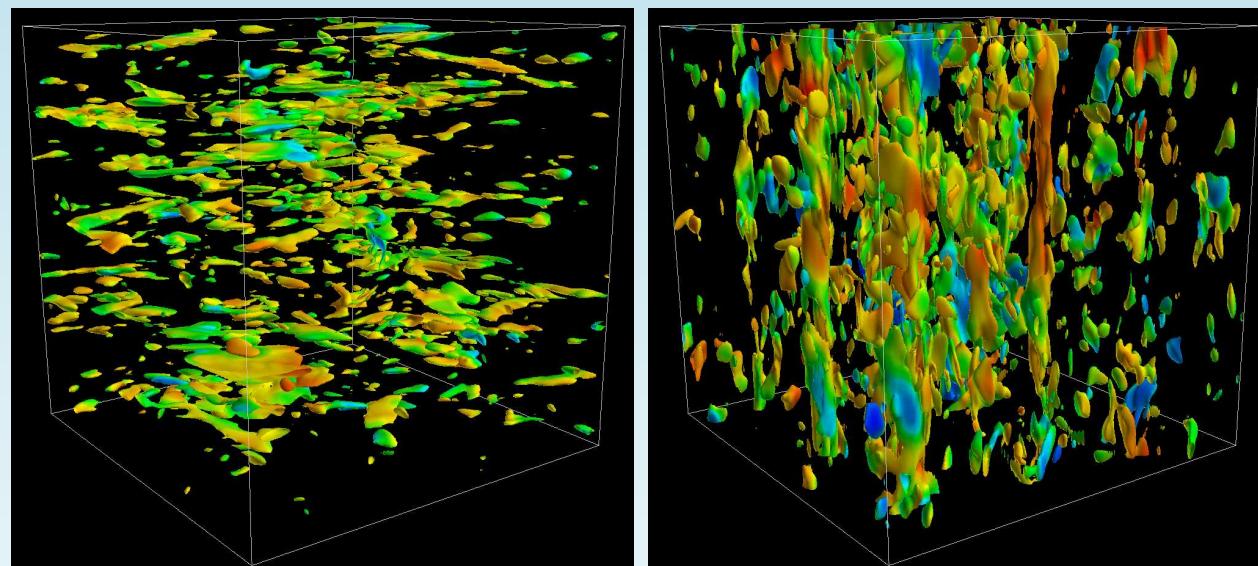
Calculs sur supercalculateur Nec SX6. Visualisations sur bi-opteron (8Go de RAM pour voir 256^3 ...)





Oscillating Mechanisms

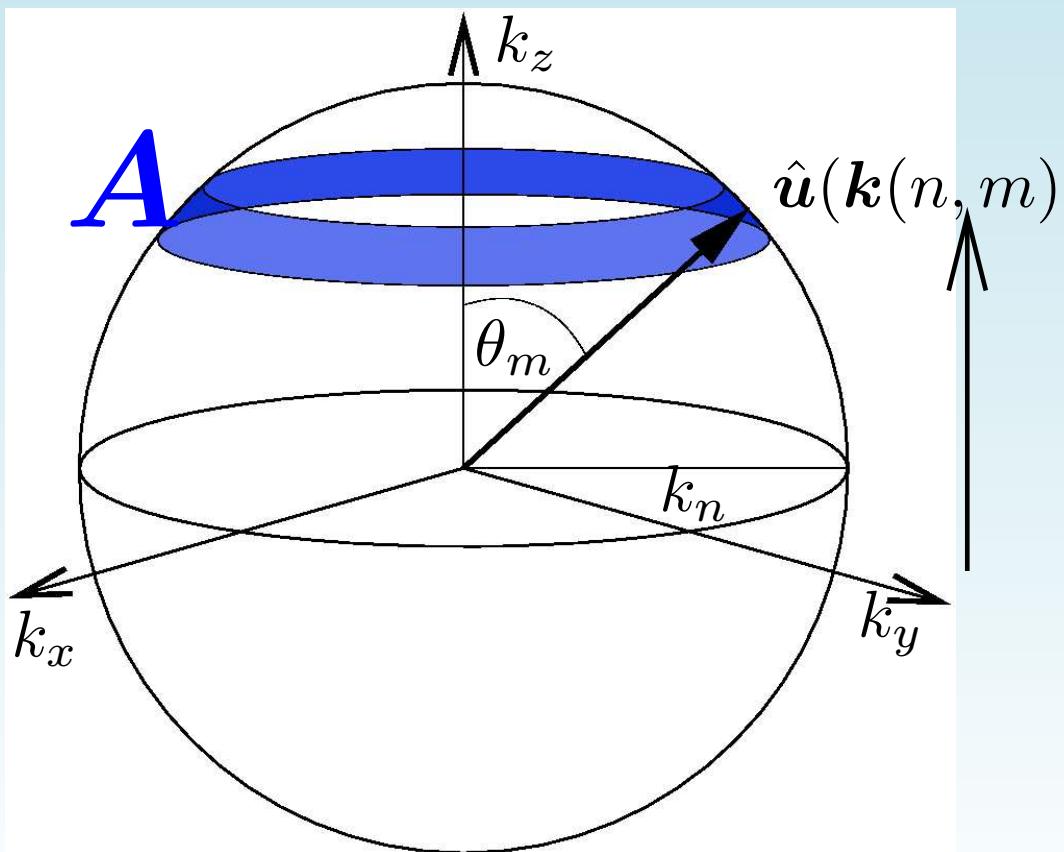
DNS, Res: 256^3 , $Re_\lambda \approx 150$



Dispersion relation: $\sigma = \sqrt{N^2 \sin^2 \theta + (2\Omega)^2 \cos^2 \theta}$

Statistics: Angular Spectra

$$E(k_n, \theta_m) \propto \sum_{\mathbf{k} \in A} \hat{u}^*(\mathbf{k}) \cdot \hat{u}(\mathbf{k})$$

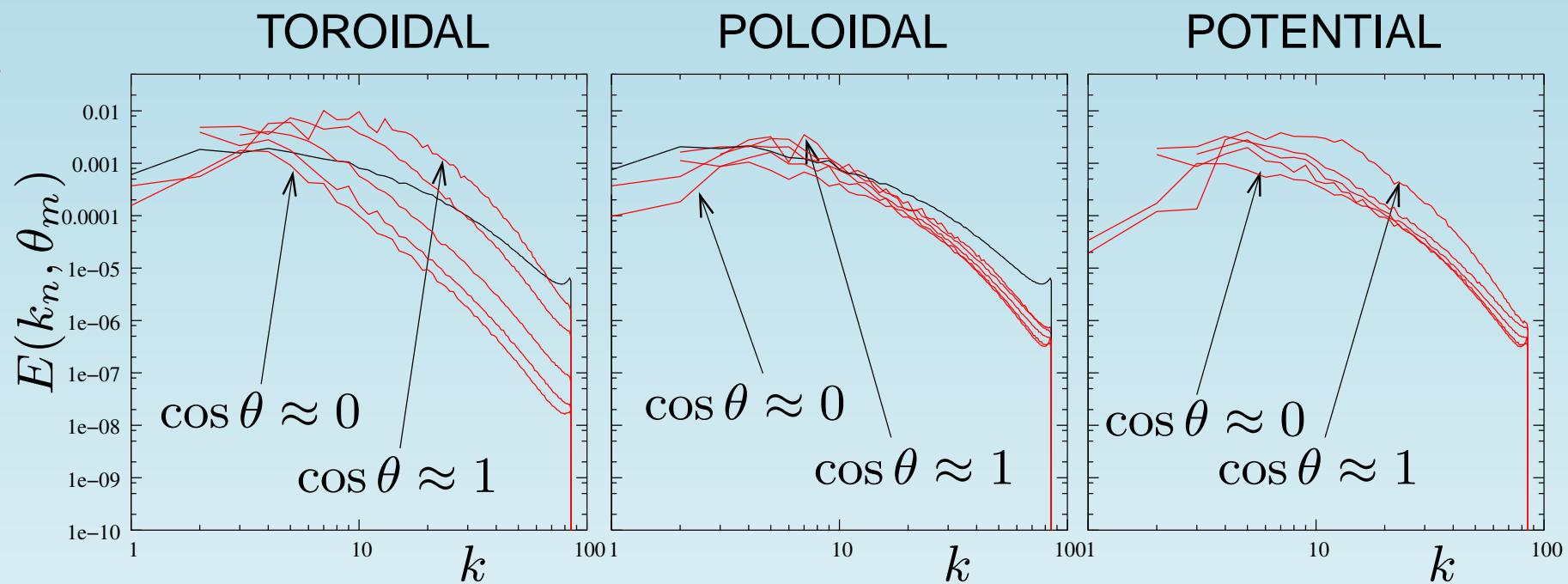


$\alpha = 0.1$

DNS

$$\frac{tN}{2\pi} = 20$$

$$\frac{t\Omega}{\pi} = 2$$

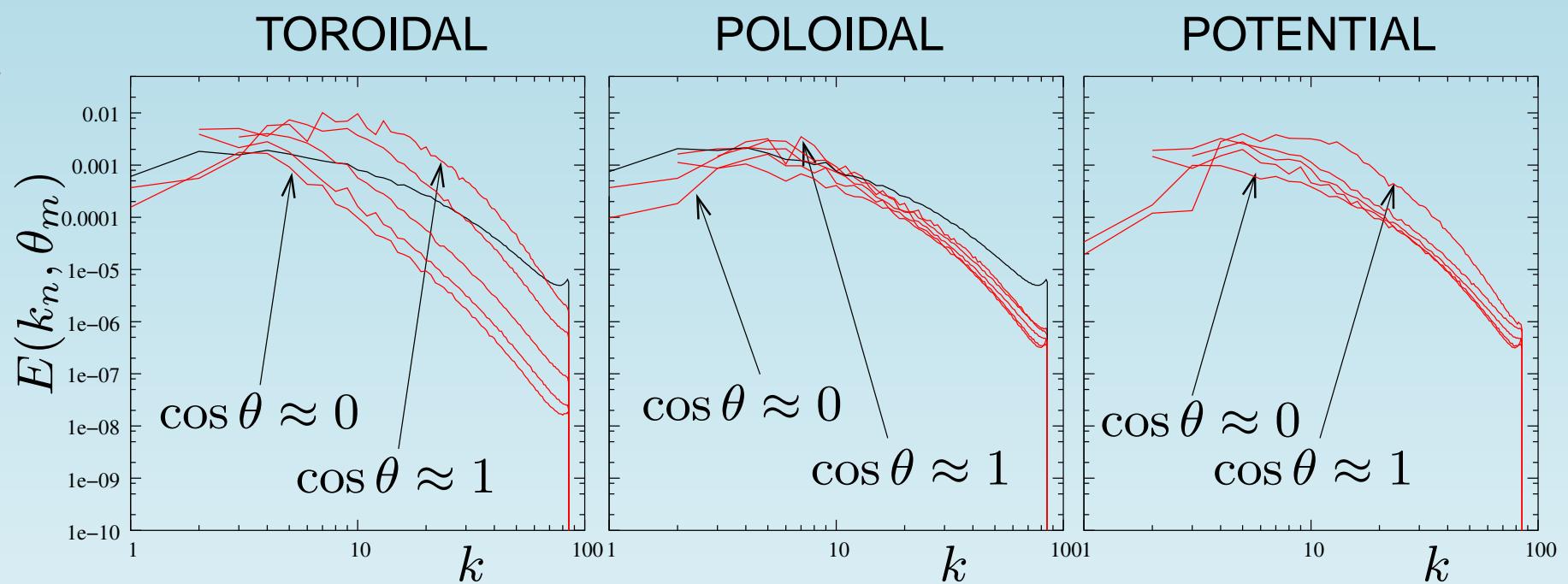


$\alpha = 0.1$

DNS

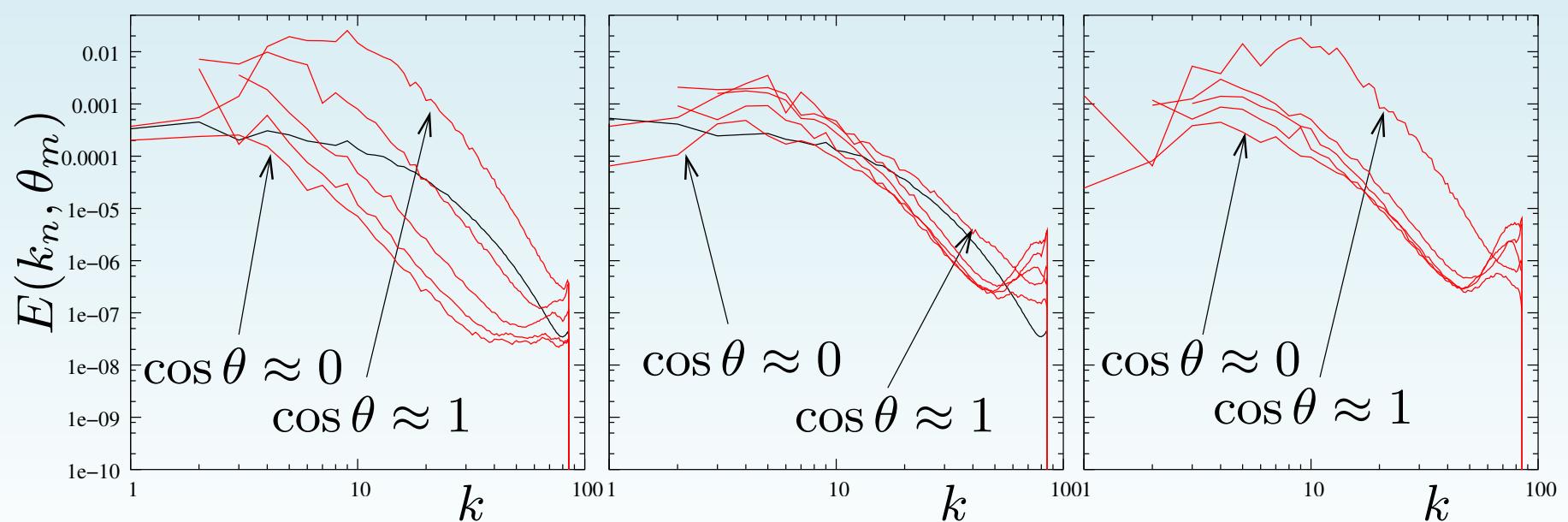
$$\frac{tN}{2\pi} = 20$$

$$\frac{t\Omega}{\pi} = 2$$



$$\frac{tN}{2\pi} = 80$$

$$\frac{t\Omega}{\pi} = 8$$

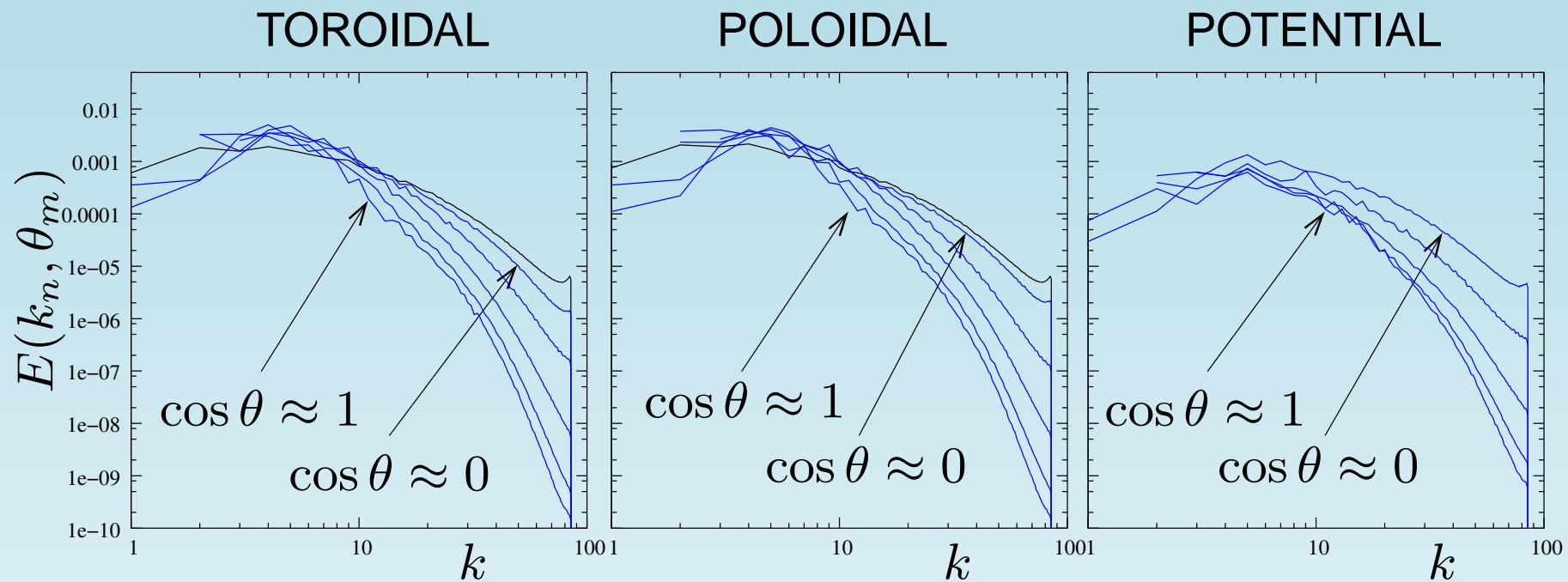


$\alpha = 10$

DNS

$$\frac{tN}{2\pi} = 2$$

$$\frac{t\Omega}{\pi} = 20$$

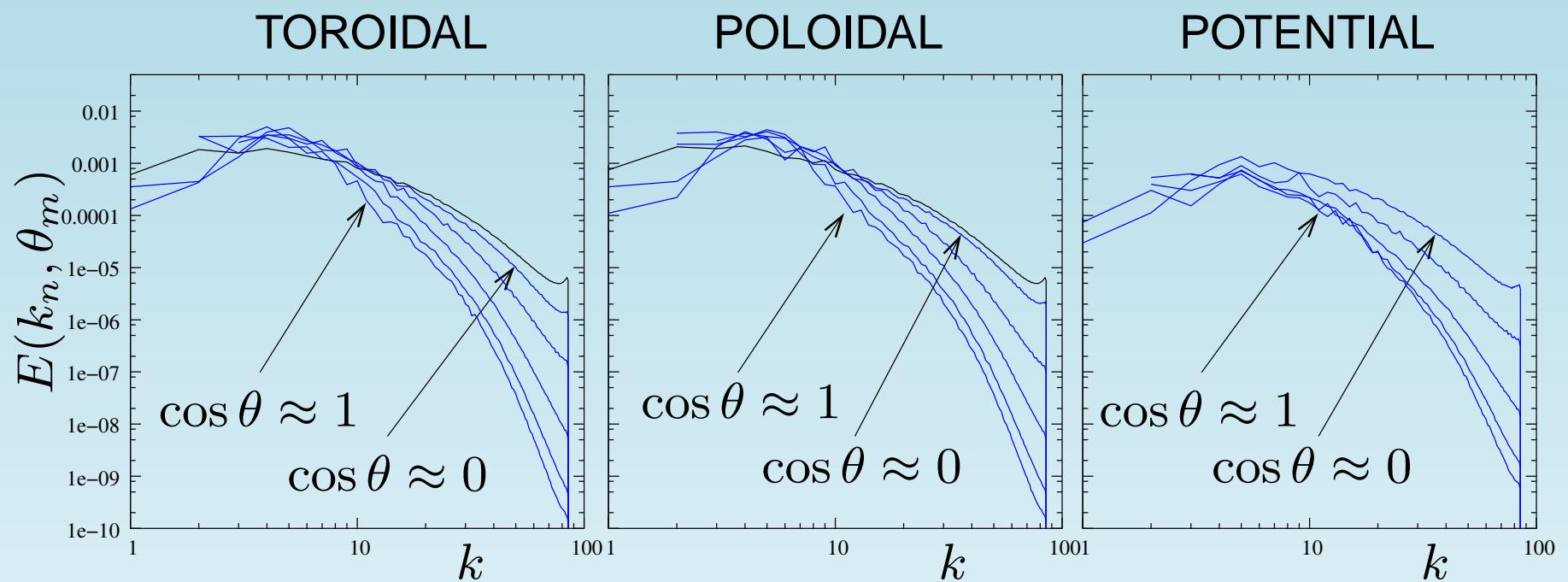


$\alpha = 10$

DNS

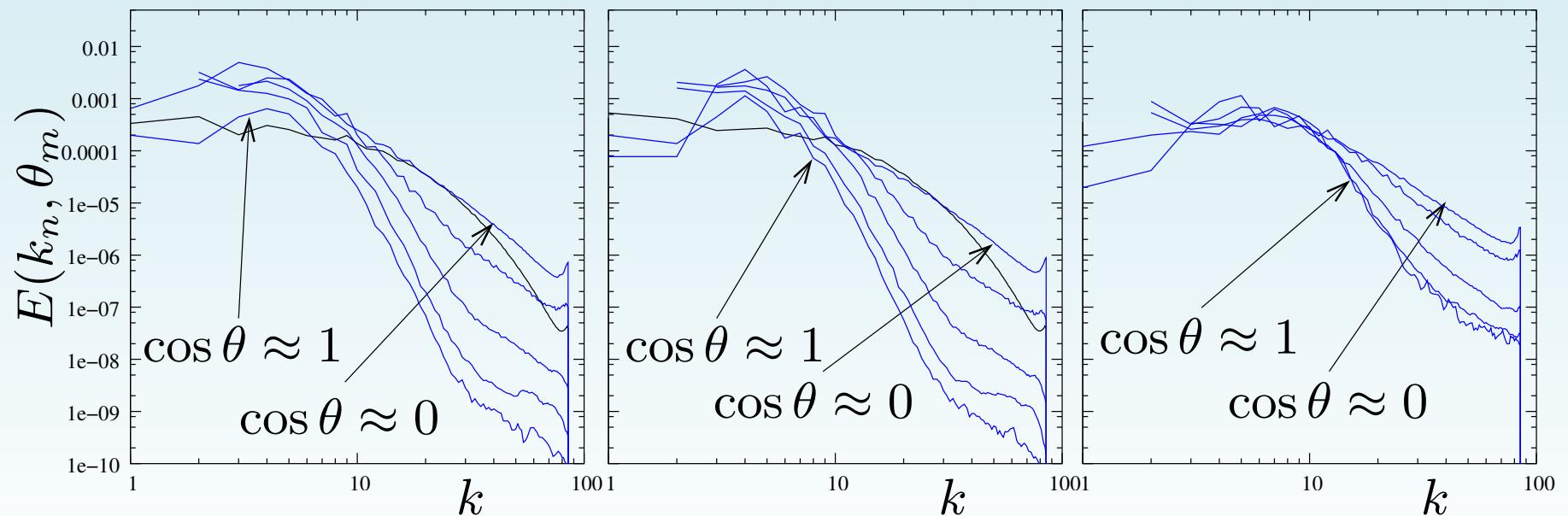
$$\frac{tN}{2\pi} = 2$$

$$\frac{t\Omega}{\pi} = 20$$

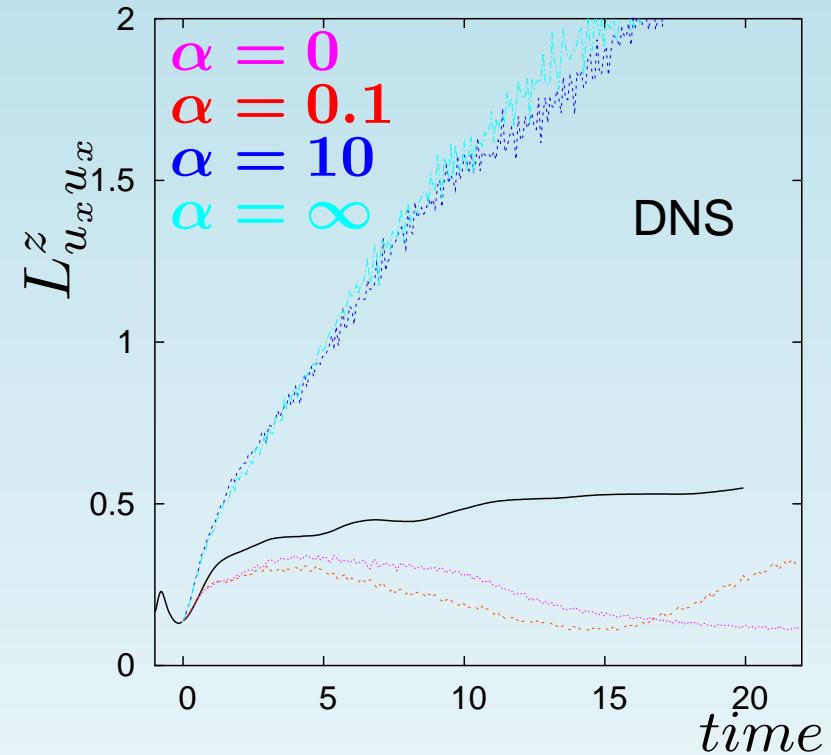
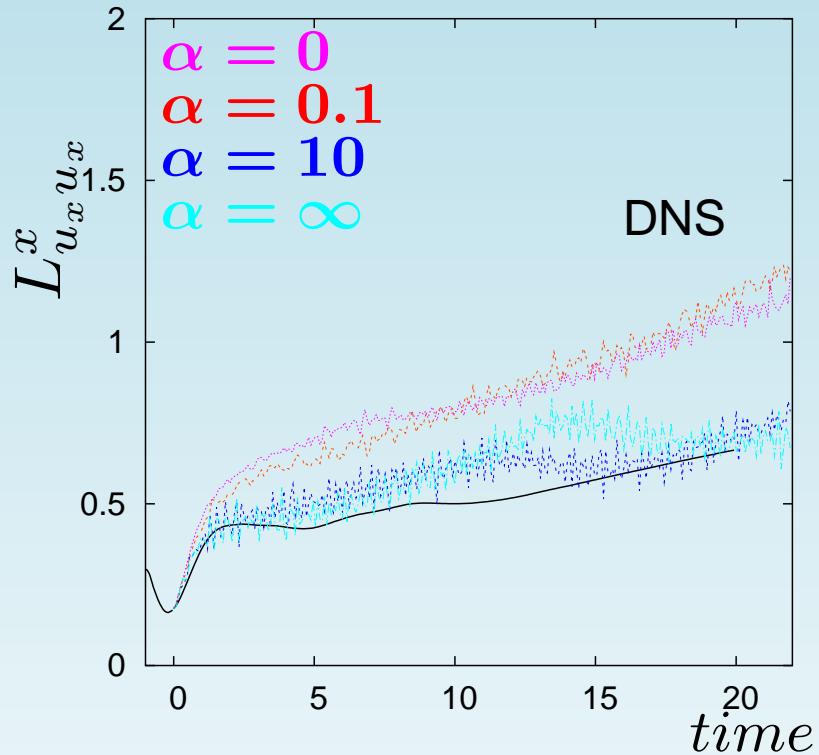


$$\frac{tN}{2\pi} = 11$$

$$\frac{t\Omega}{\pi} = 110$$



Statistics: Anisotropy in velocity correlations

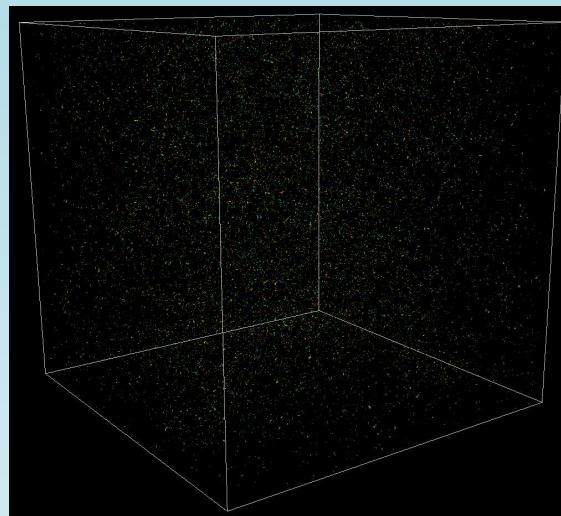
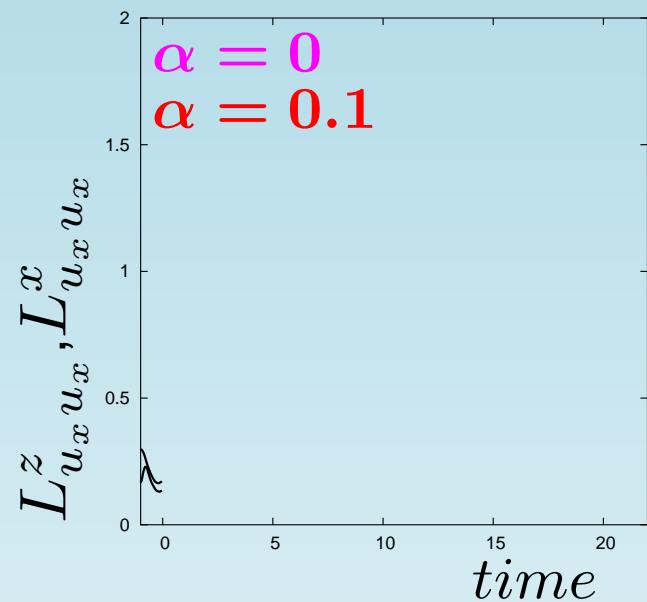


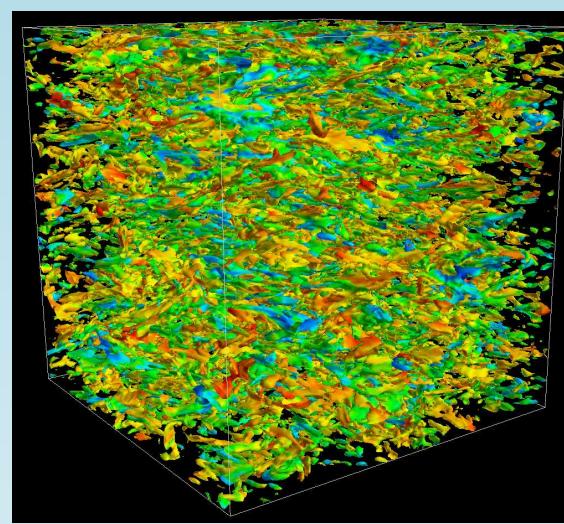
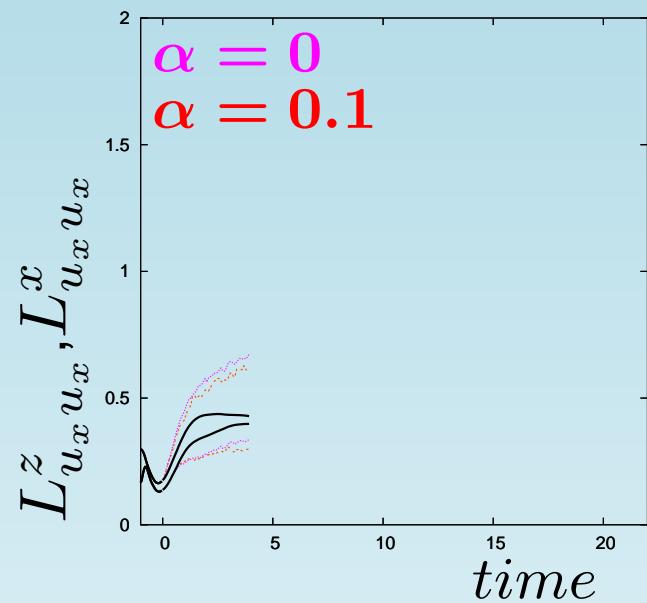
Horizontal correlation of horizontal velocity:

$$L_{u_x u_x}^x(t) = \left\langle \int_{-L/2}^{L/2} e^{-ik_x x} u_x^2(x, k_y, k_z) dx \right\rangle_{k_x=0}$$

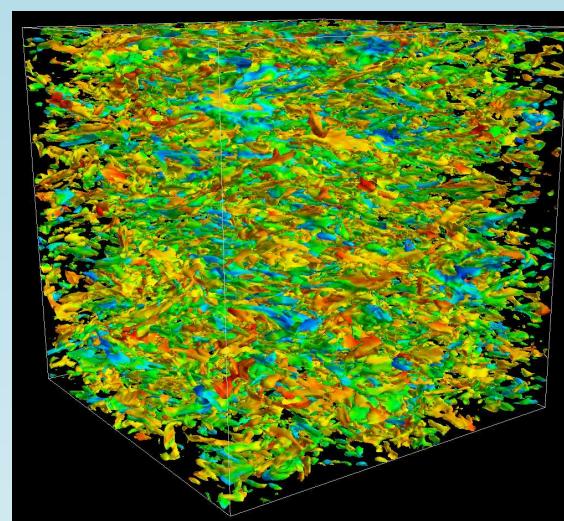
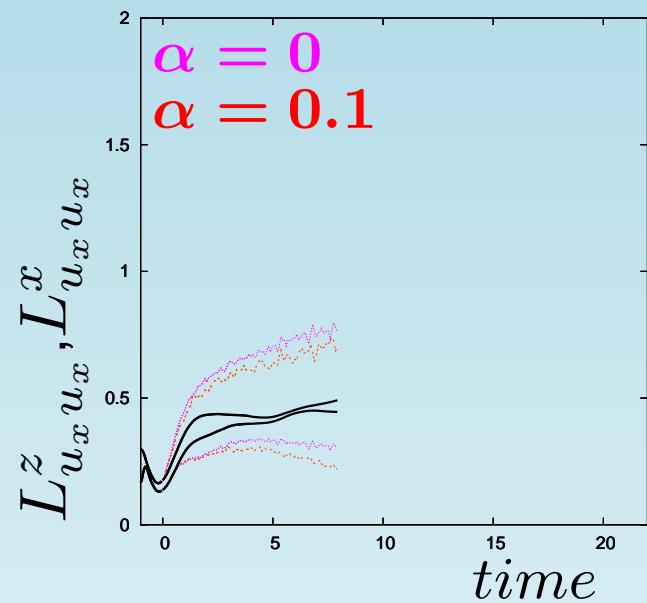
Vertical correlation of horizontal velocity:

$$L_{u_x u_x}^z(t) = \left\langle \int_{-L/2}^{L/2} e^{-ik_z z} u_x^2(k_x, k_y, z) dz \right\rangle_{k_z=0}$$

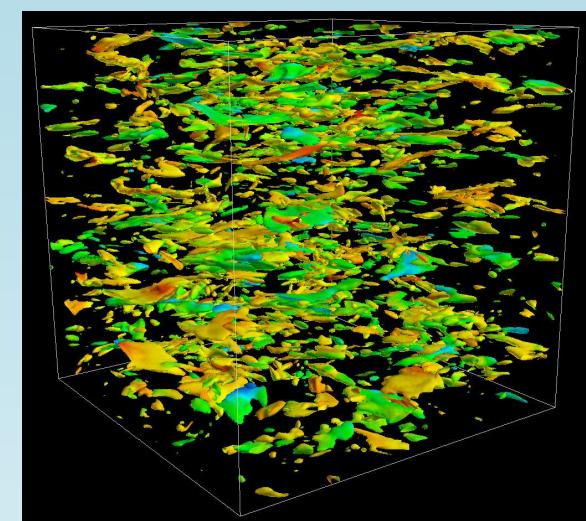




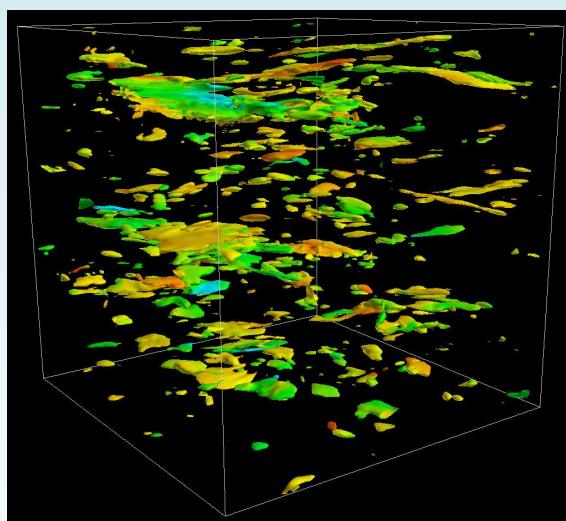
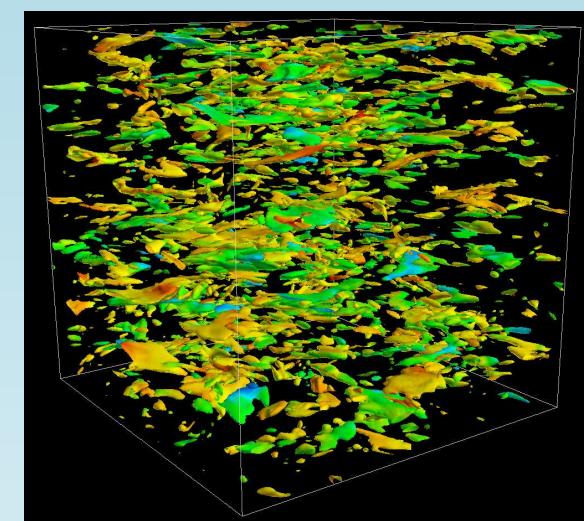
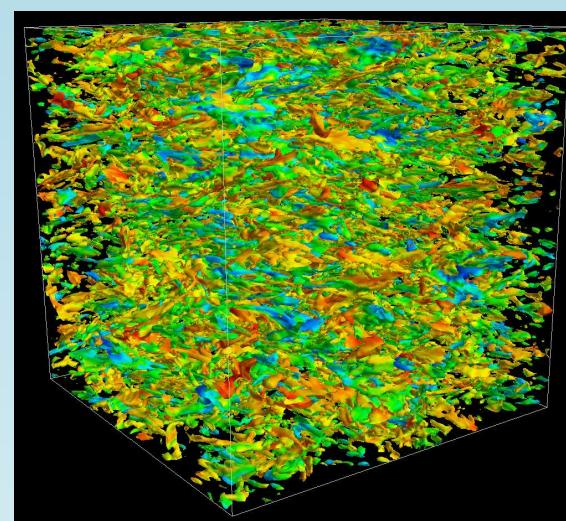
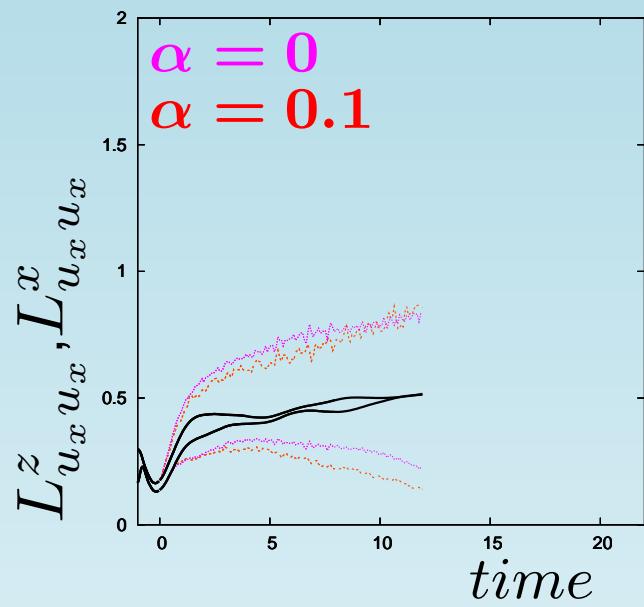
$t_{N/2\pi} = 20, t_{\Omega/\pi} = 2$



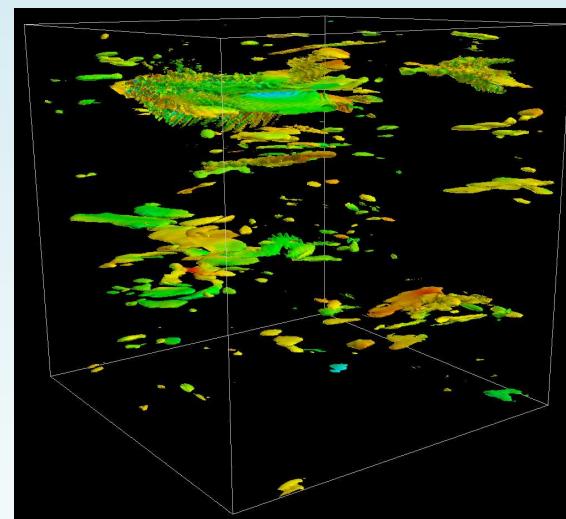
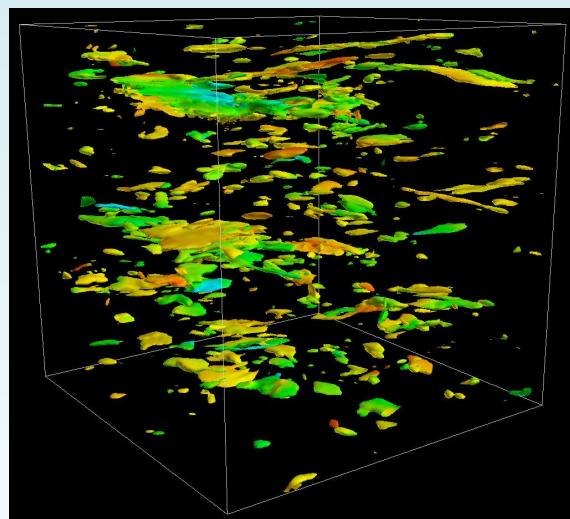
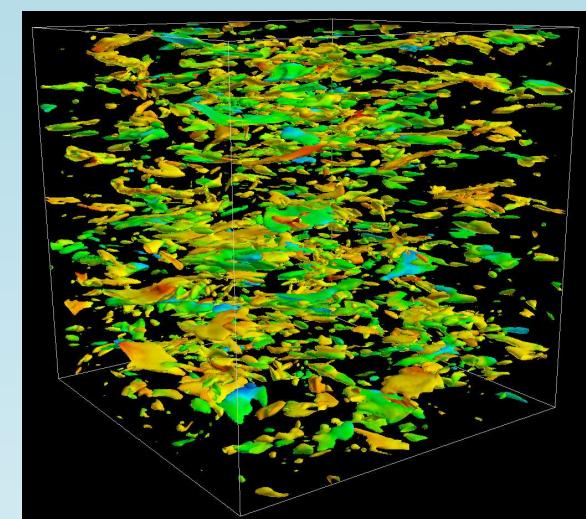
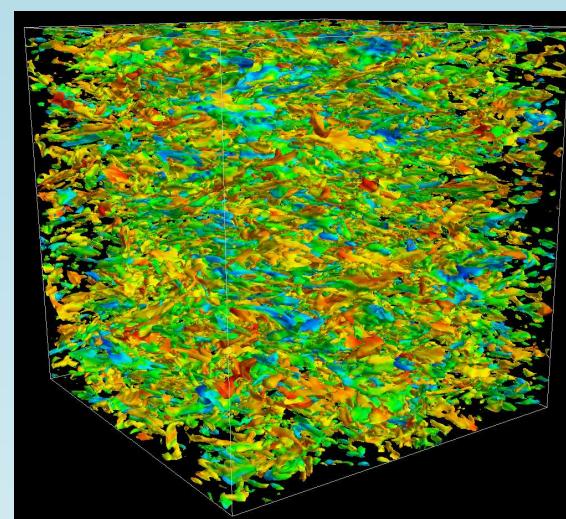
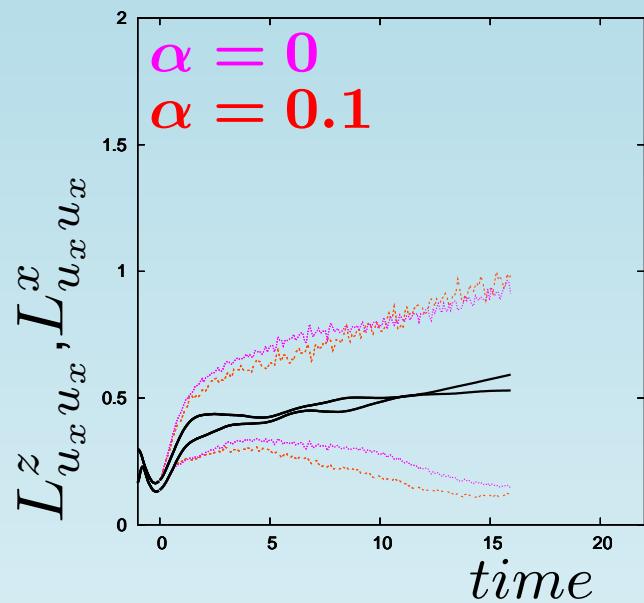
$t_{N/2\pi} = 20, t_{\Omega/\pi} = 2$



$t_{N/2\pi} = 40, t_{\Omega/\pi} = 4$

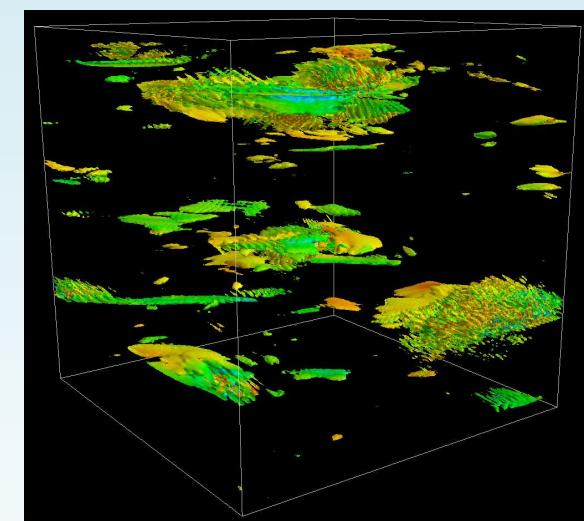
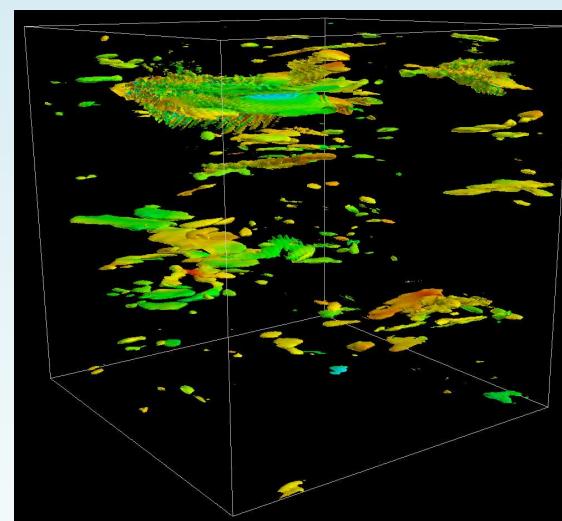
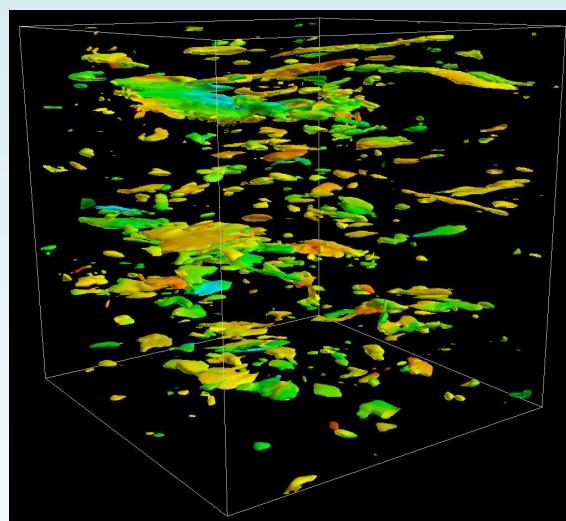
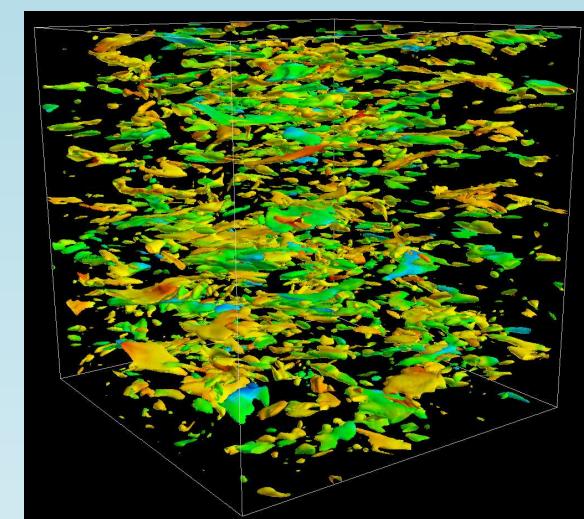
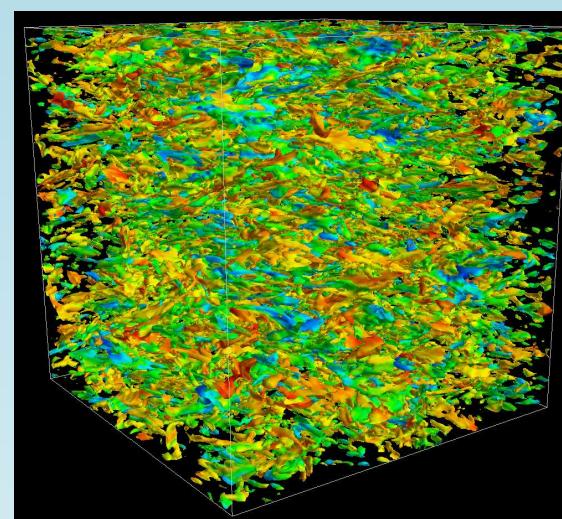
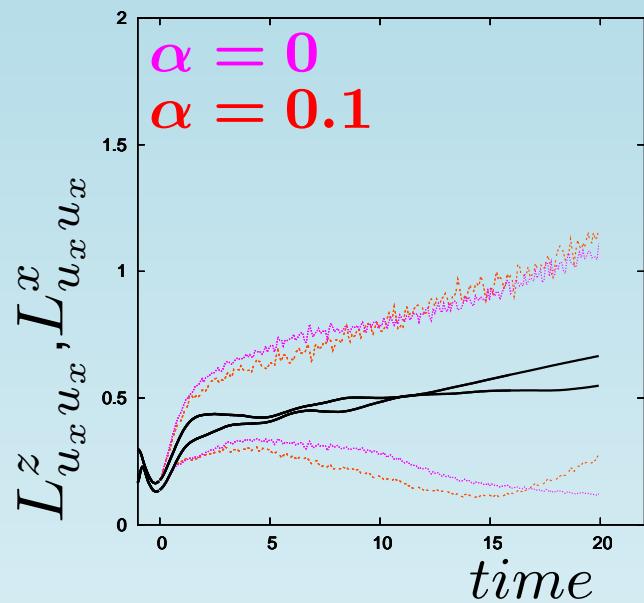


$t_{N/2\pi} = 60, t_{\Omega/\pi} = 6$



$t_{N/2\pi} = 60, t_{\Omega/\pi} = 6$

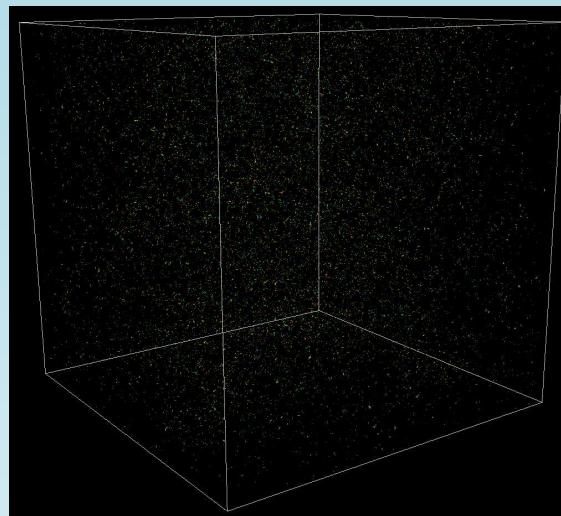
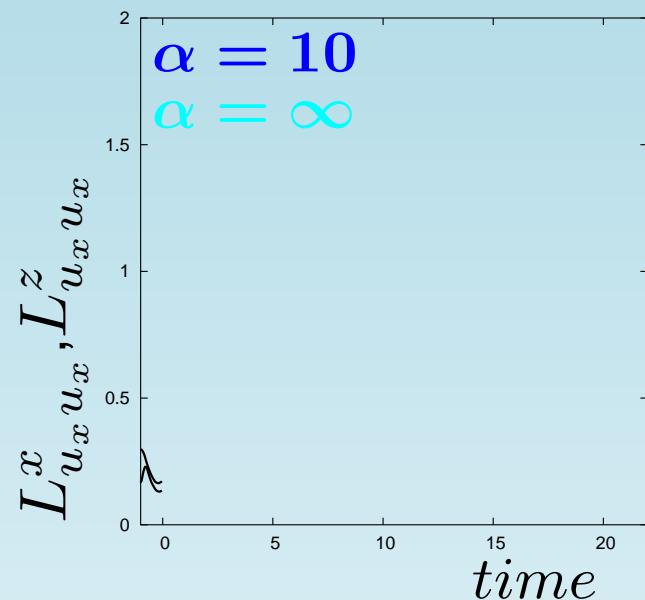
$t_{N/2\pi} = 80, t_{\Omega/\pi} = 8$

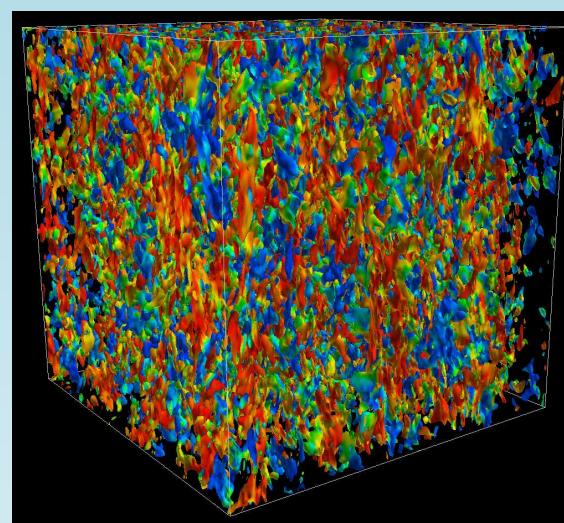
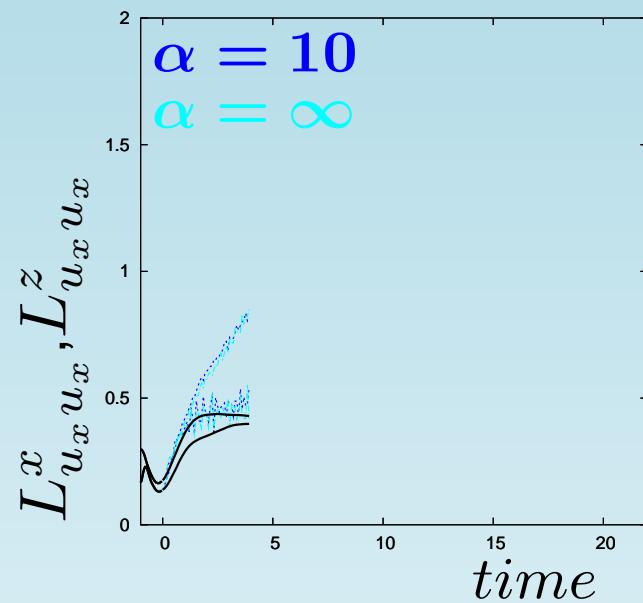


$t_{N/2\pi} = 60, t_{\Omega/\pi} = 6$

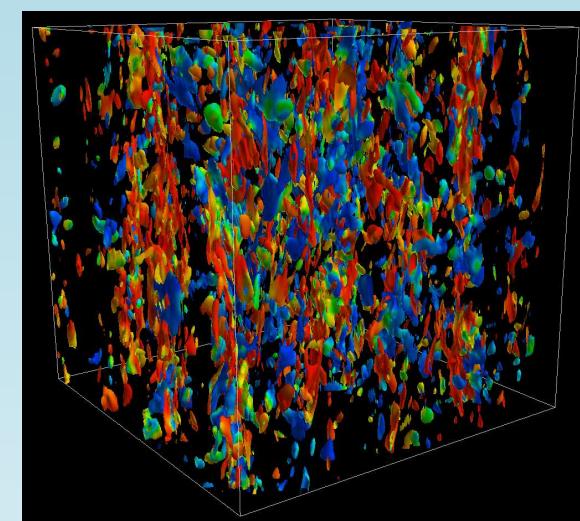
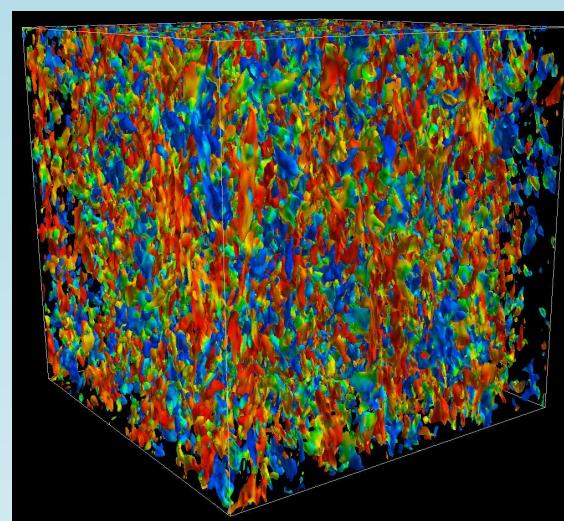
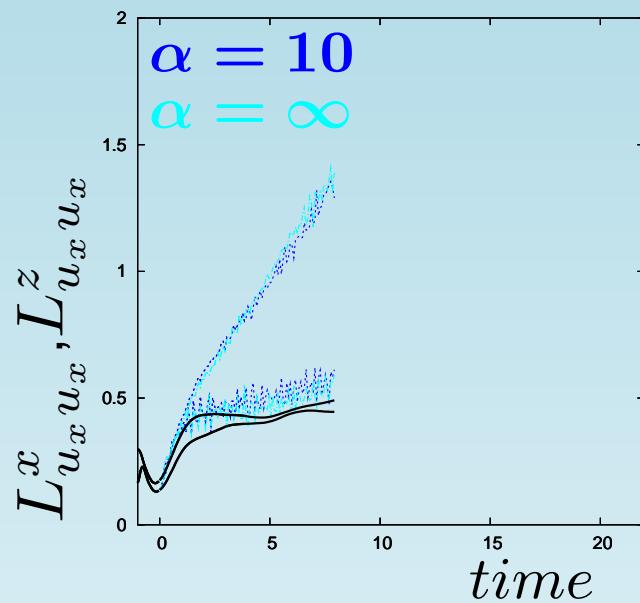
$t_{N/2\pi} = 80, t_{\Omega/\pi} = 8$

$t_{N/2\pi} = 100, t_{\Omega/\pi} = 10$

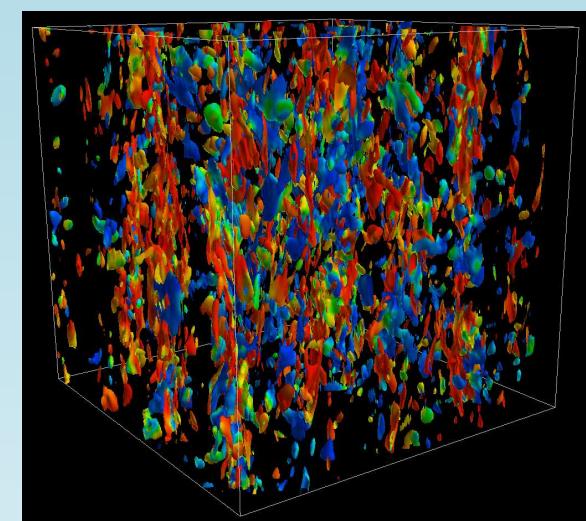
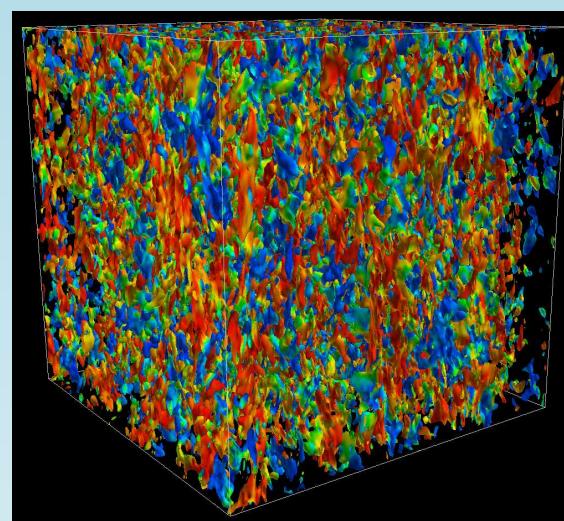
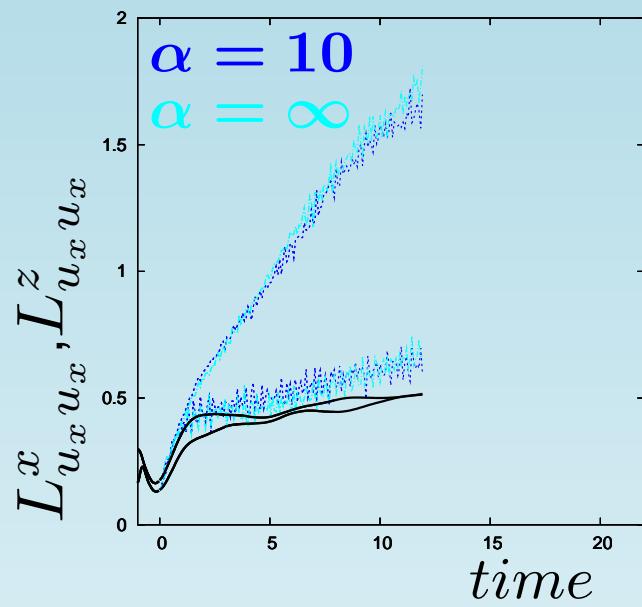




$$t_{N/2\pi} = 2, t_{\Omega/\pi} = 20$$

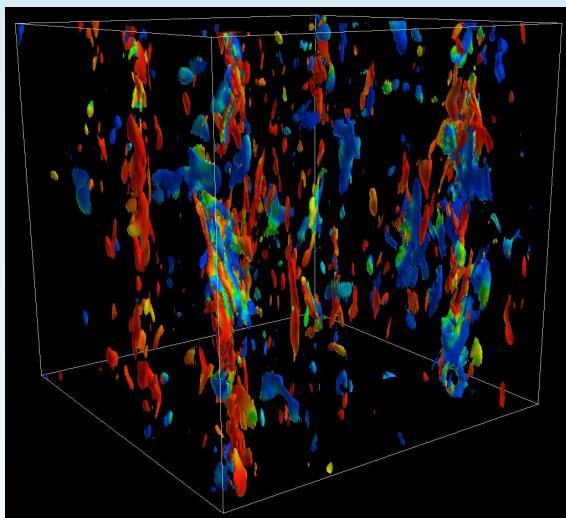


$t_{N/2\pi} = 2, t_{\Omega/\pi} = 20$ $t_{N/2\pi} = 4, t_{\Omega/\pi} = 40$

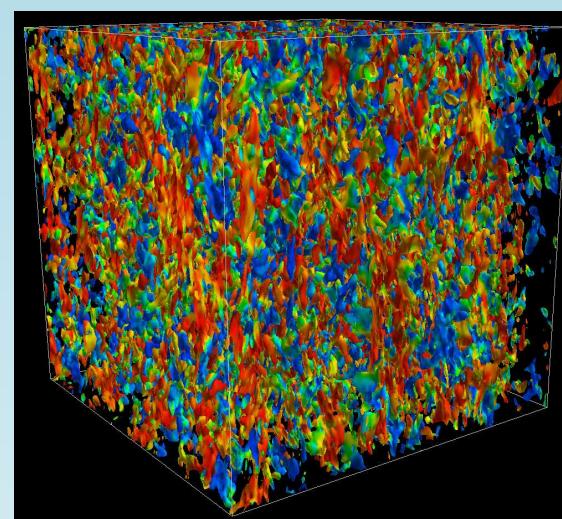
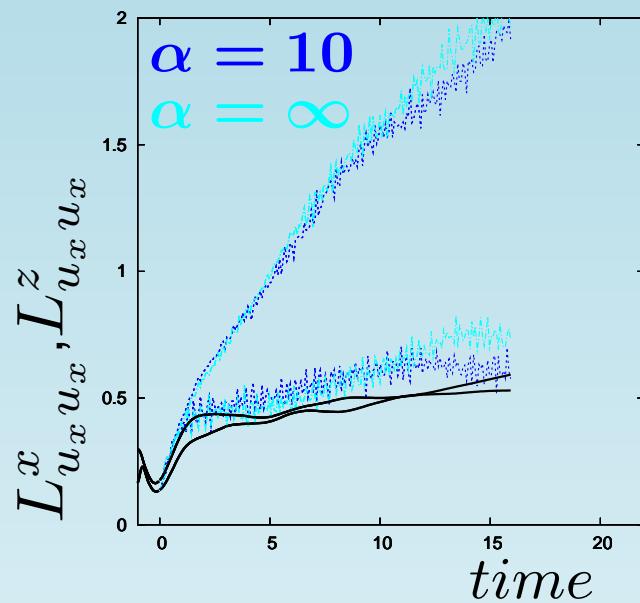


$t_{N/2\pi} = 2, t_{\Omega/\pi} = 20$

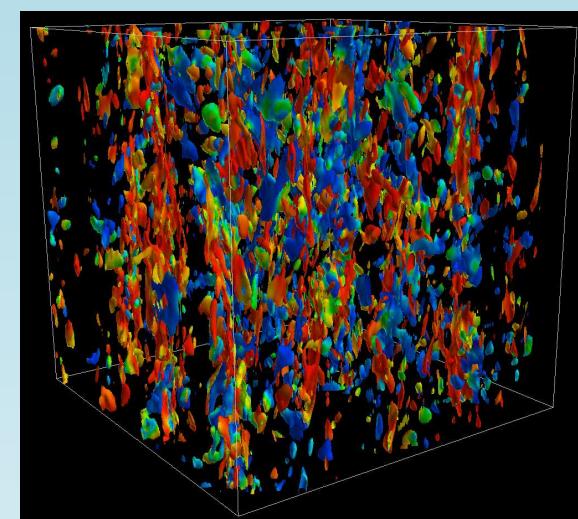
$t_{N/2\pi} = 4, t_{\Omega/\pi} = 40$



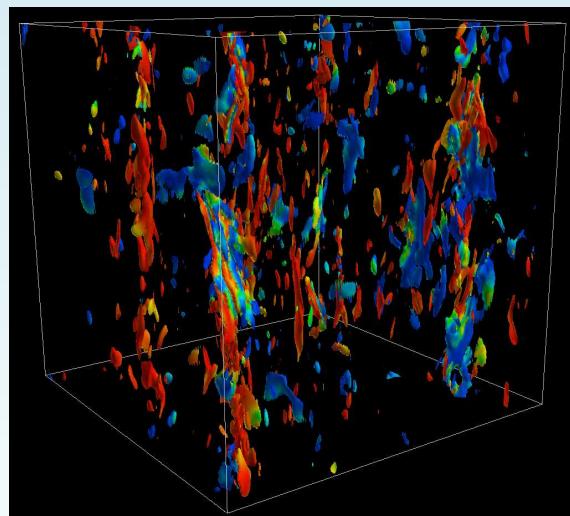
$t_{N/2\pi} = 6, t_{\Omega/\pi} = 60$



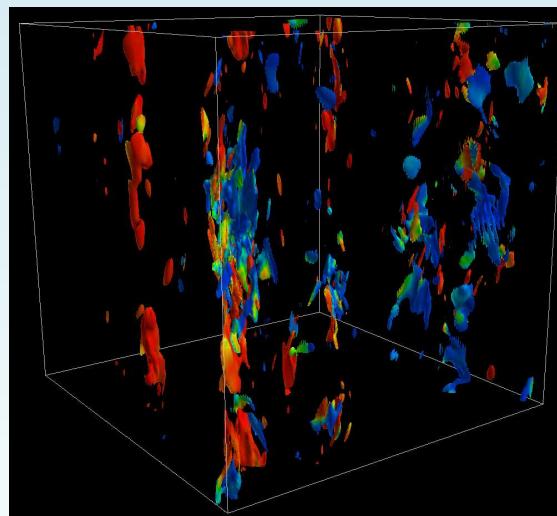
$t_{N/2\pi} = 2, t_{\Omega/\pi} = 20$



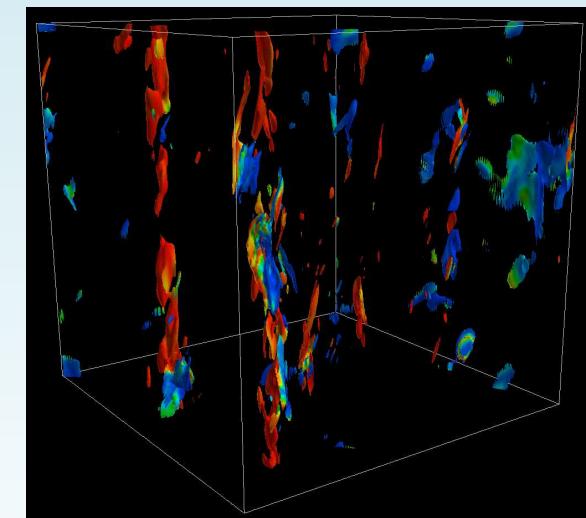
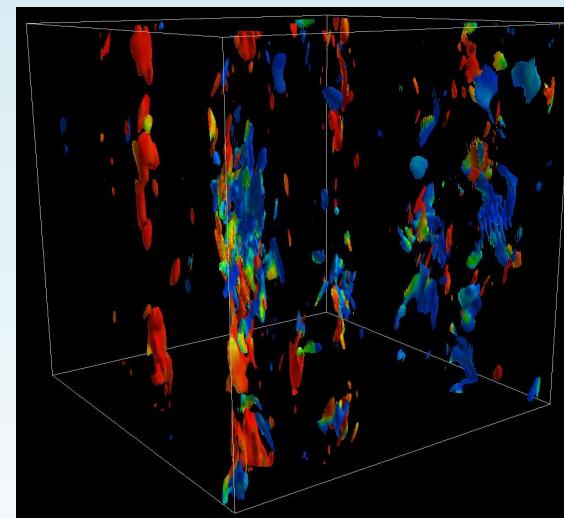
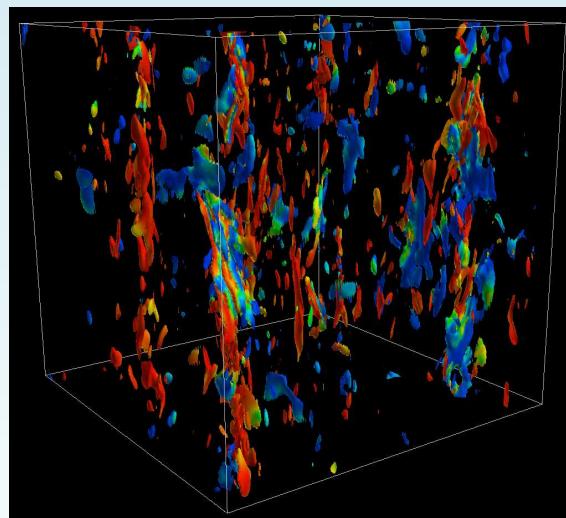
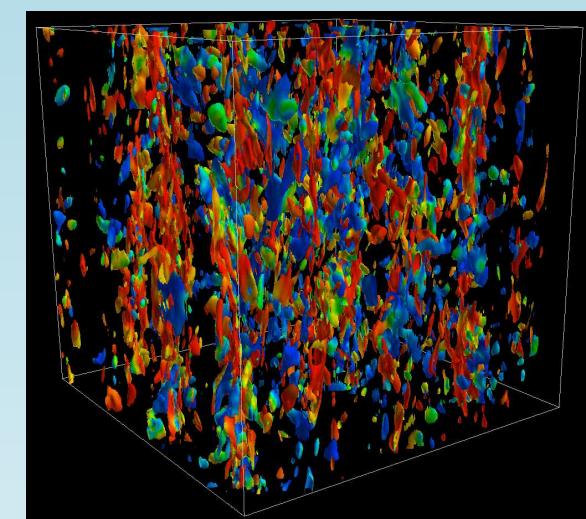
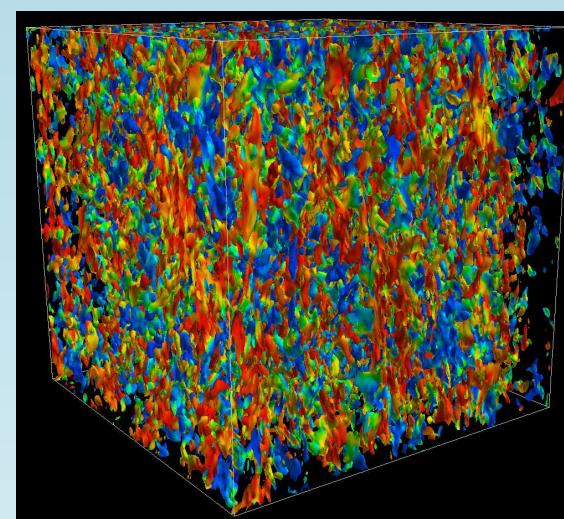
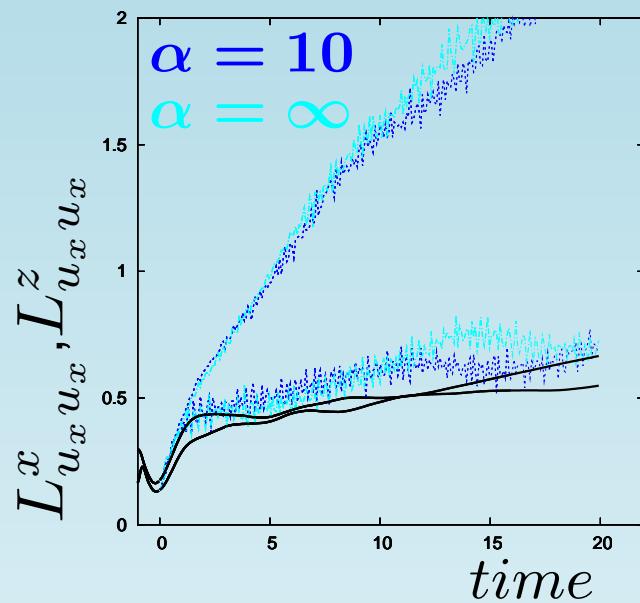
$t_{N/2\pi} = 4, t_{\Omega/\pi} = 40$



$t_{N/2\pi} = 6, t_{\Omega/\pi} = 60$



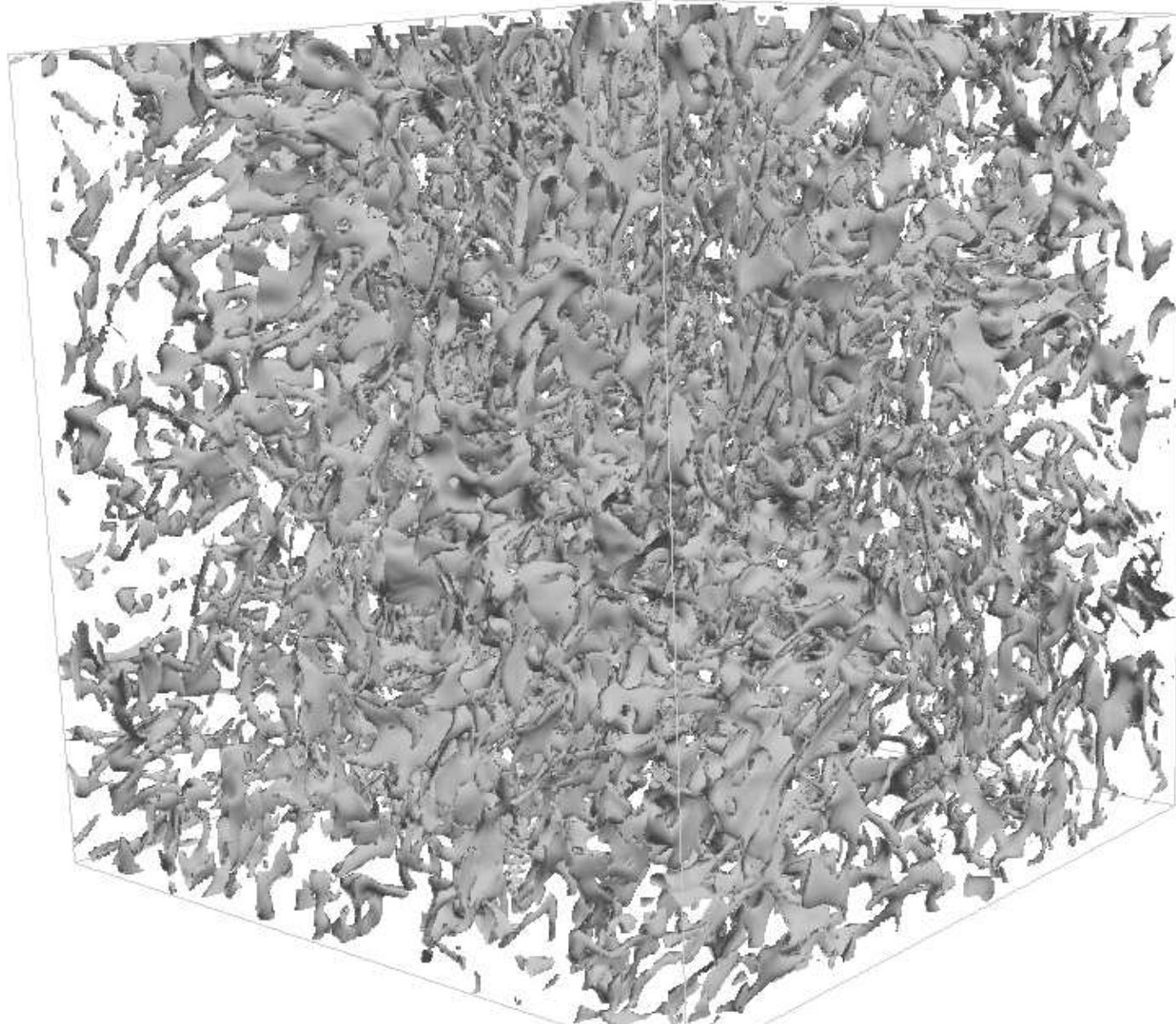
$t_{N/2\pi} = 8, t_{\Omega/\pi} = 80$



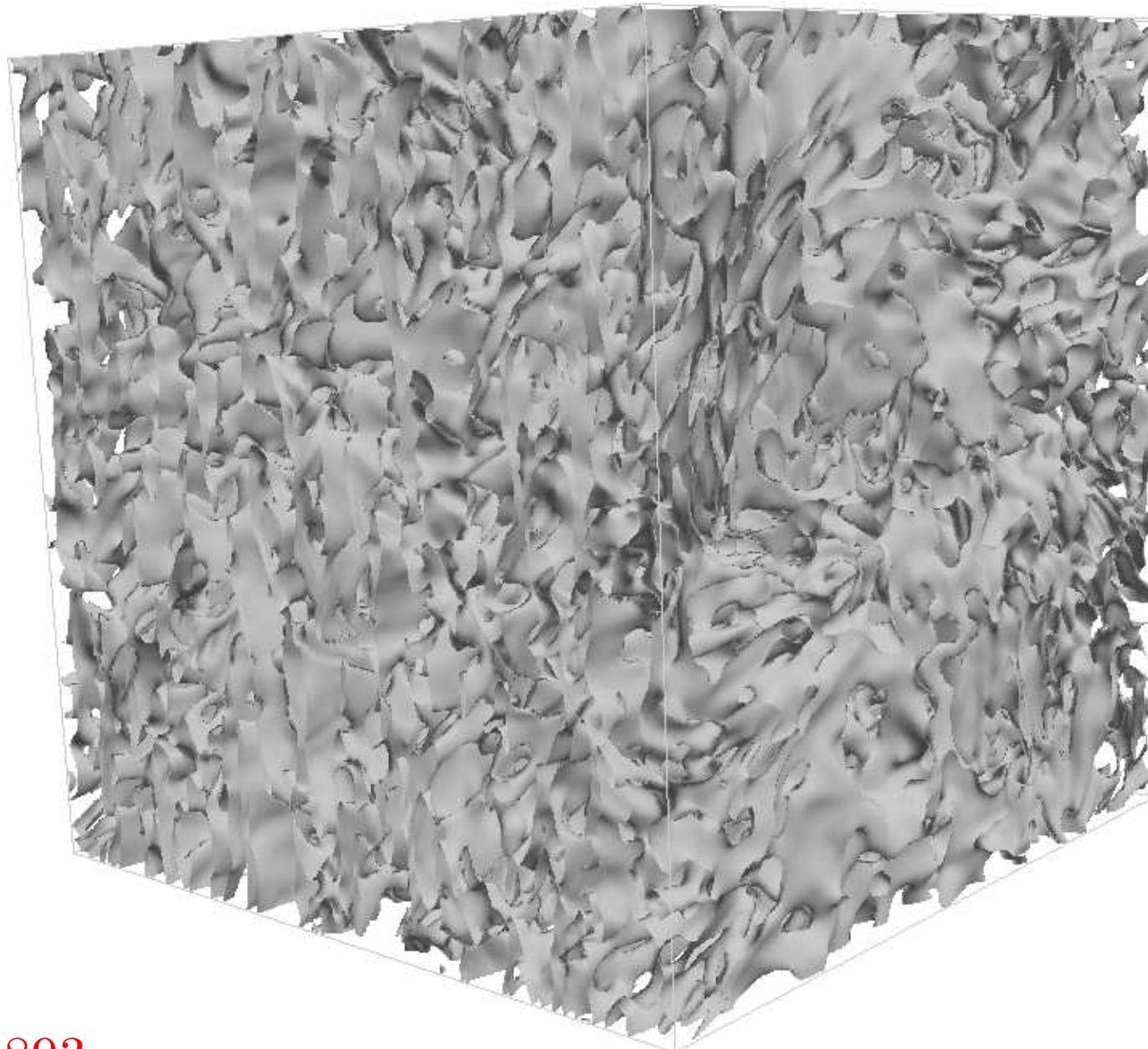
$t_{N/2\pi} = 6, t_{\Omega/\pi} = 60$

$t_{N/2\pi} = 8, t_{\Omega/\pi} = 80$

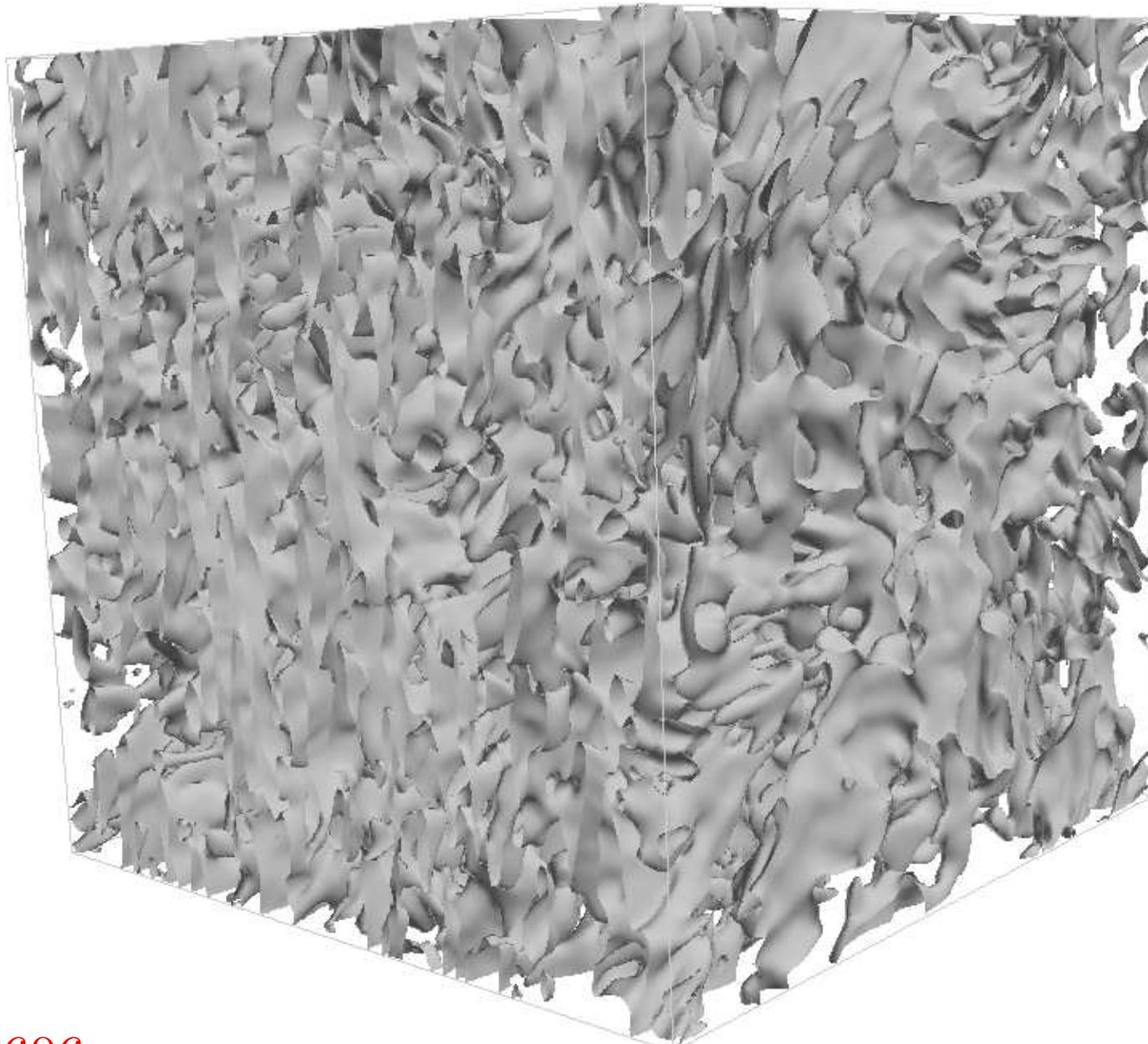
$t_{N/2\pi} = 10, t_{\Omega/\pi} = 100$



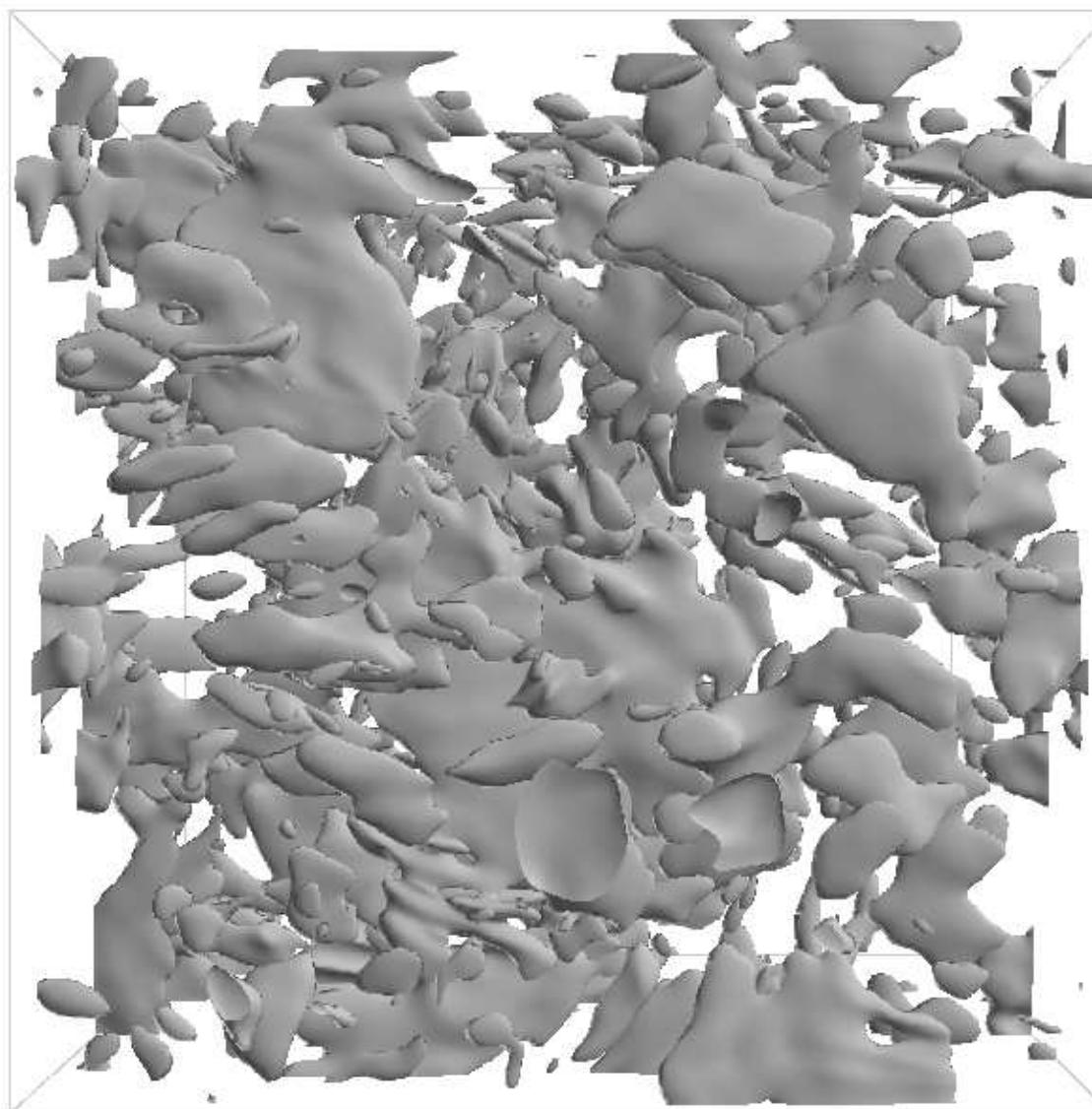
$$\omega = \nabla \times \mathbf{u}; \omega^2 = 0.383 \in [0.00189; 6.775] \text{ (stratifié)}$$



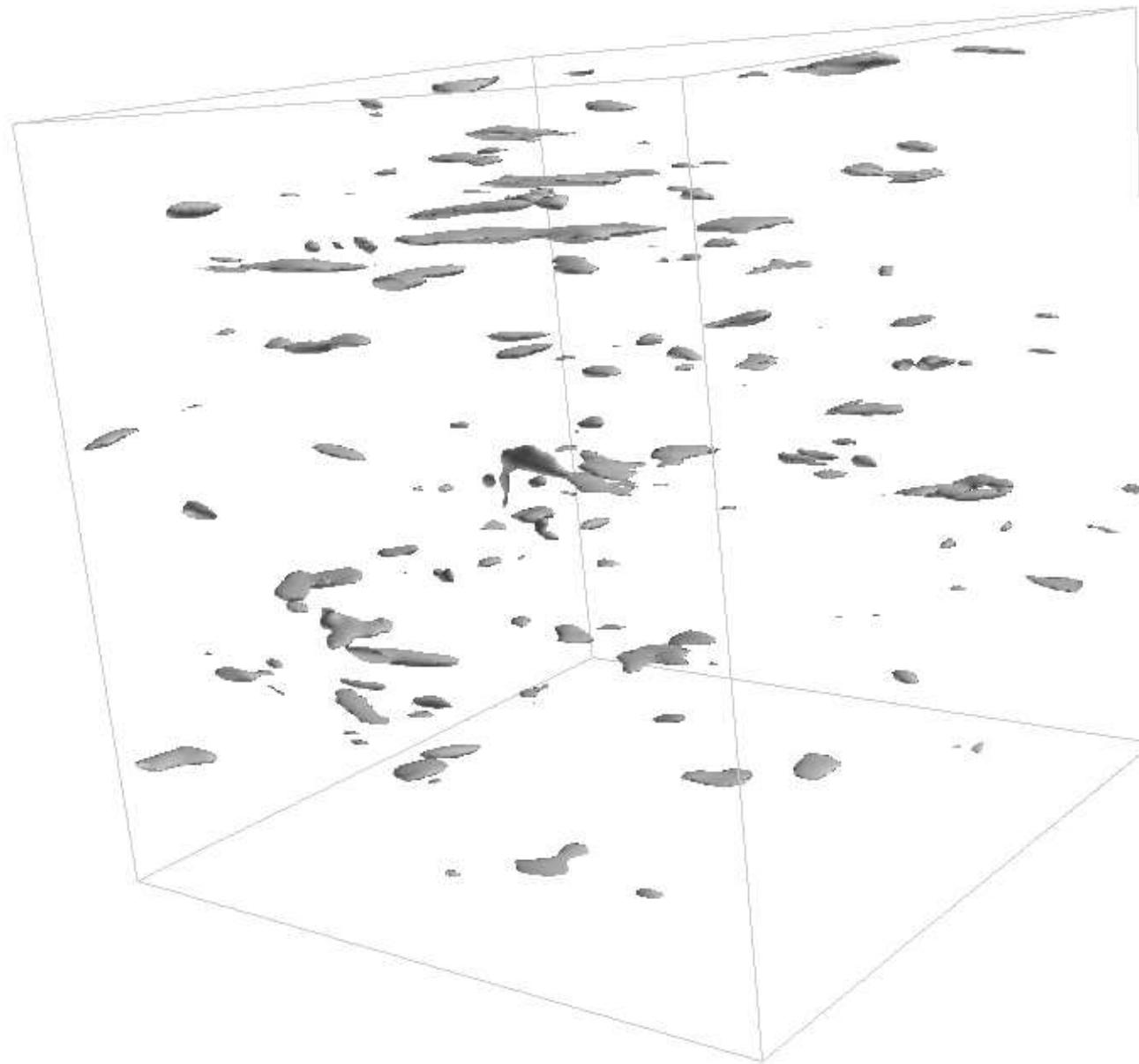
$$\omega^2 = 0.893$$



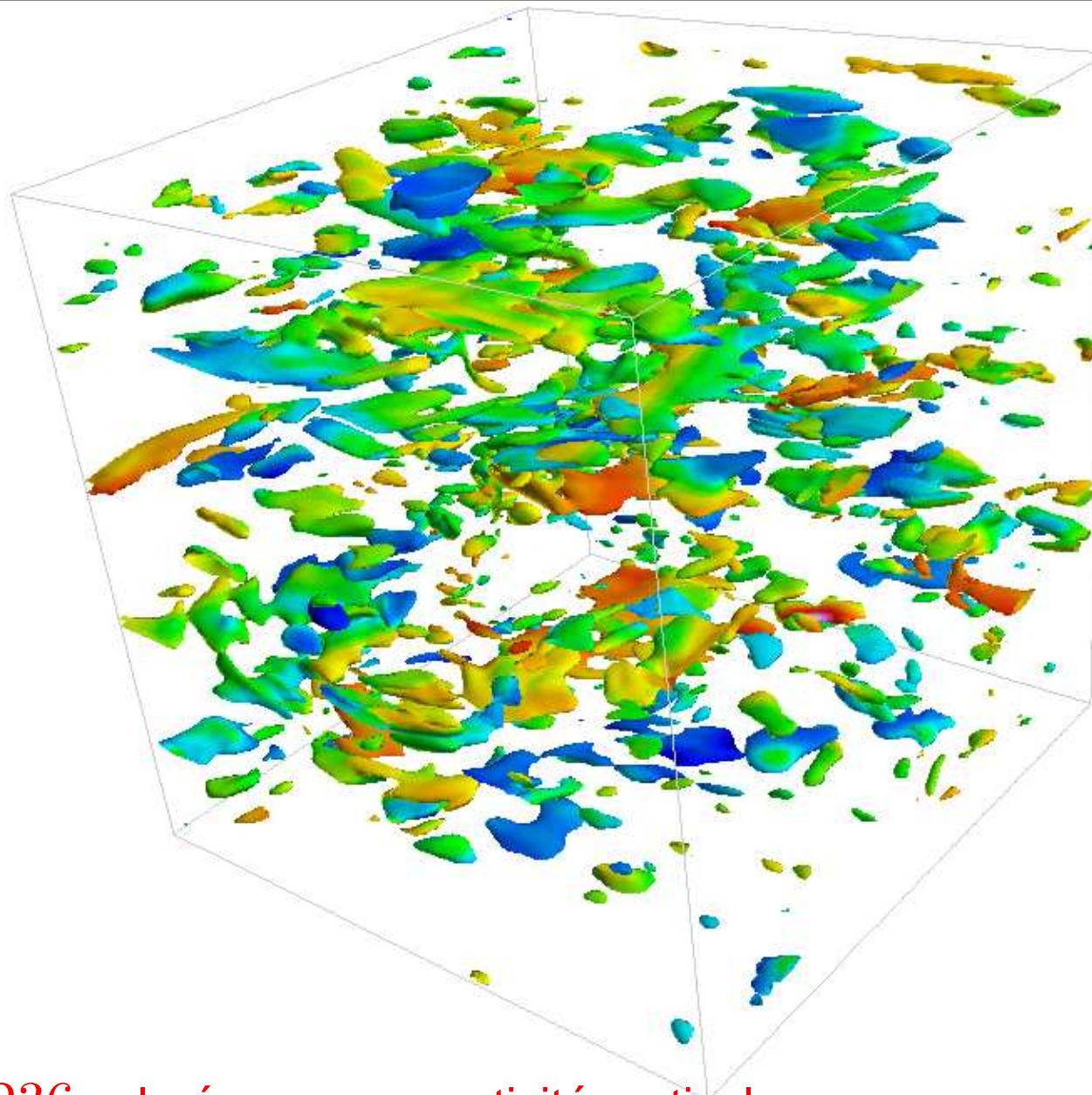
$$\omega^2 = 1.606$$



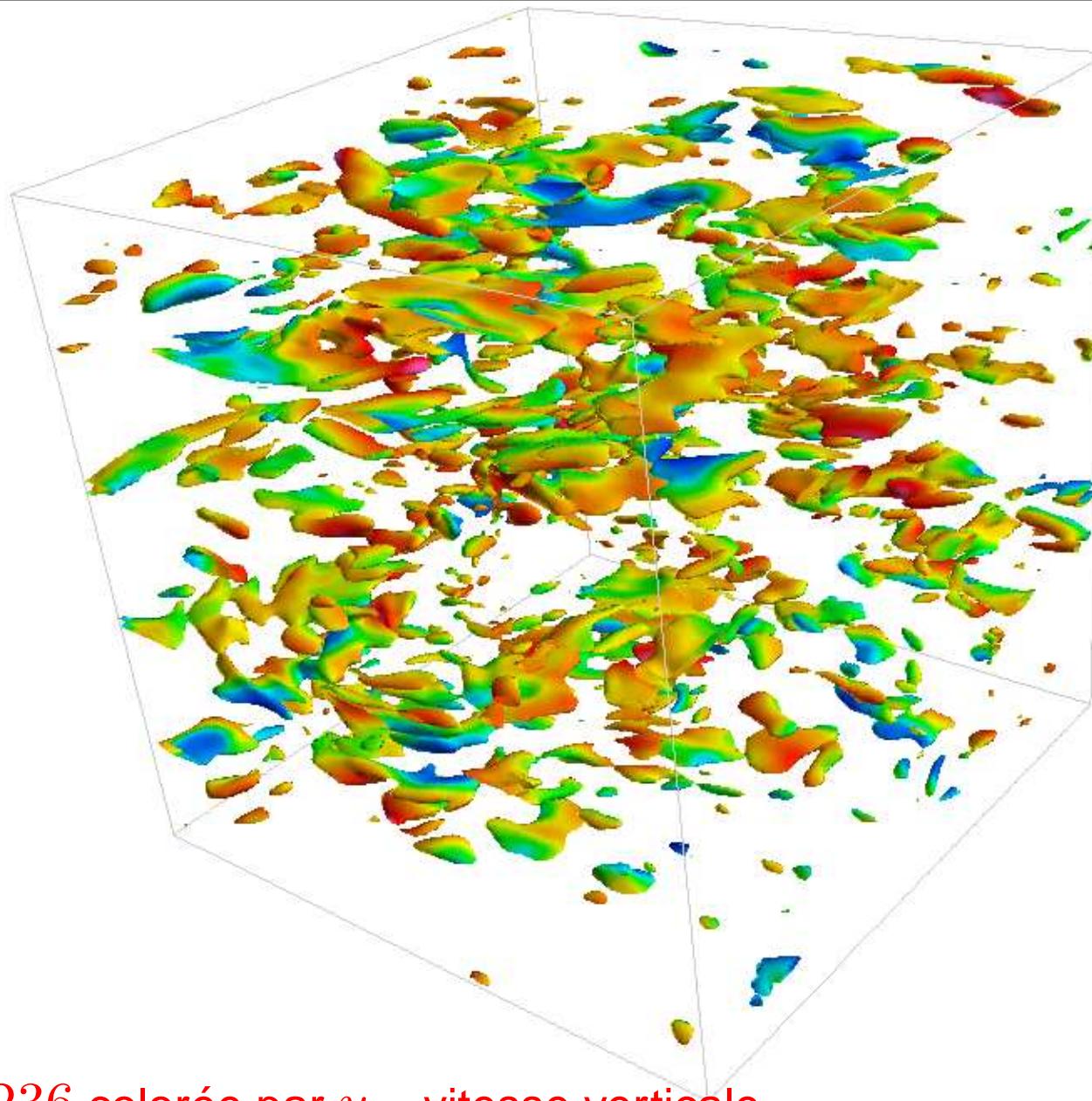
$$\omega^2 = 3.236$$



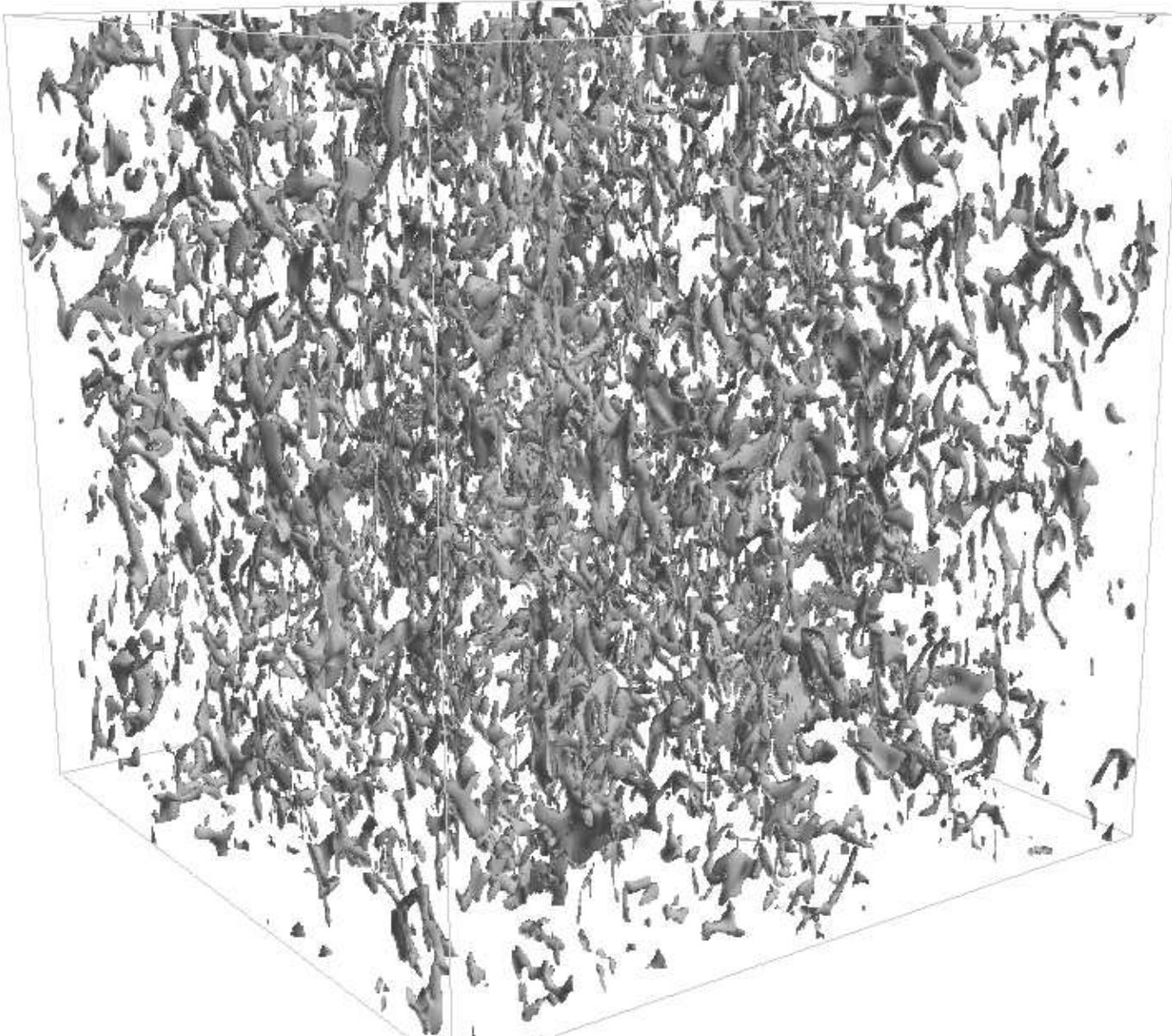
$$\omega^2 = 4.152$$



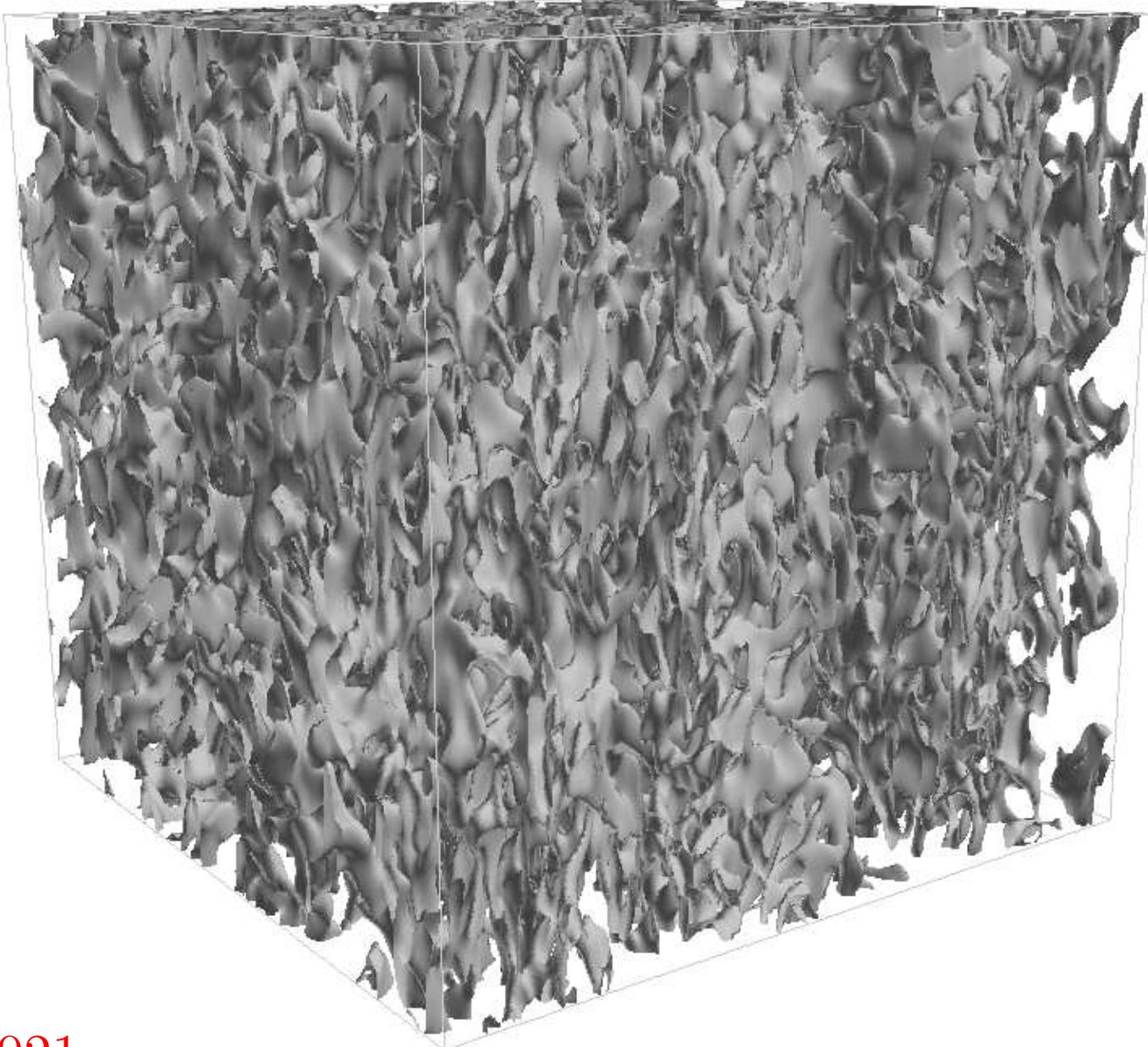
$\omega^2 = 3.236$ colorée par ω_z , vorticité verticale



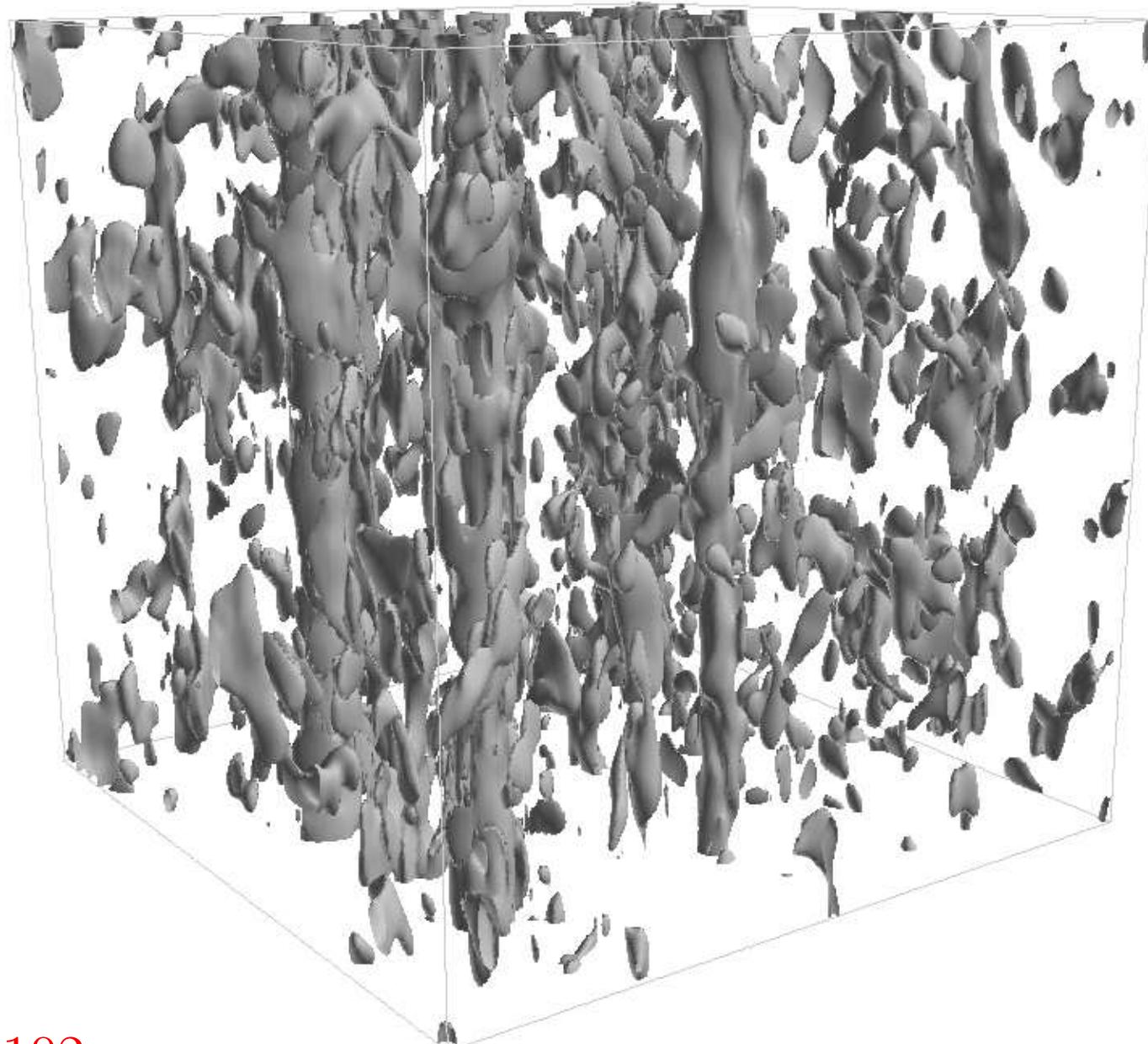
$\omega^2 = 3.236$ colorée par u_z , vitesse verticale



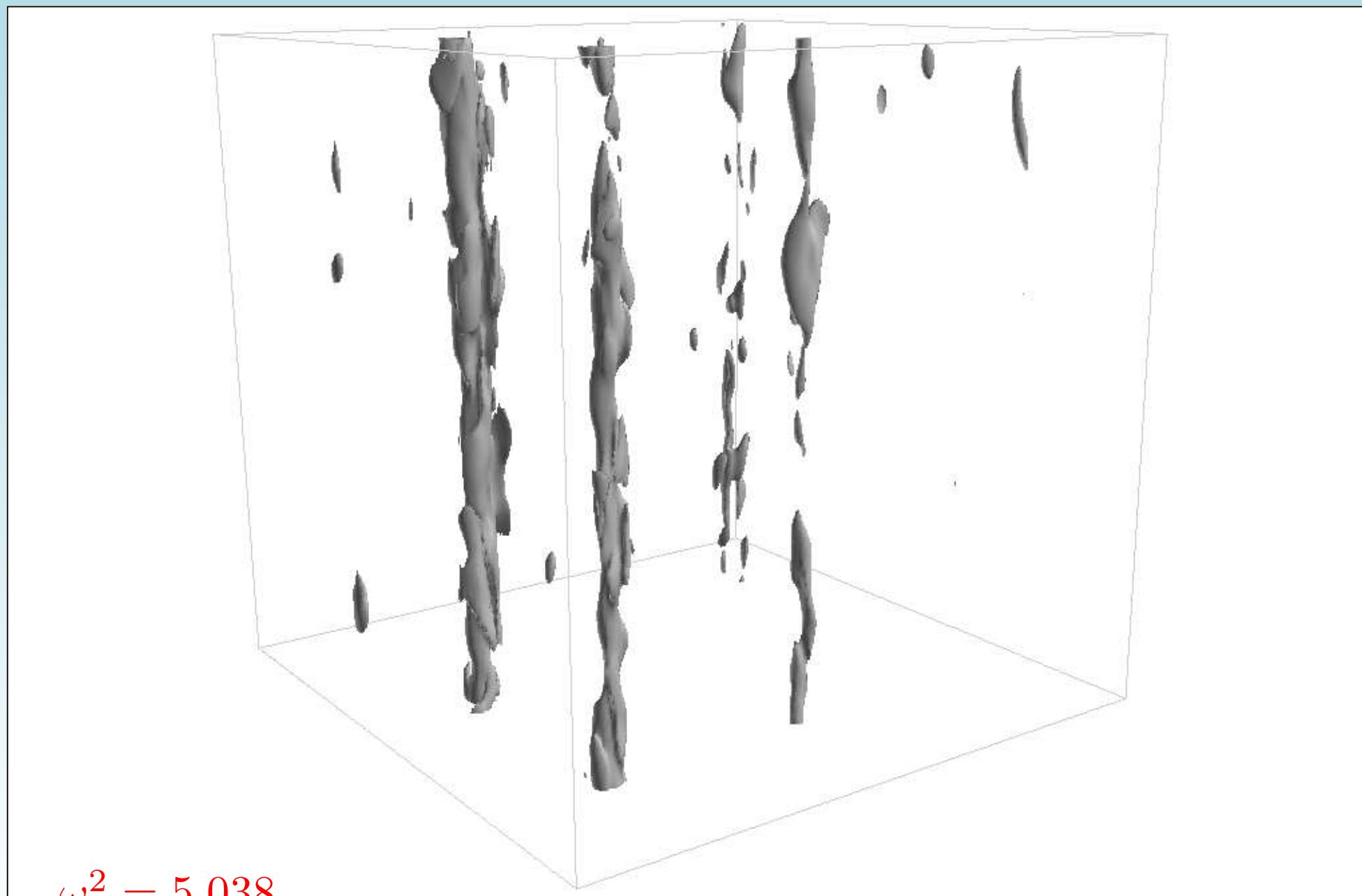
$$\omega^2 = 0.392 \in [0.00458; 12.88] \text{ (rotation)}$$



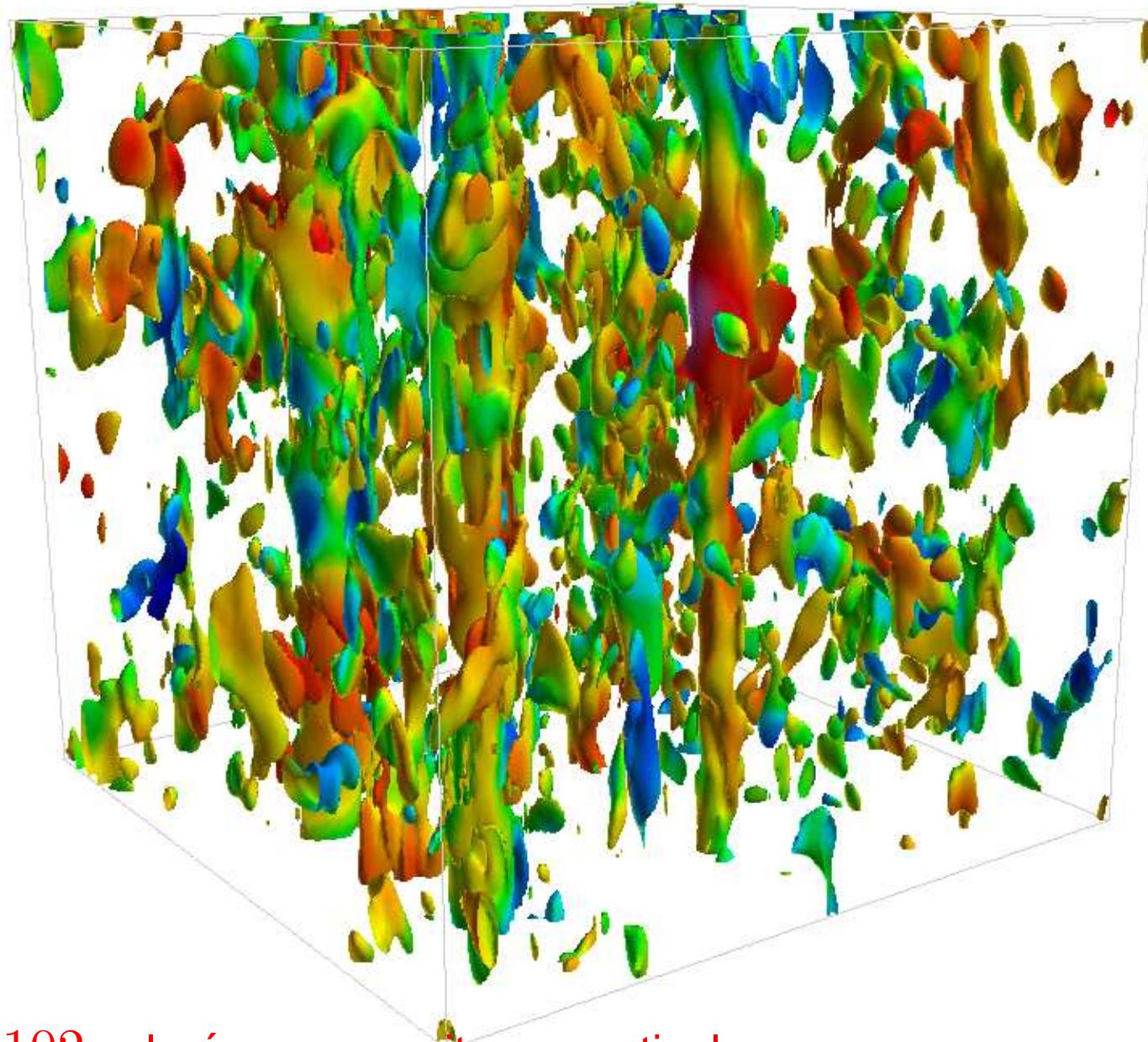
$$\omega^2 = 1.021$$



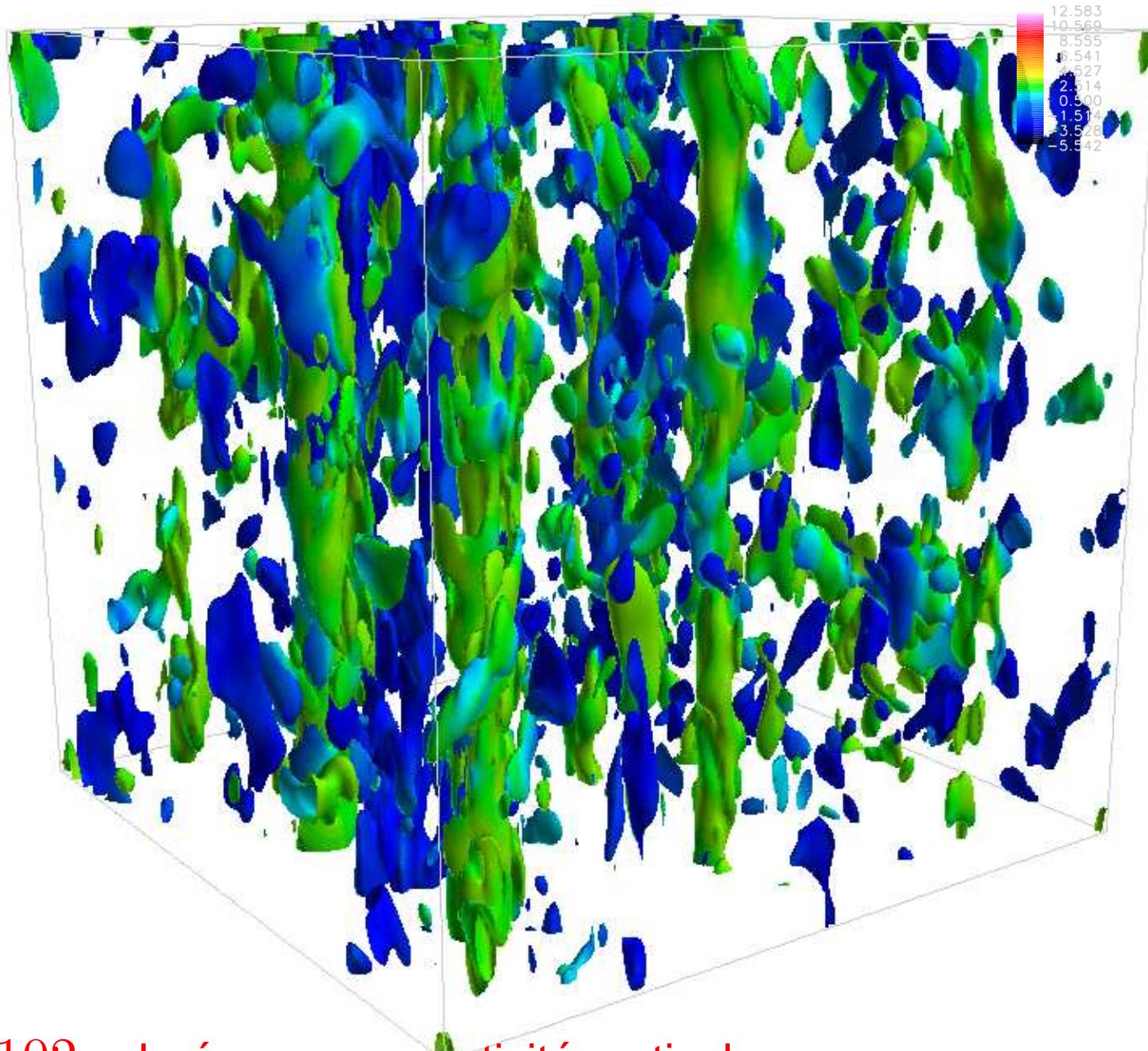
$$\omega^2 = 3.102$$



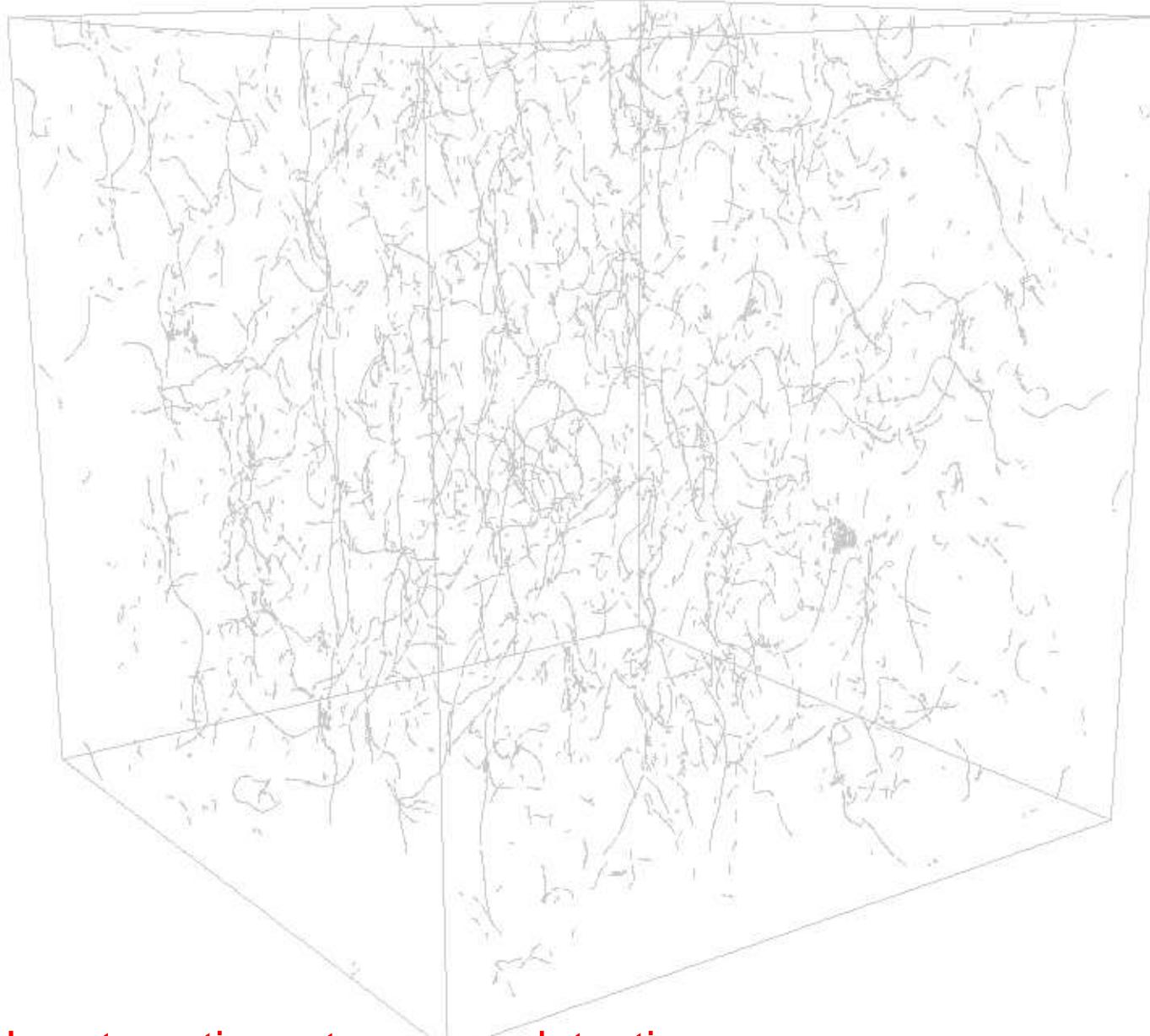
$$\omega^2 = 5.038$$



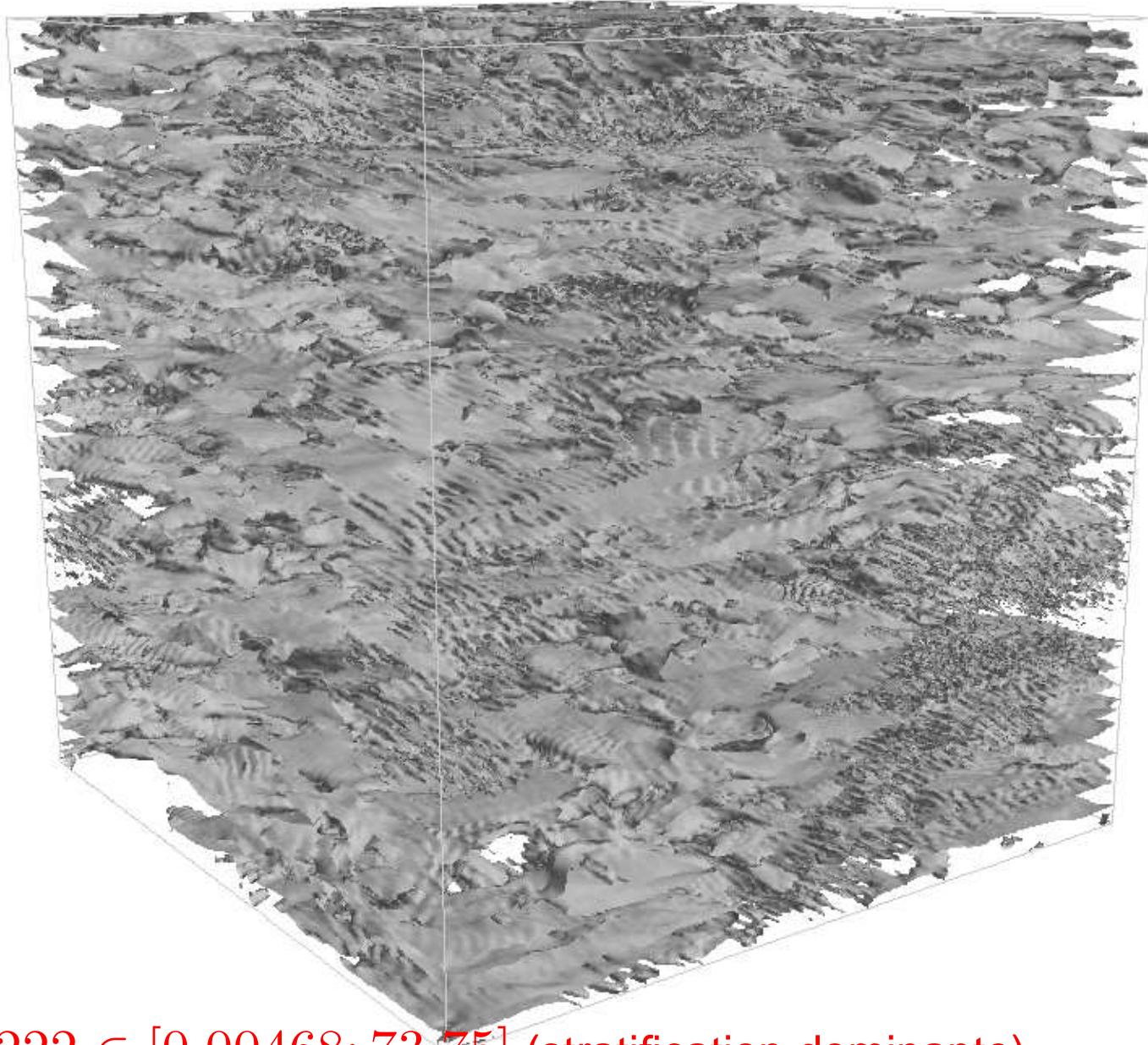
$\omega^2 = 3.102$ colorée par u_z , vitesse verticale



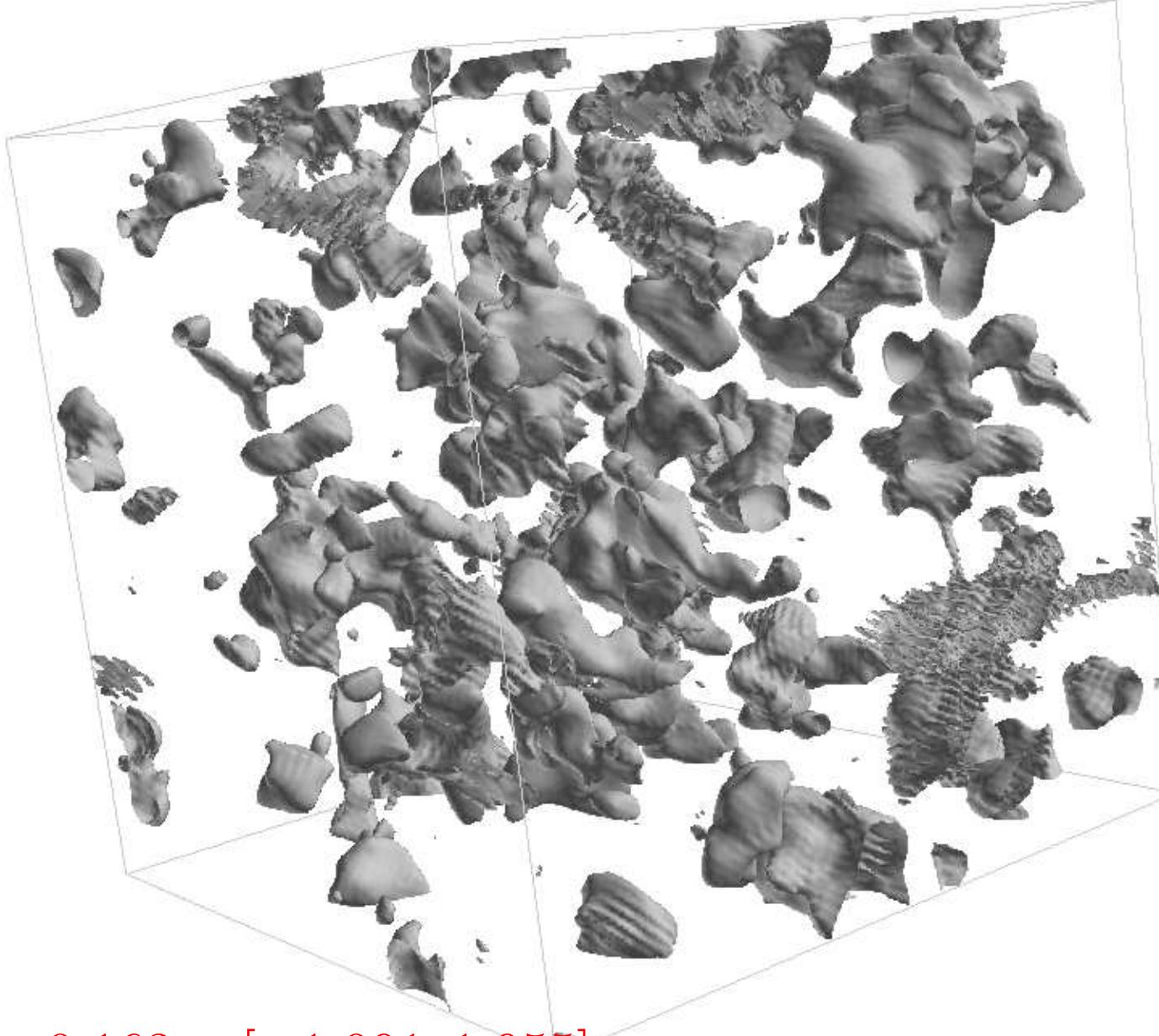
$\omega^2 = 3.102$ colorée par ω_z , vorticité verticale



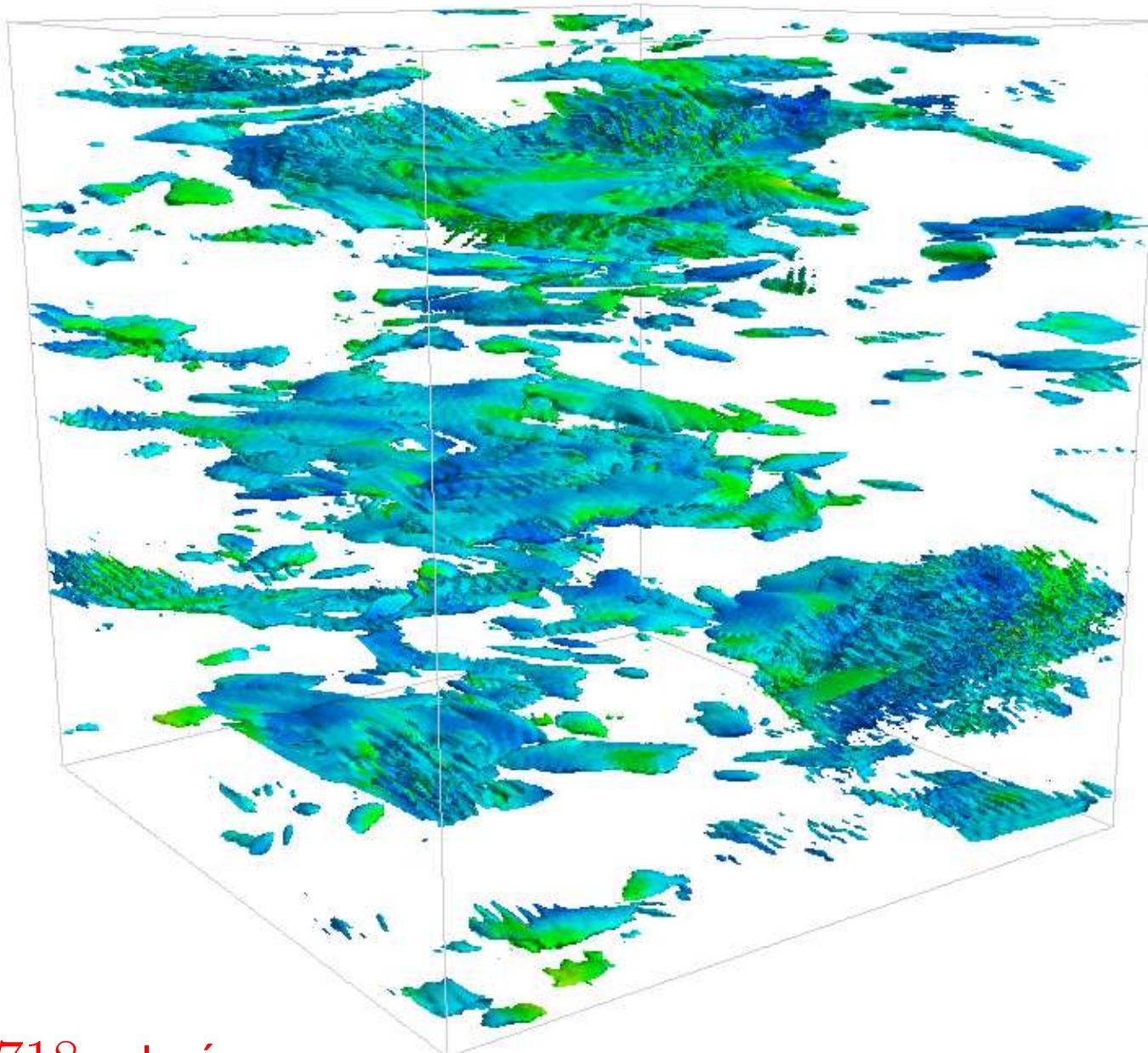
Fieldview's automatic vortex cores detection



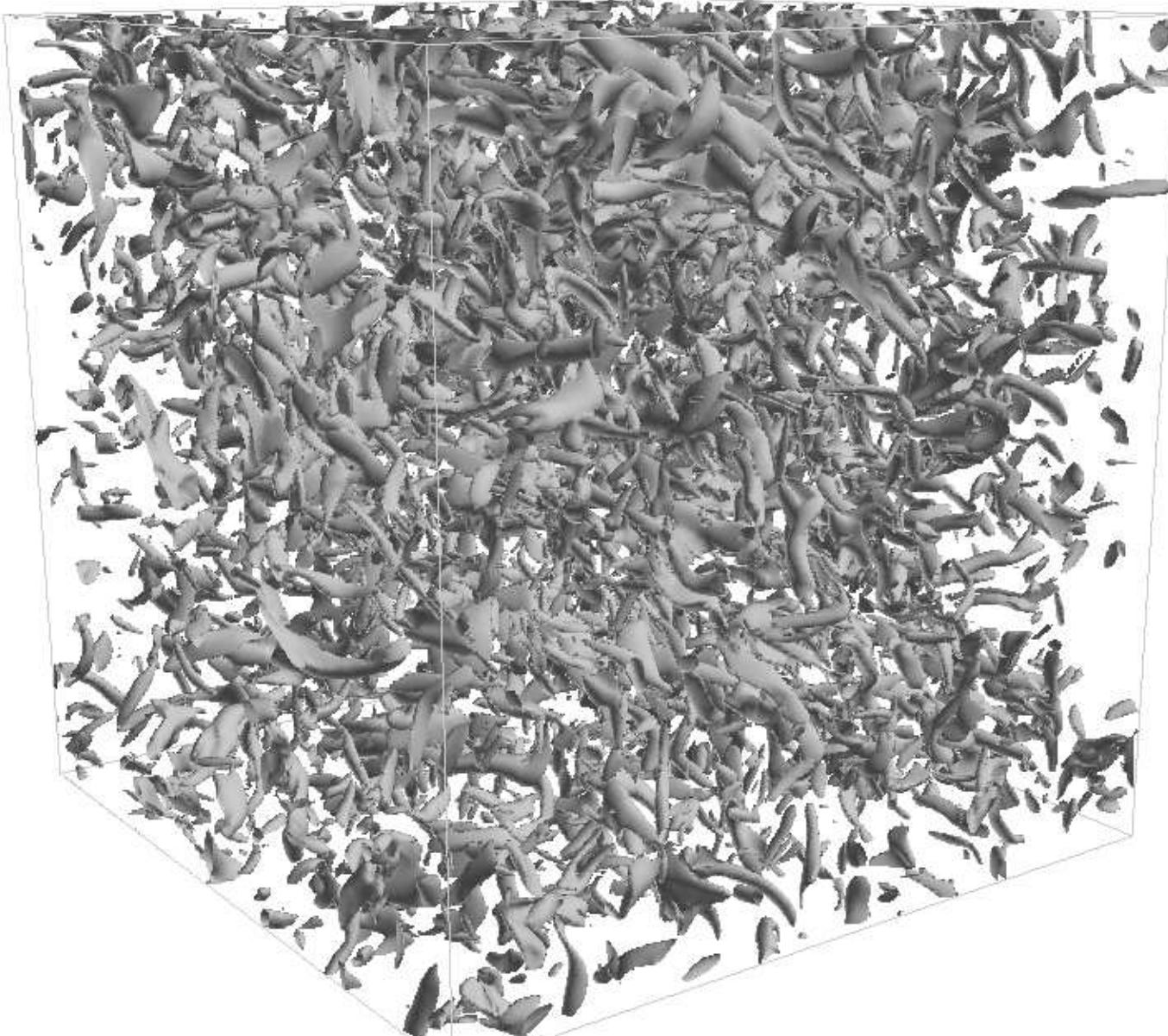
$$\omega^2 = 2.222 \in [0.00468; 73.75] \text{ (stratification dominante)}$$



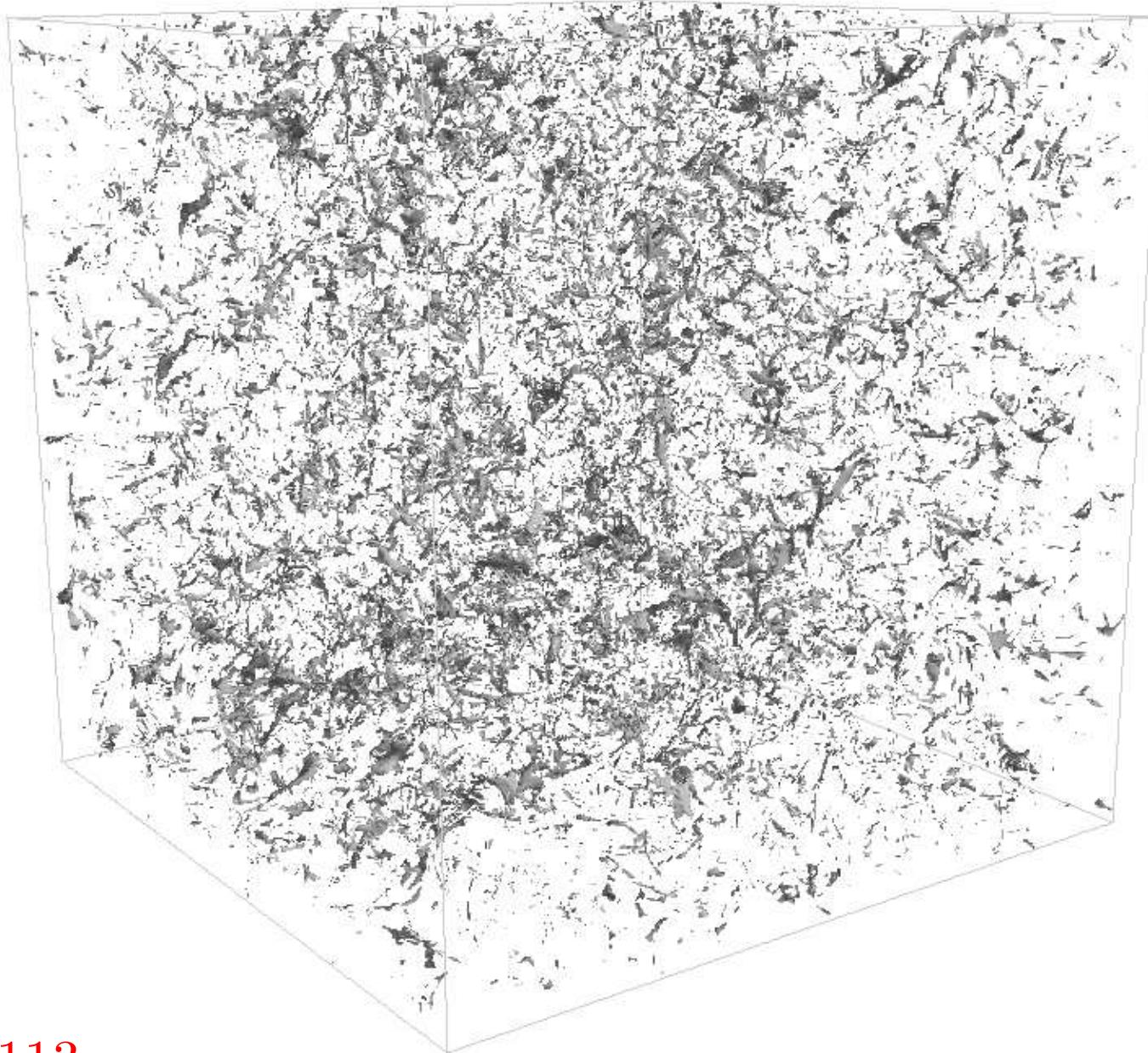
$$u_z = -0.163 \in [-1.091; 1.955]$$



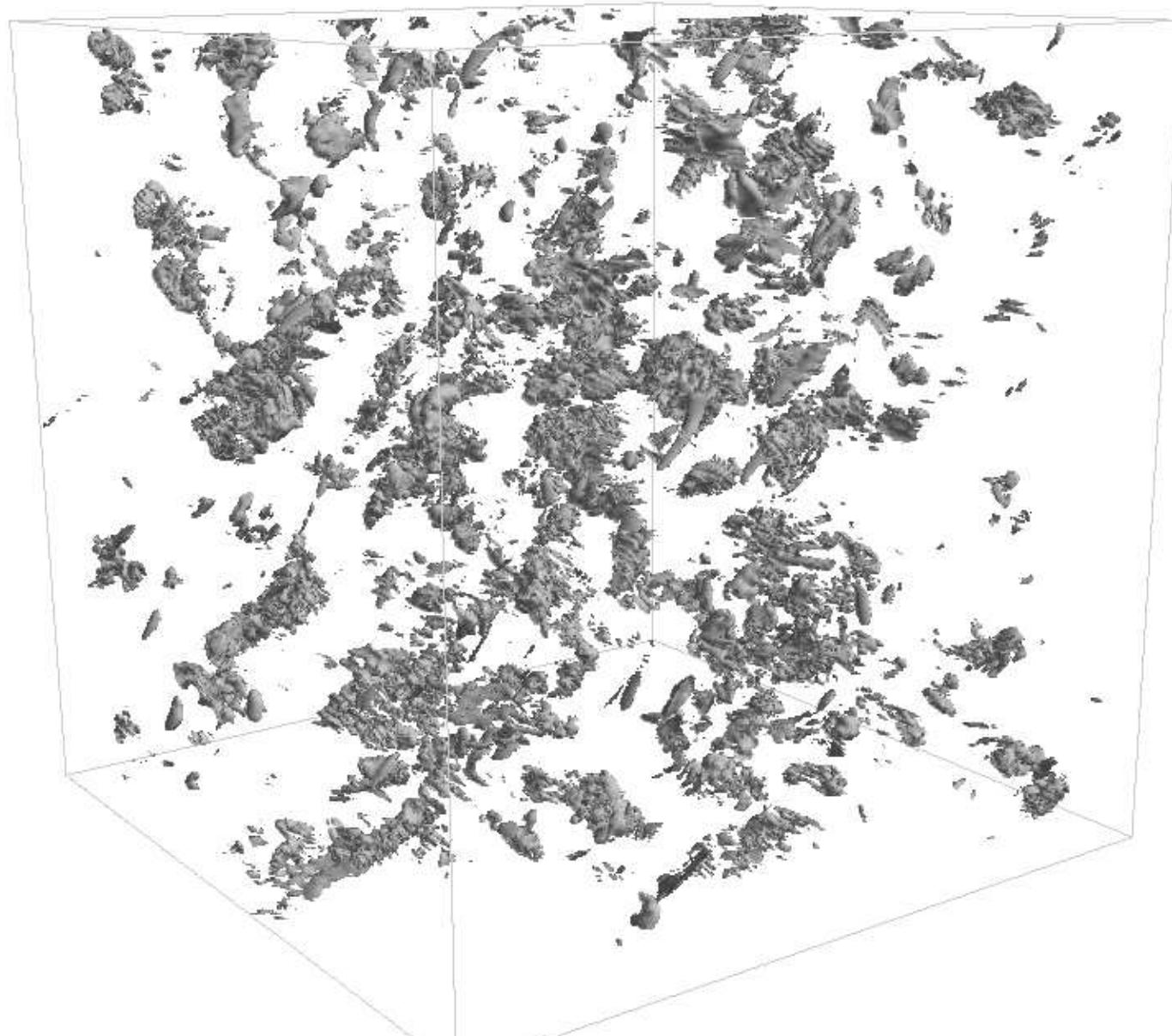
$\omega^2 = 4.718$ colorée par u_z



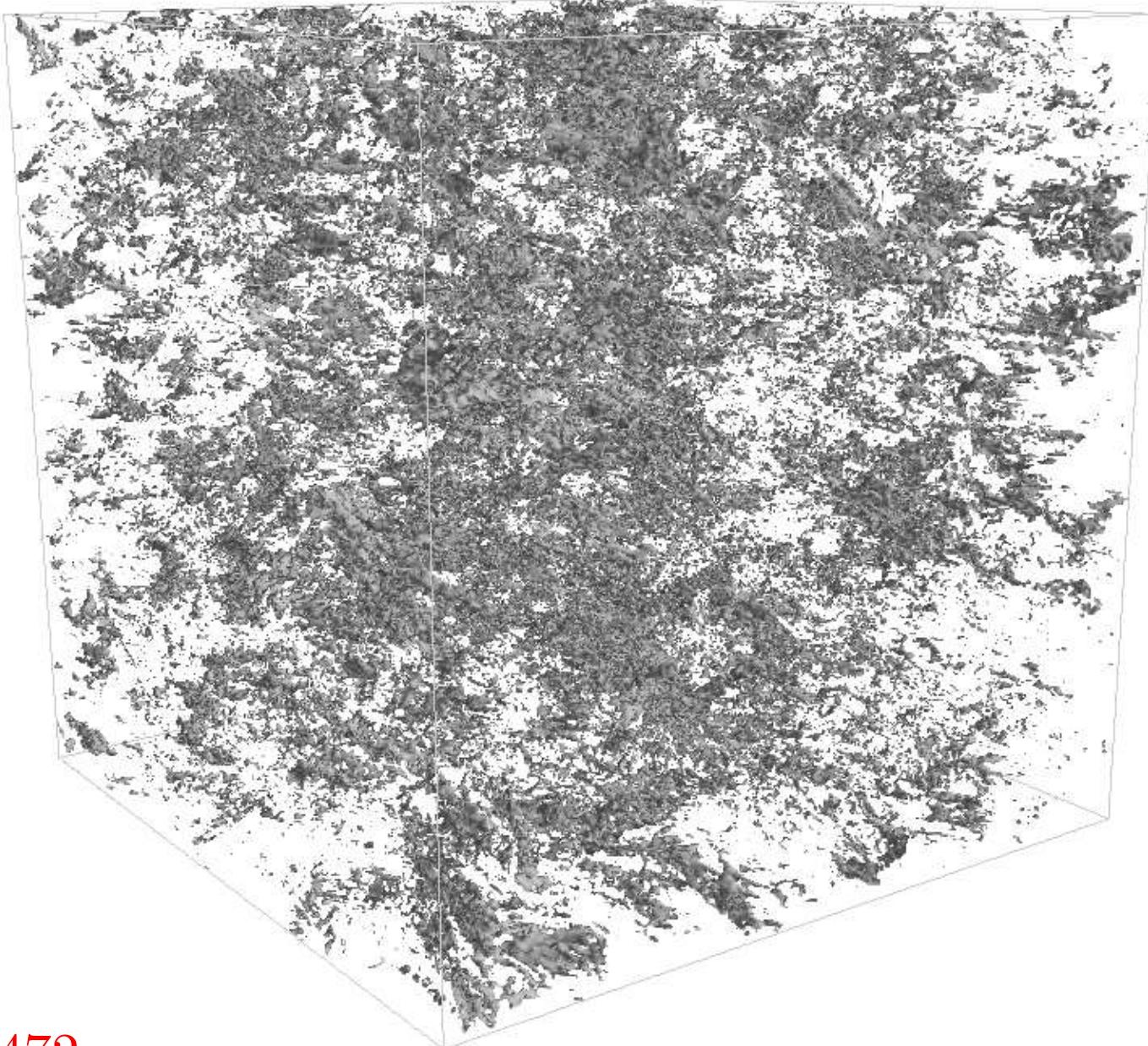
$$\omega^2 = 2.01 \in [0.00138; 7.425] \text{ (neutre)}$$



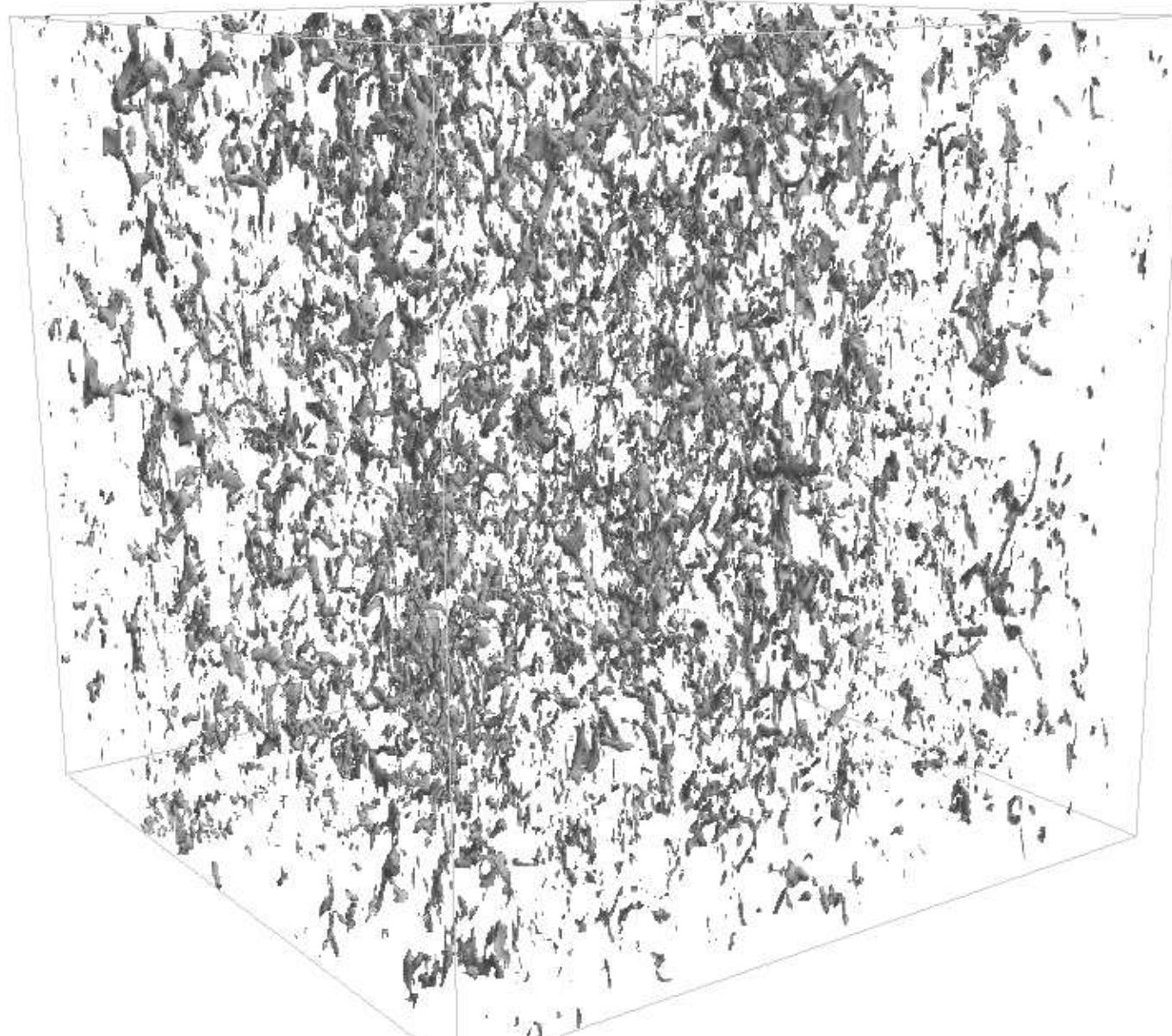
$$\omega^2 = 0.113$$



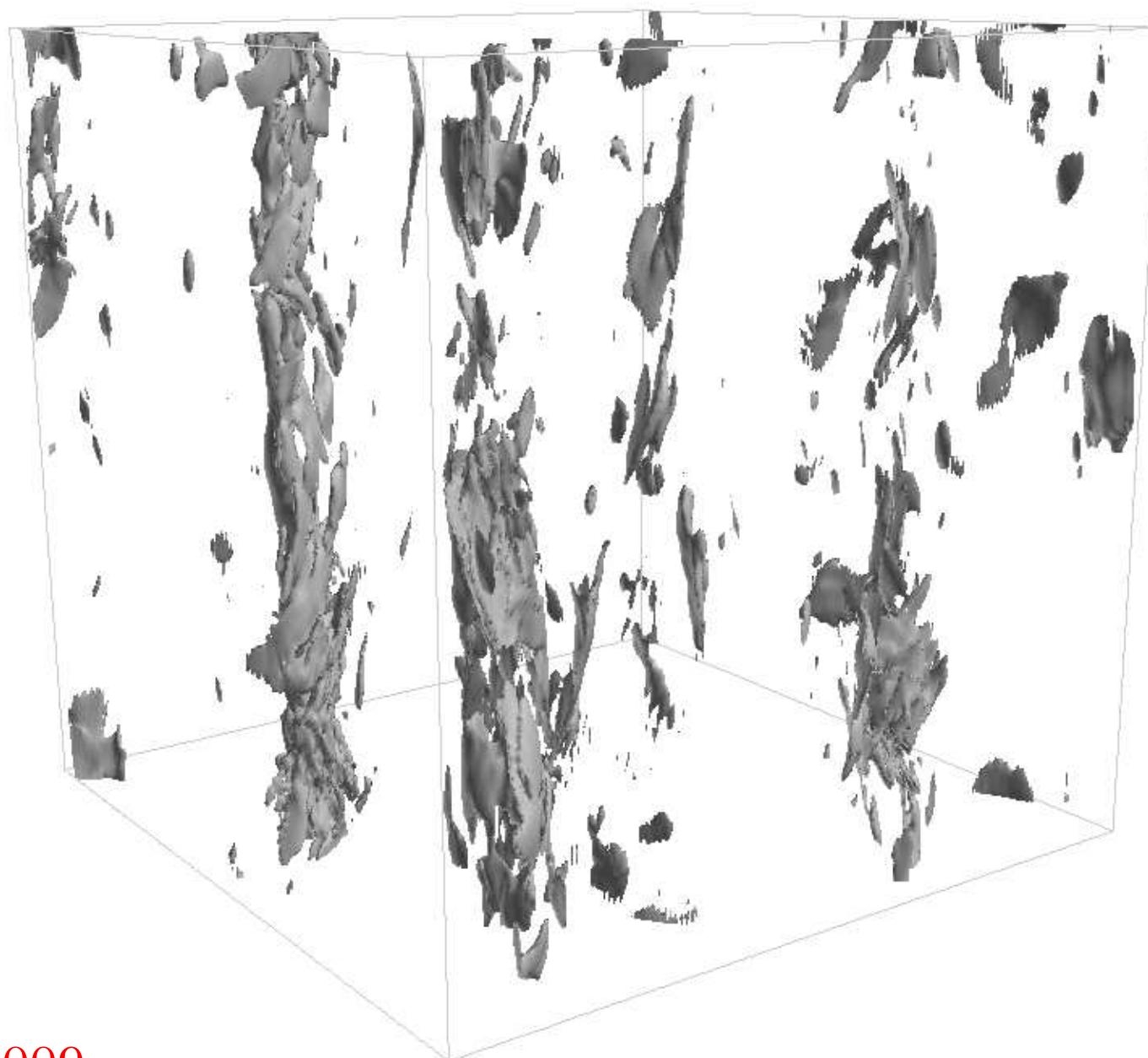
$$\omega^2 = 11.333 \in [0.00324; 62.39] \text{ (non dispersif)}$$



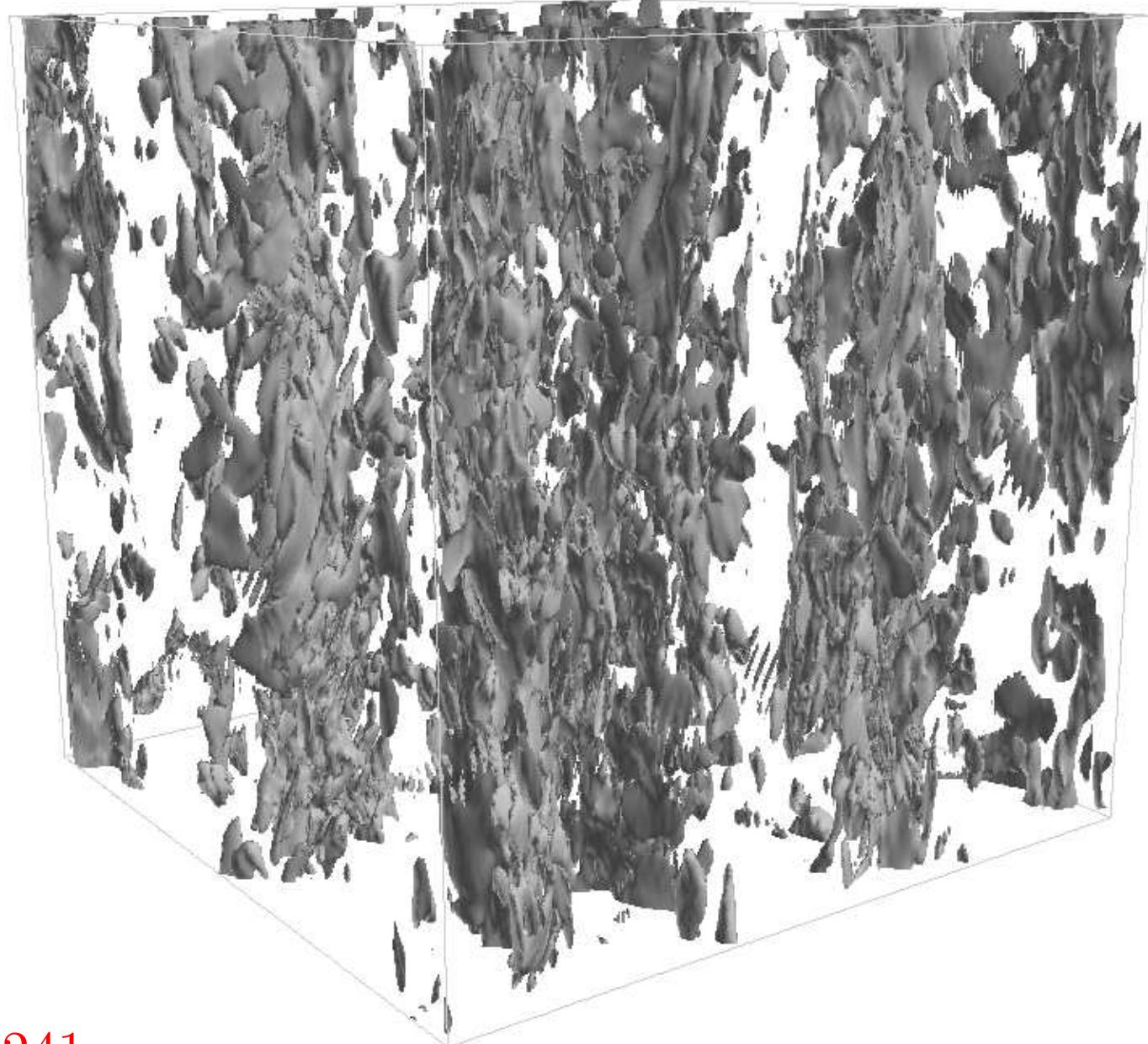
$$\omega^2 = 0.472$$



$$\omega^2 = 0.295 \in [0.000738; 26.122] \text{ (rotation dominante)}$$



$$\omega^2 = 5.009$$



$$\omega^2 = 3.241$$

Trajectoires tridimensionnelles des particules fluides

Définies par :

$$\frac{dx}{dt} = \mathbf{u}$$

différentes des lignes de courant en général.

Visualisation des lignes de courant dans une “boîte” de turbulence stratifiée. f2.mpg

Turbulence inhomogène

Turbulence diffusive en rotation entre deux parois planes

Lionel Lollini, 1998

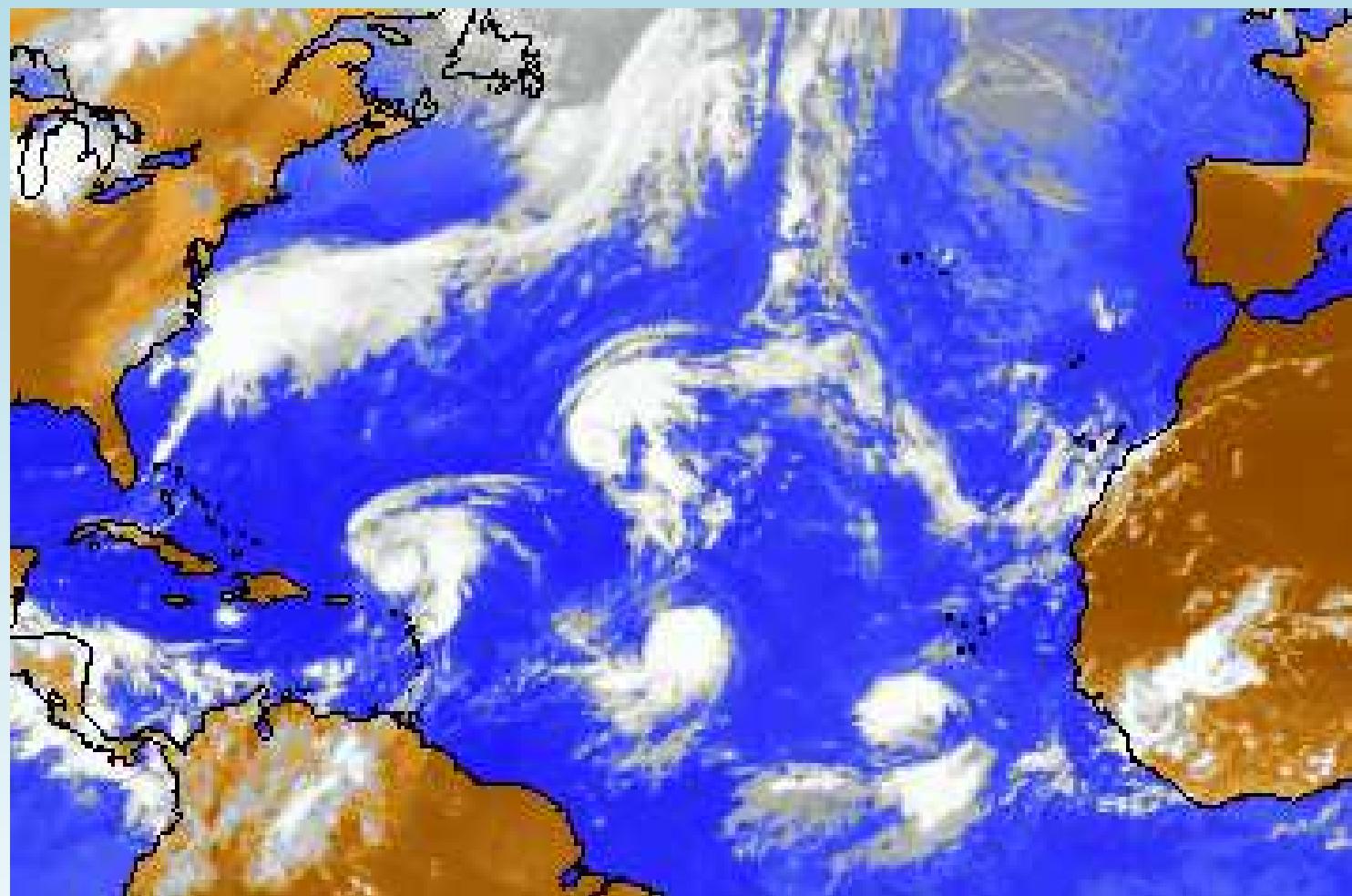


Figure 3: Cyclonic activity over the North Atlantic Ocean area in 1995. The four cyclones were christened Humberto, Iris, Karen and Luis (Source Météo-France).

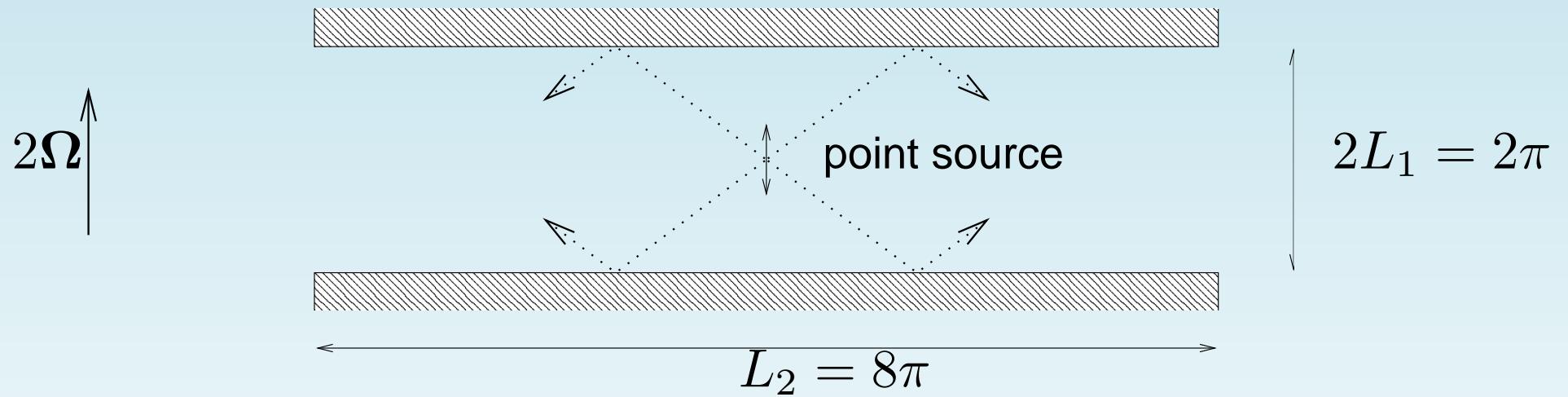


Figure 4: Sketch of the numerical experiment geometry used for generating the inertial waves.

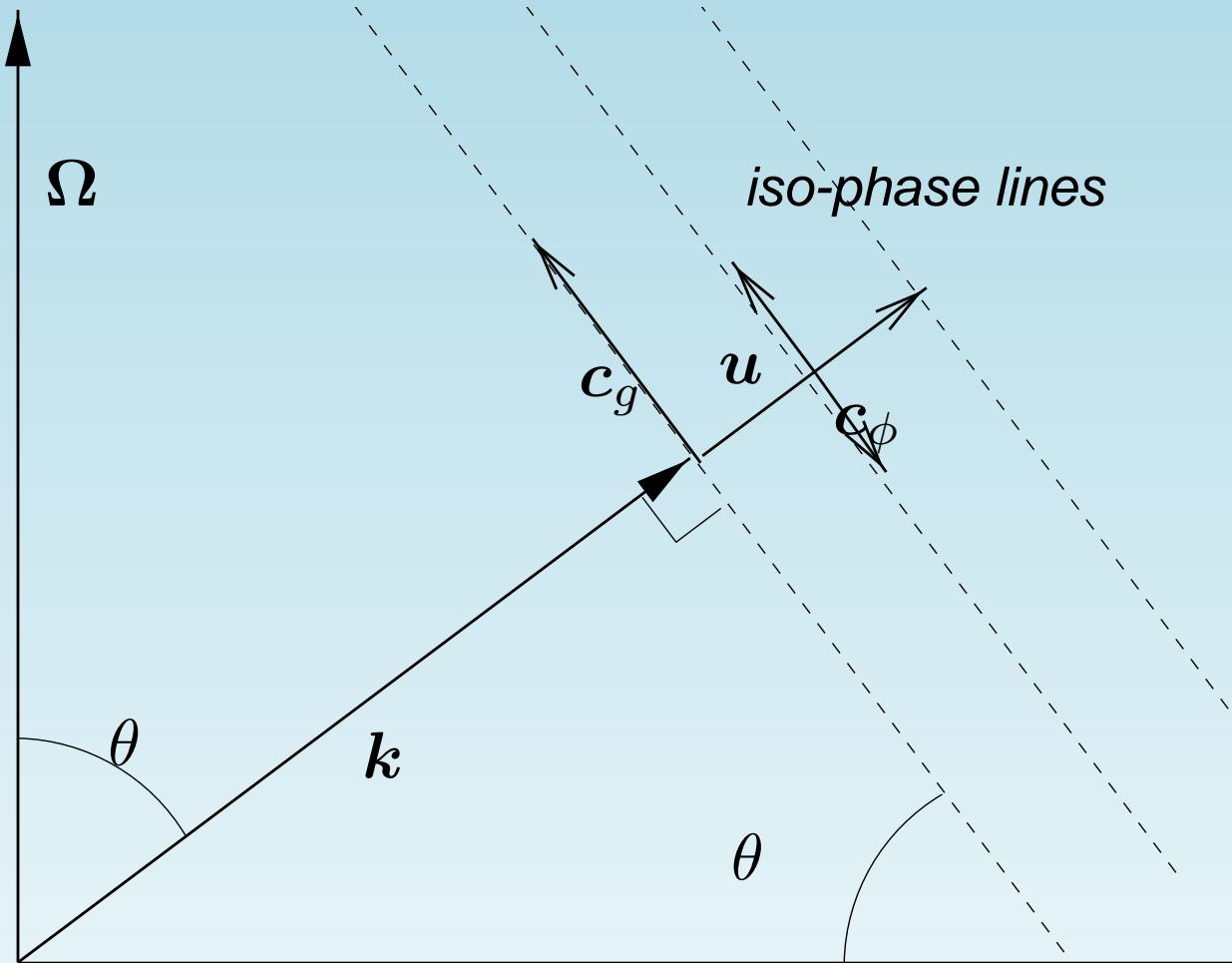


Figure 5: Dynamics of inertial waves. Dashes indicate lines of constant phase, aligned with the group velocity of the waves, and the velocity vector.

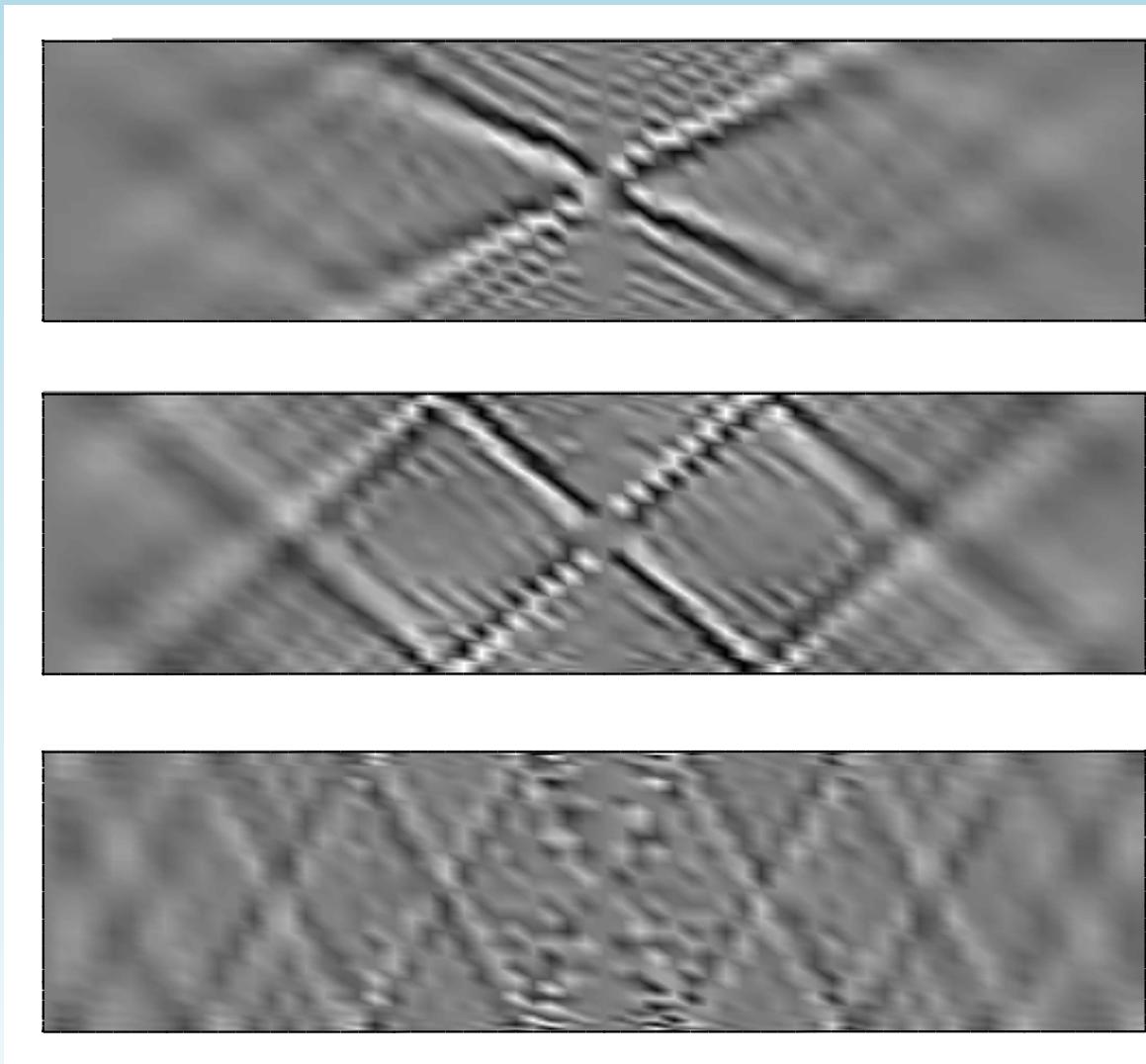


Figure 6: Two-dimensional plot of the lines of constant phase for the inertial waves (the wavevector is orthogonal to these lines).



Figure 7: Photograph in the experiment by McEwan. The tank is cylindrical, with its axis aligned to the axis of rotation.

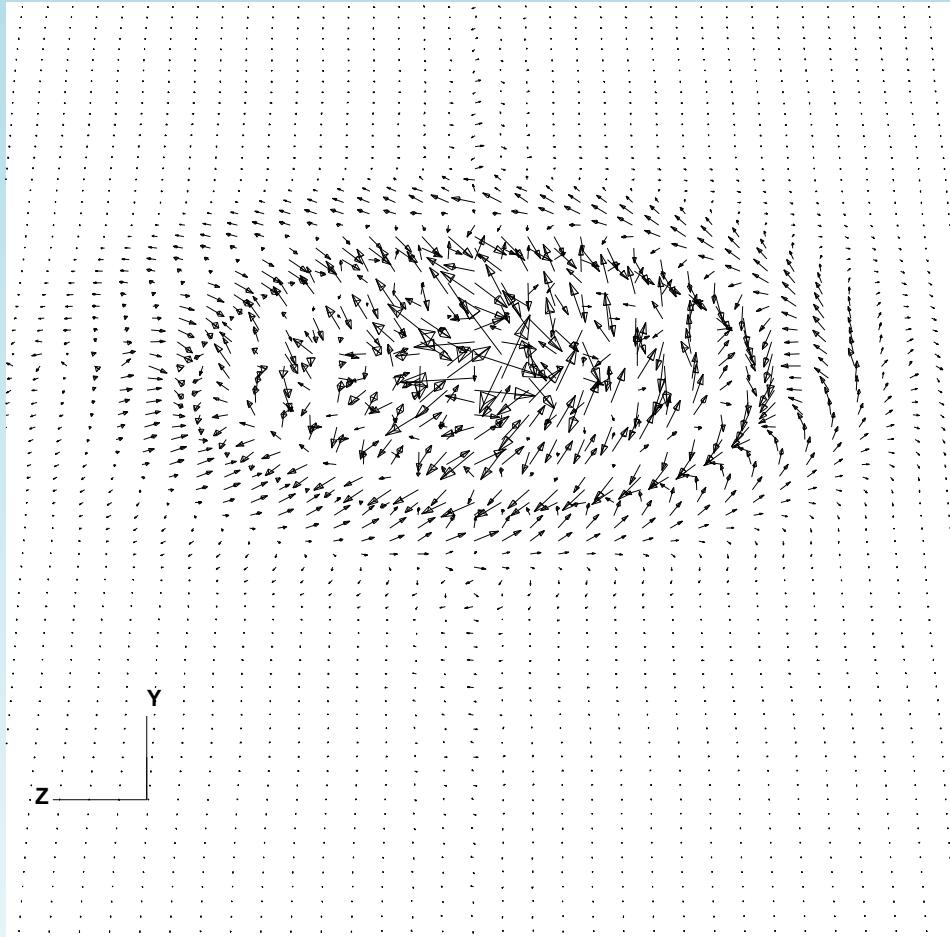
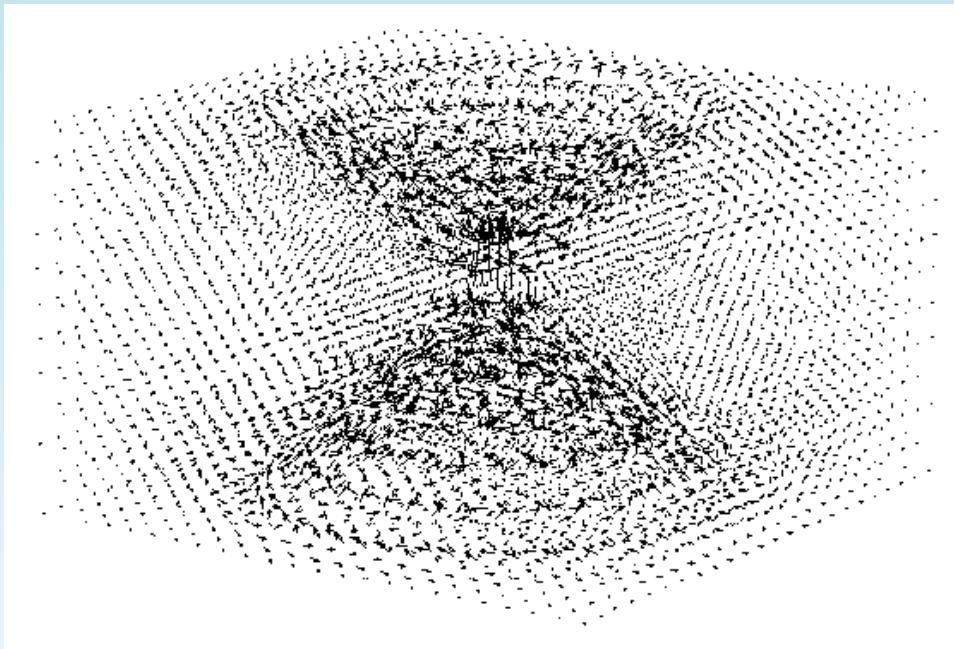


Figure 8: (a) Three-dimensional plot of the velocity field in the case $2\Omega/n = 1.33$. (b) Closer view of the velocity field for this very case, in a (y, z) plane away from the grid. The rotation of the velocity vector (phase change) can be observed along the radial directions.

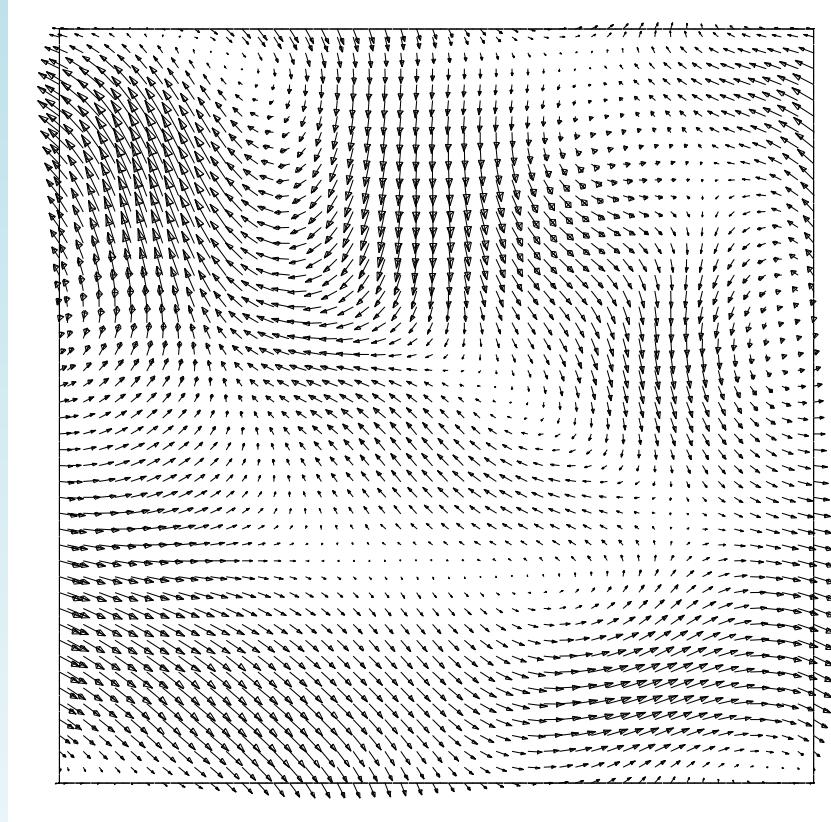


Figure 9: Velocity field in a plane of the simulation NRH at $t/\tau(0) = 20$, showing that only large scale variability remains in the fbw after such a long decay.

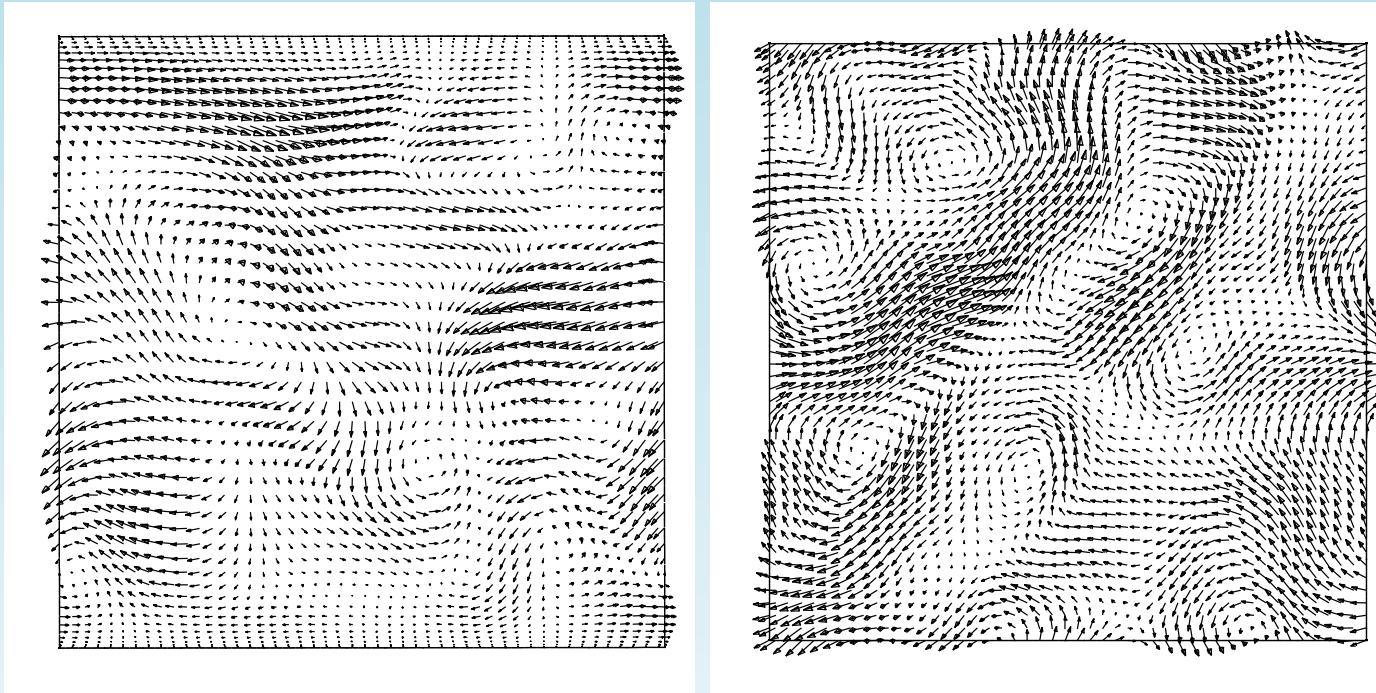


Figure 10: Velocity field for simulation NRI at $t/\tau(0) \approx 90$, (a) in an inhomogeneous (vertical) plane; (b) in a homogeneous (horizontal) plane

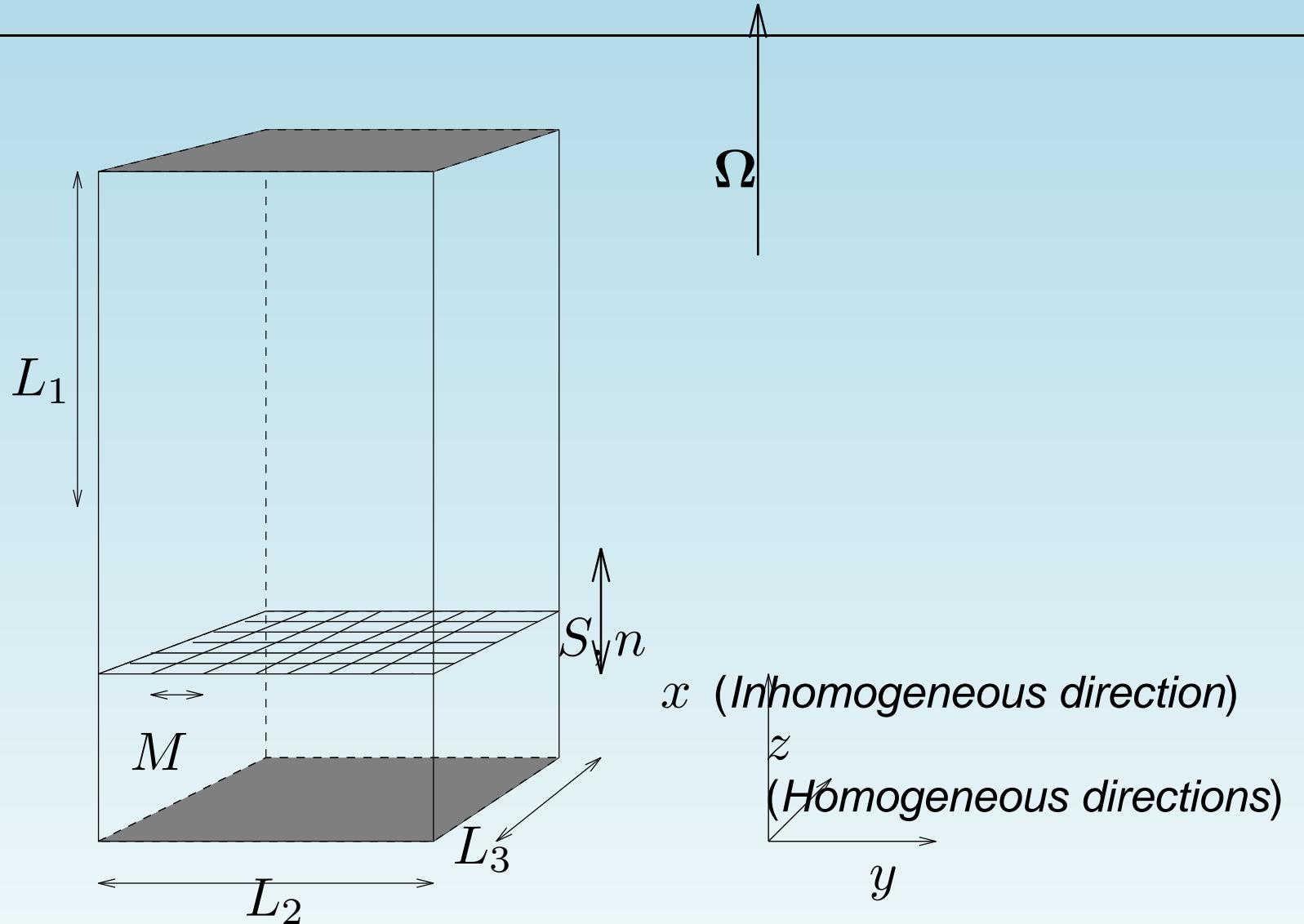


Figure 11: Scheme of the computational domain in the numerical experiment of rotating forced turbulence. The grid is located closer to the bottom wall, as in the experiment by Hopfinger, Browand and Gagne.

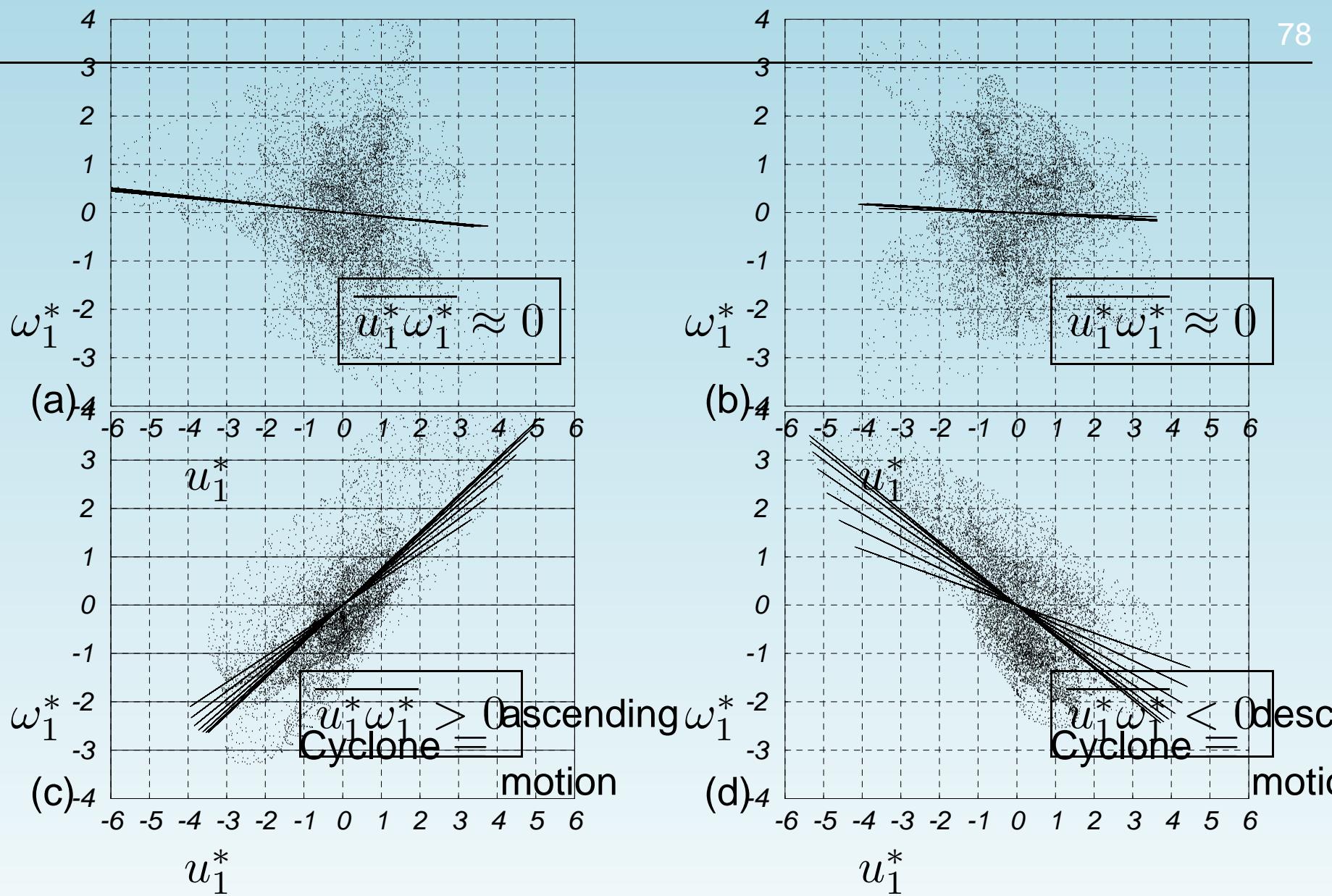


Figure 12: Correlation between vertical components of velocity and vorticity; (a) close to the bottom wall for run NRI; (b) close to the upper wall for run NRI; (c) close to the bottom wall for run RI; (d) close to the upper wall for run RI.

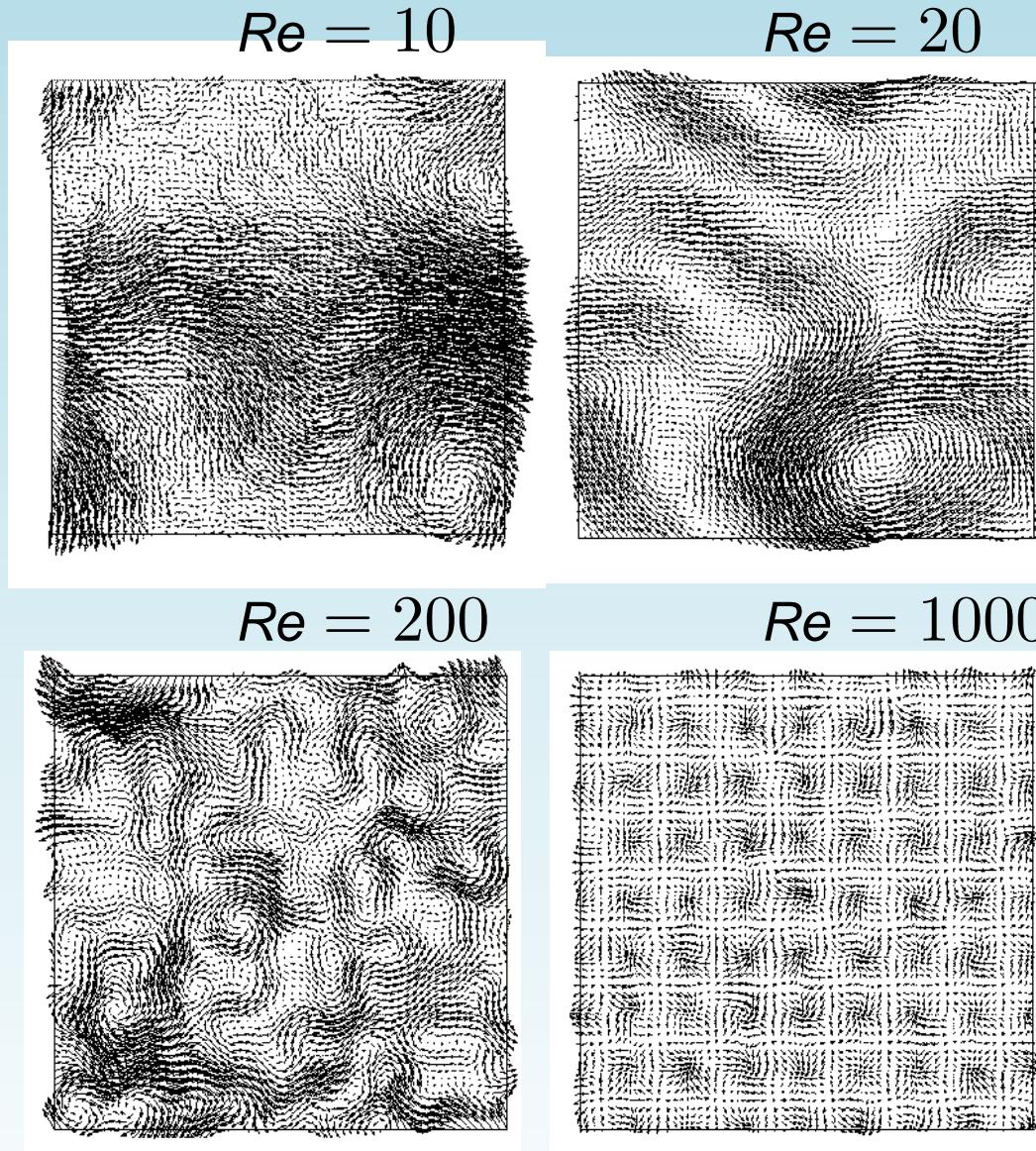


Figure 13: Plot of the velocity field at fixed Rossby number $Ro = 0.25$ with variable Reynolds number.

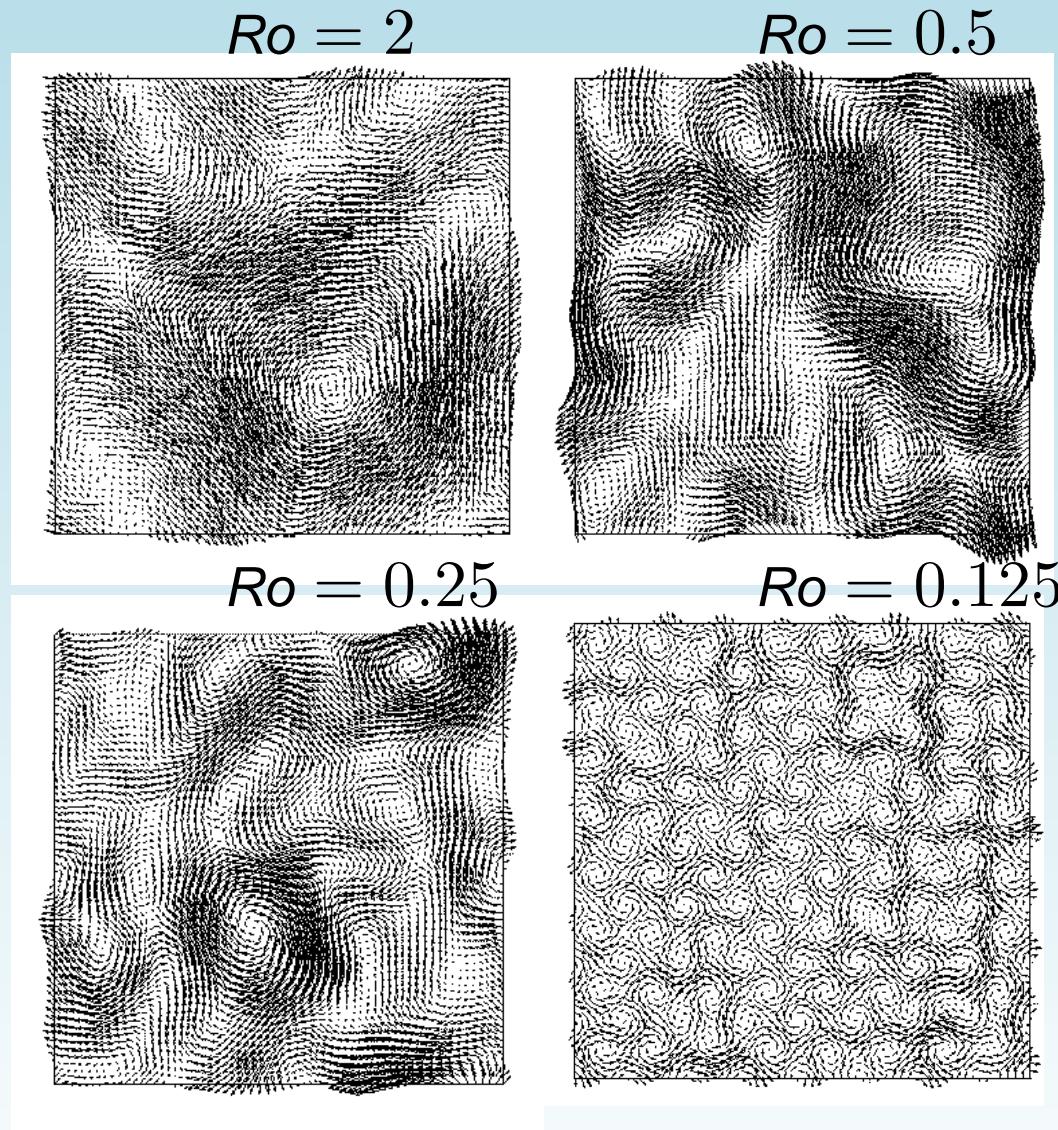


Figure 14: Plot of the velocity field at fixed Reynolds number $Re = 100$ and variable Rossby number.

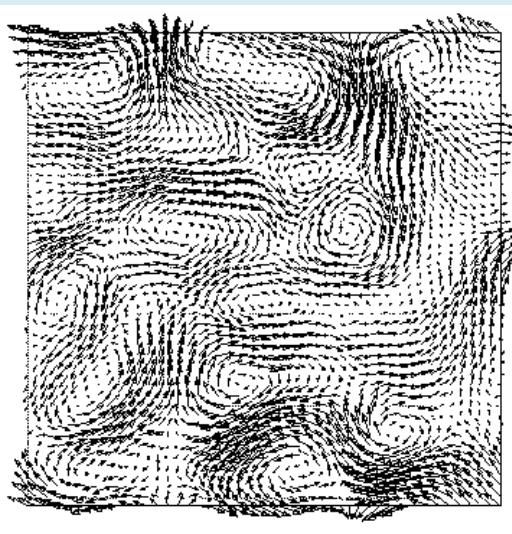
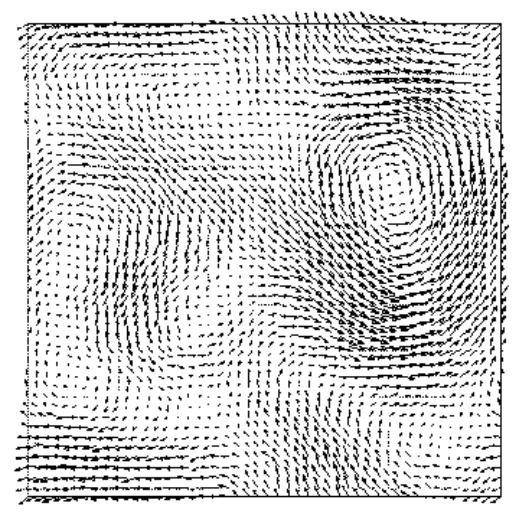


Figure 15: Qualitative comparisons between flow visualisation in (a) the experiment by Hopfinger,

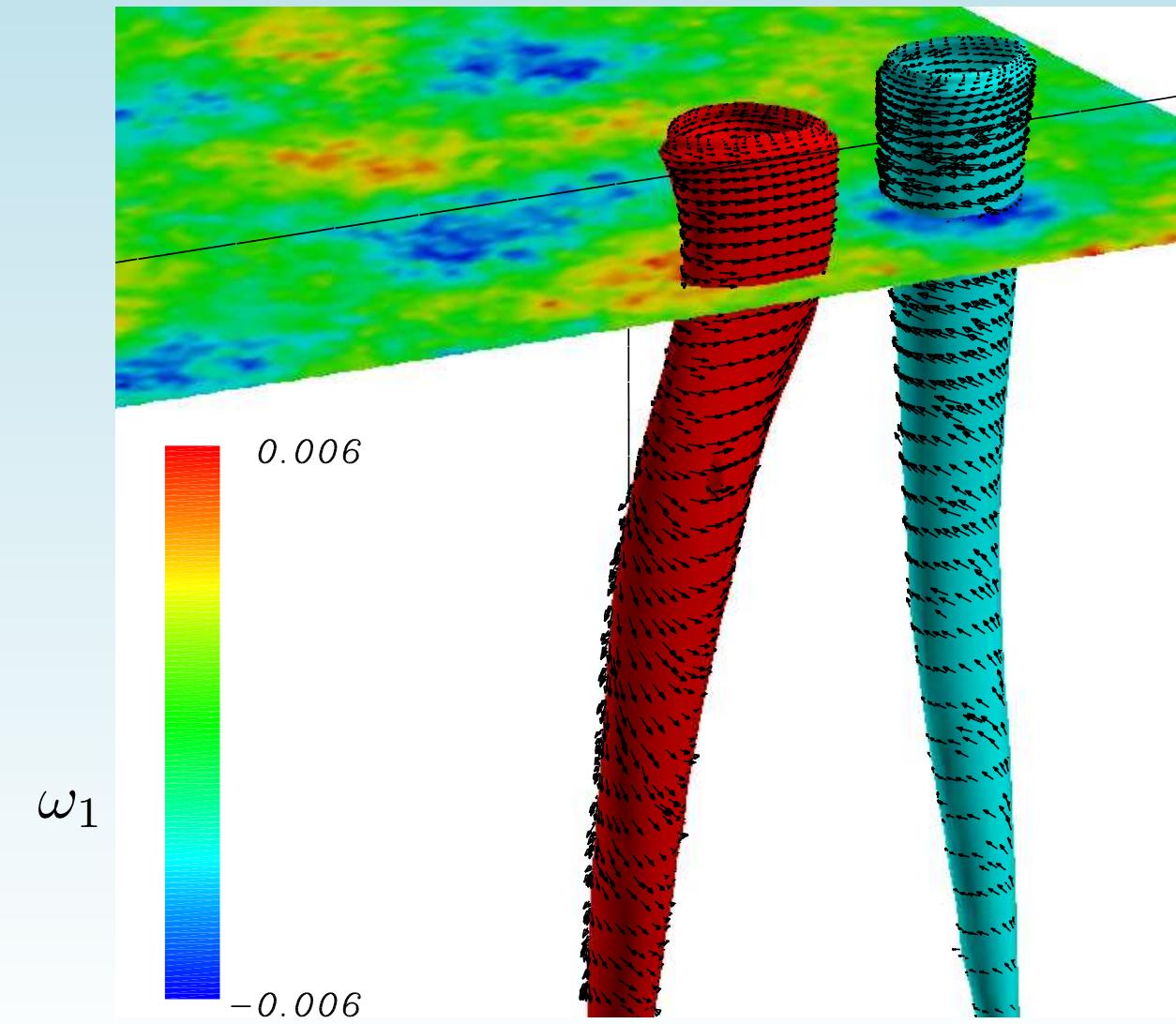


Figure 16: Close view of two structures identified using the NAM criterion in run R3 close to

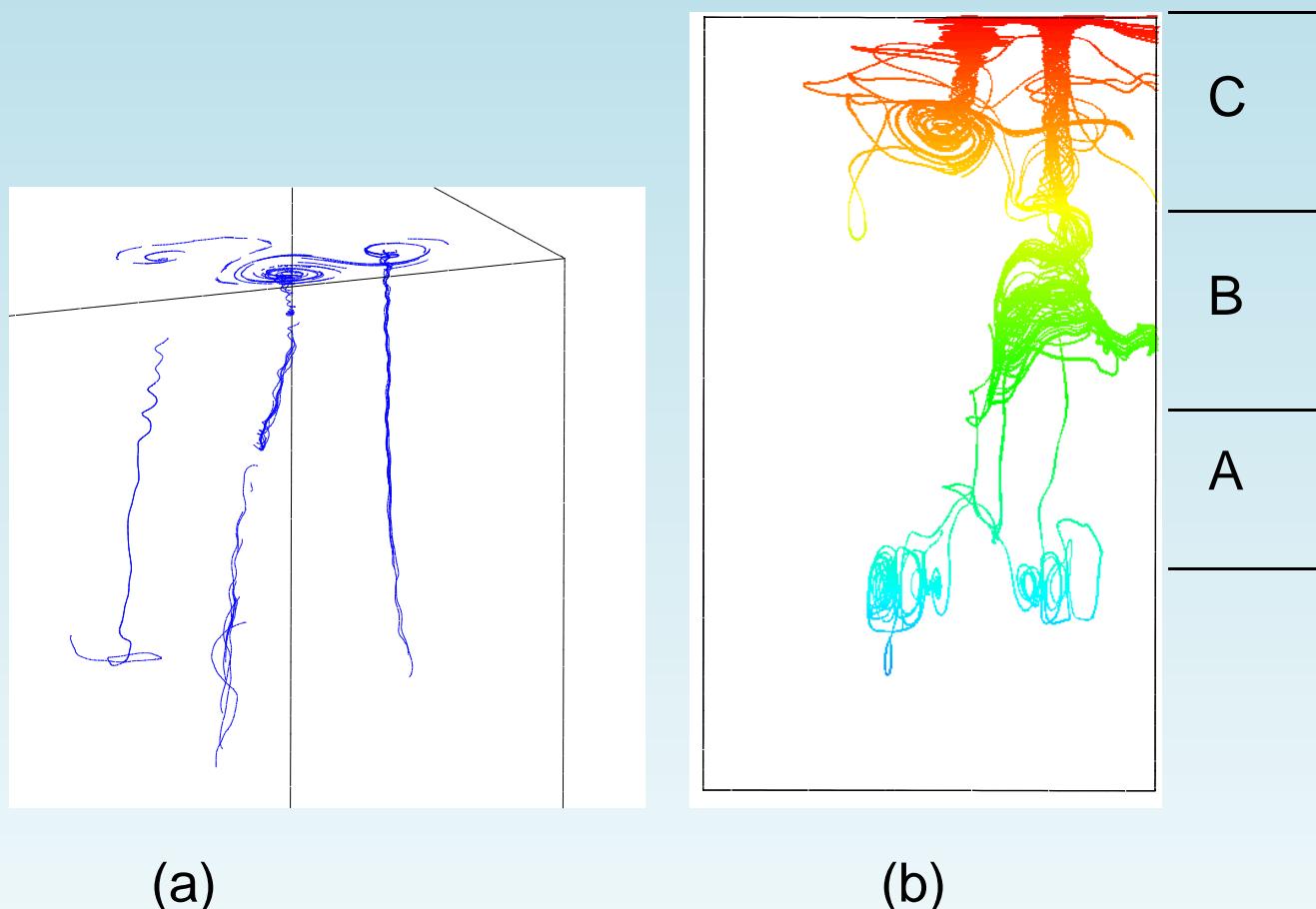


Figure 17: (a) materialisation of three vortices by particle paths computed in the instantaneous velocity field. The central vortex is cyclonic. (b) regions of the flow identified by the behaviour of particle paths for run R3 (the view is projected onto a vertical plane).

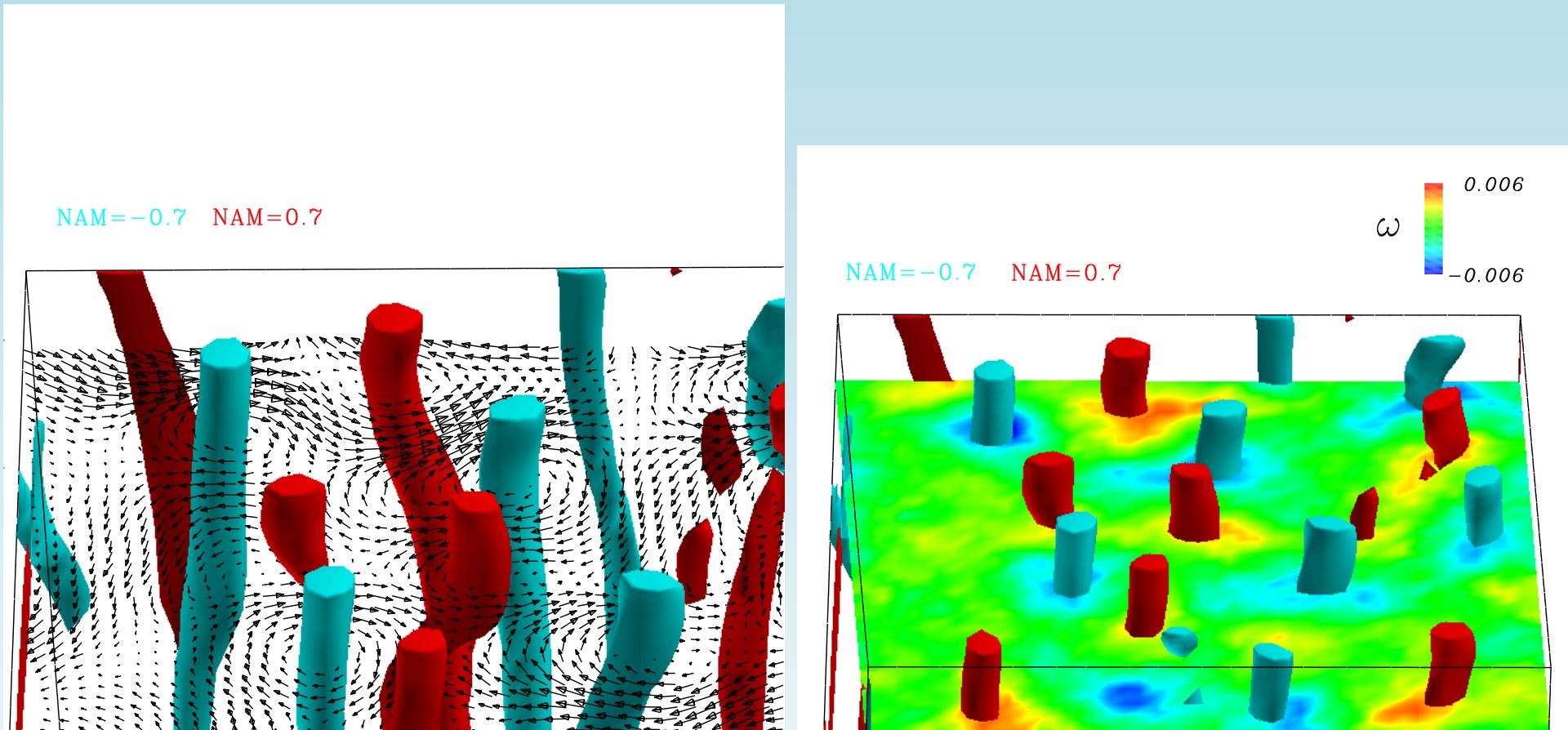


Figure 18: For run R3: (a) plot of the velocity field in a horizontal plane, and of the three-dimensional structures identified with the N.A.M. criterion. (b) of iso-lines of vertical vorticity and three-dimensional structures identified by the N.A.M. criterion.

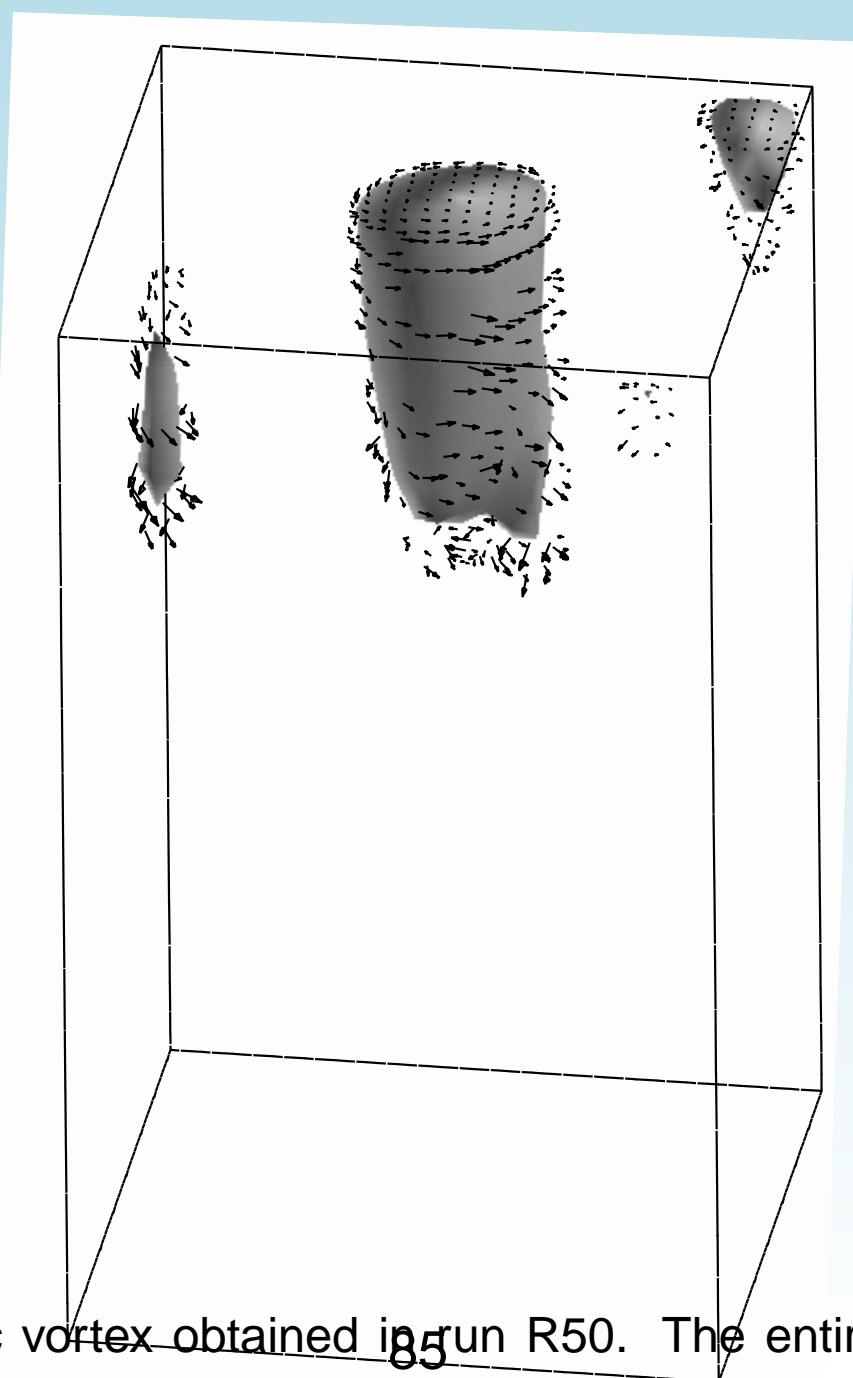
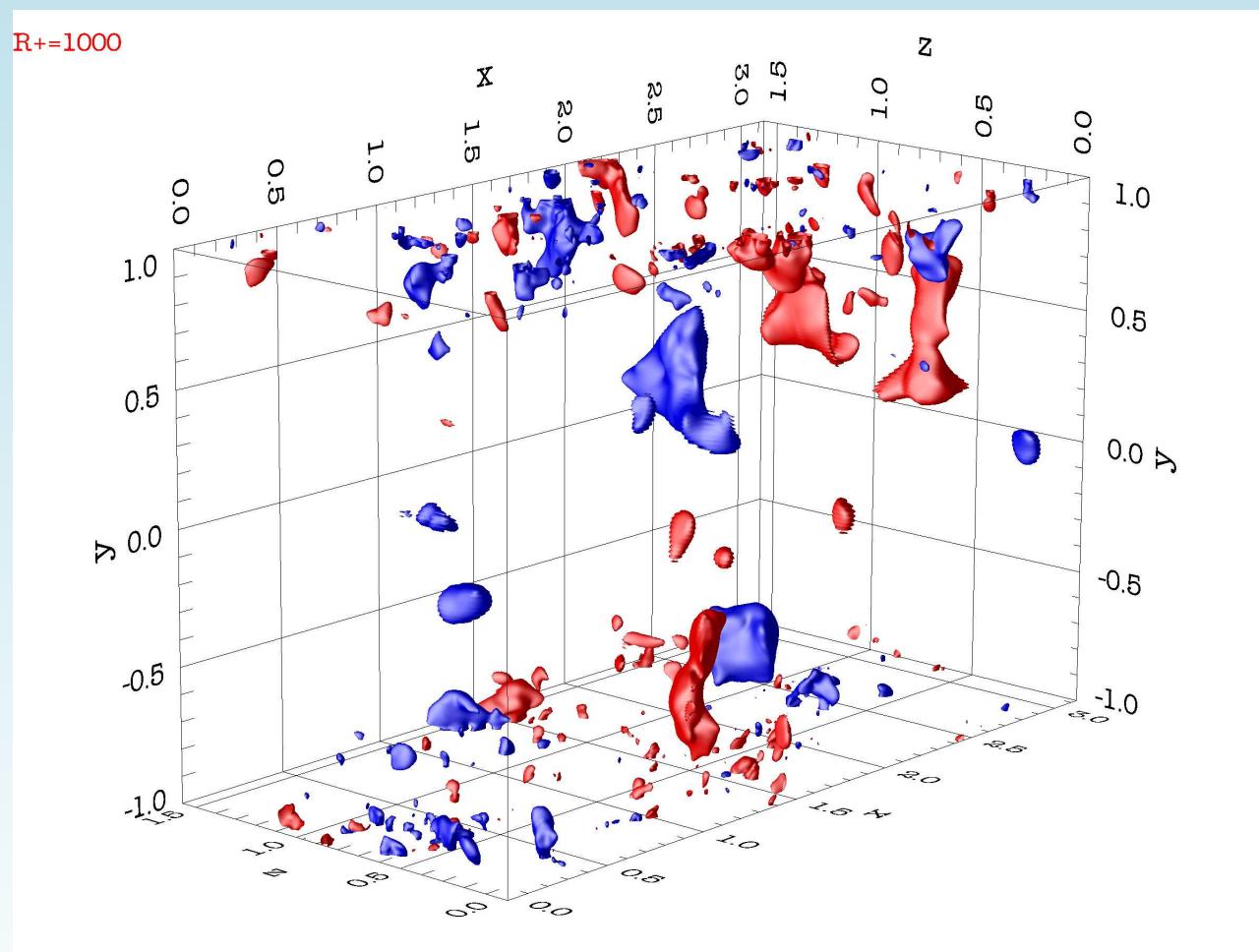


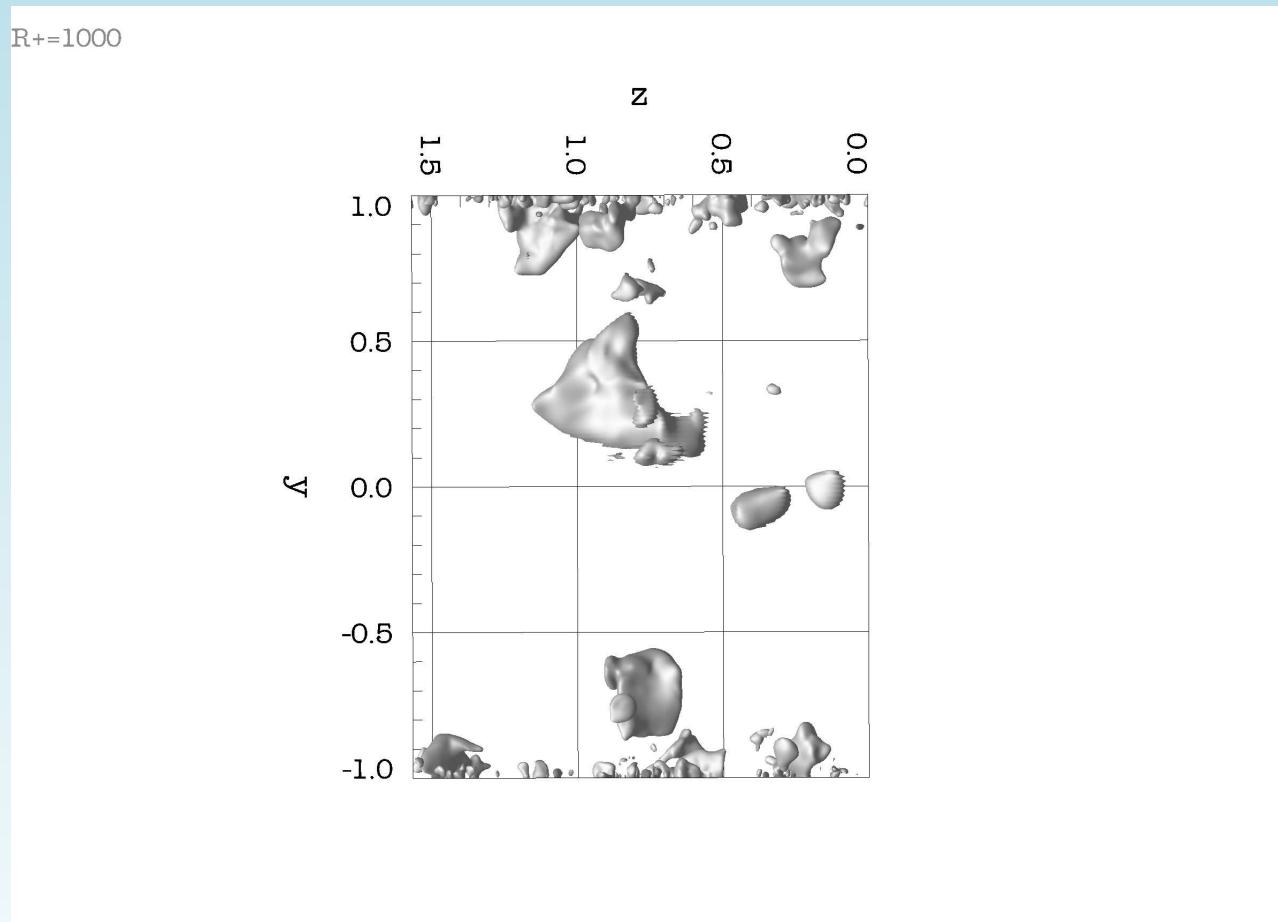
Figure 19: Isolated cyclonic vortex obtained in run R50. The entire computational domain be-

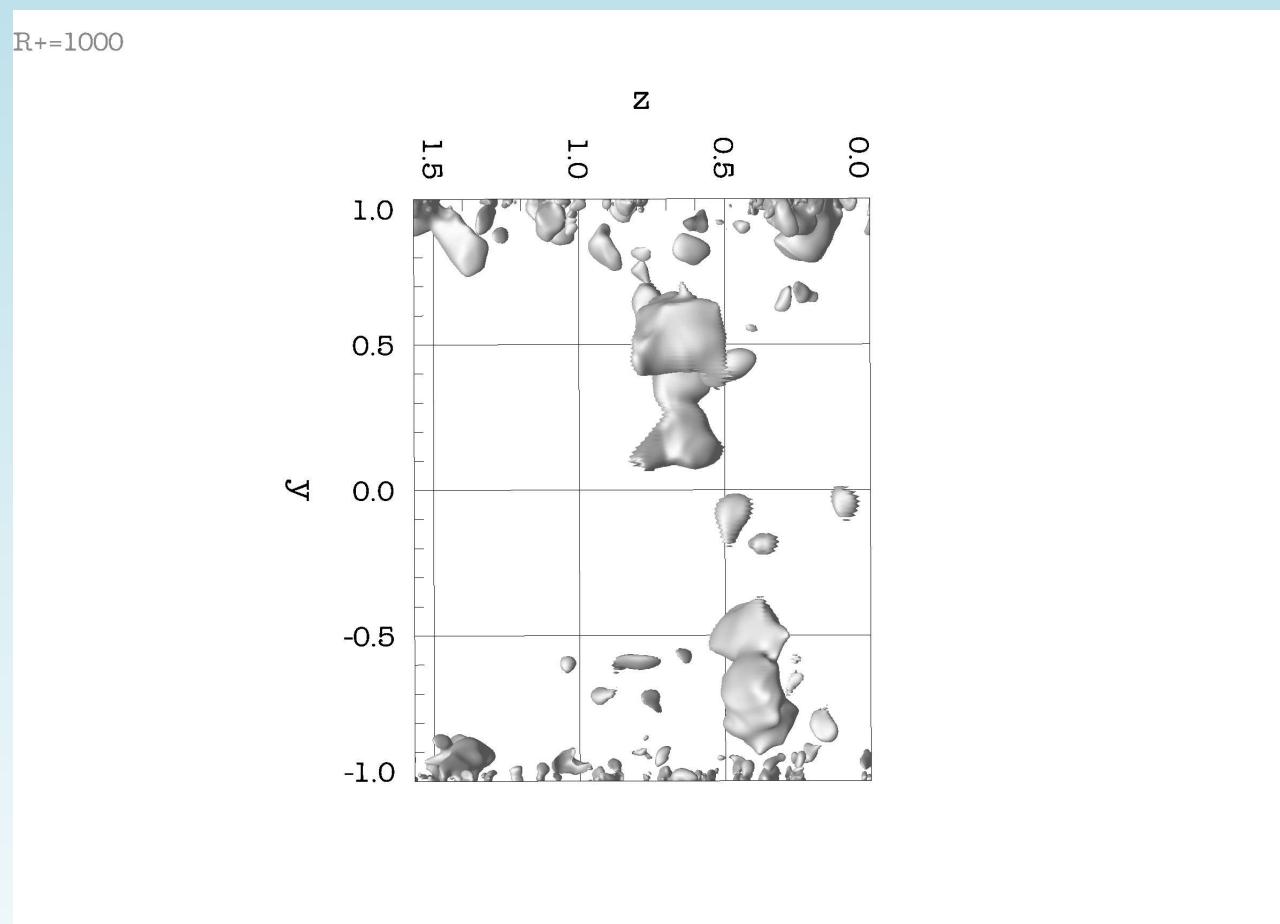
Canal plan

Faouzi Laadhari (LMFA), 2003–2005

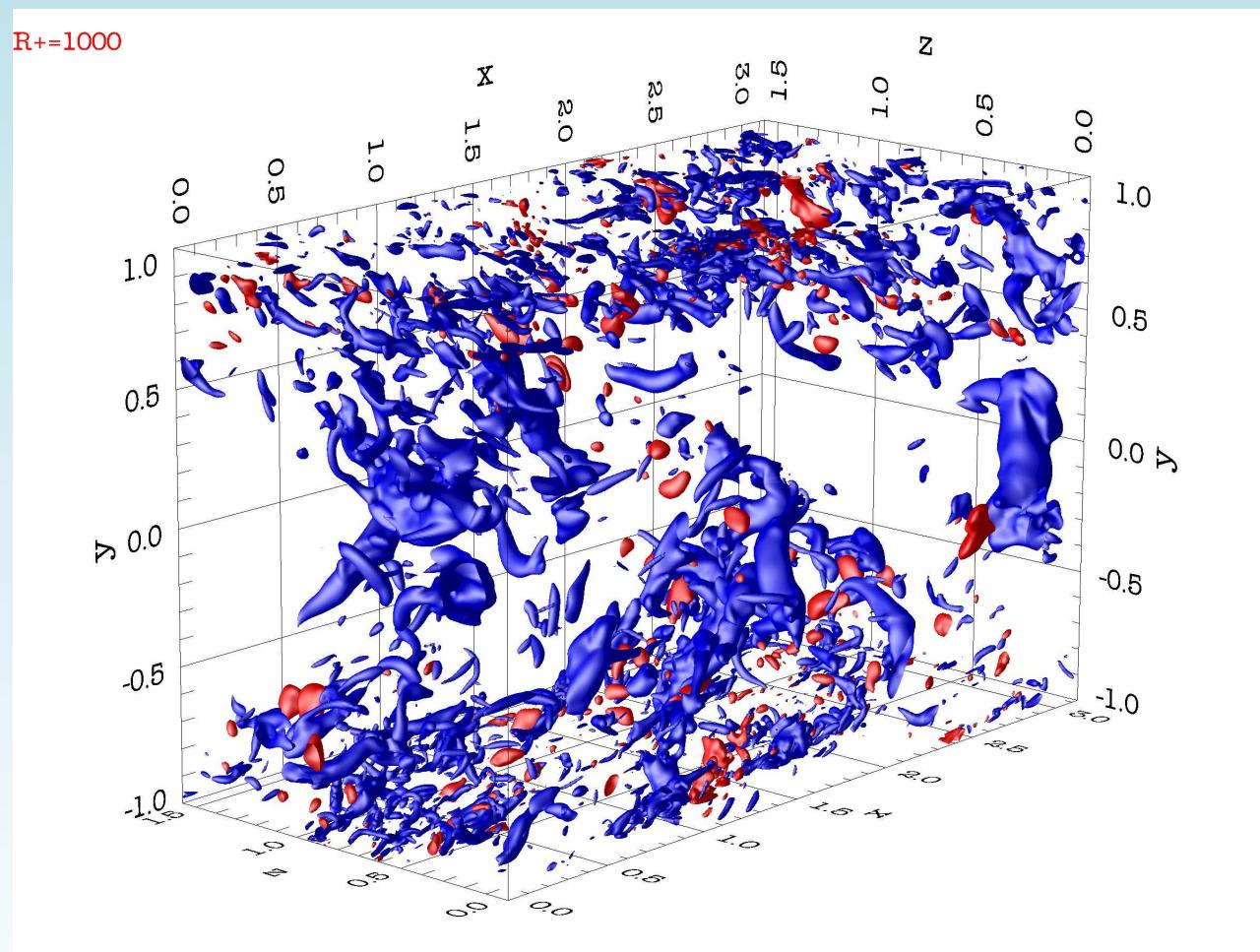
- Critère de **pression linéaire**



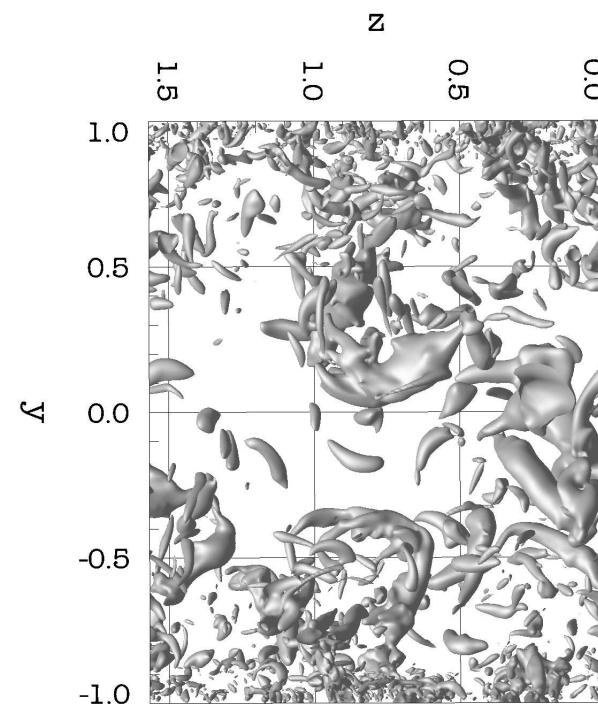




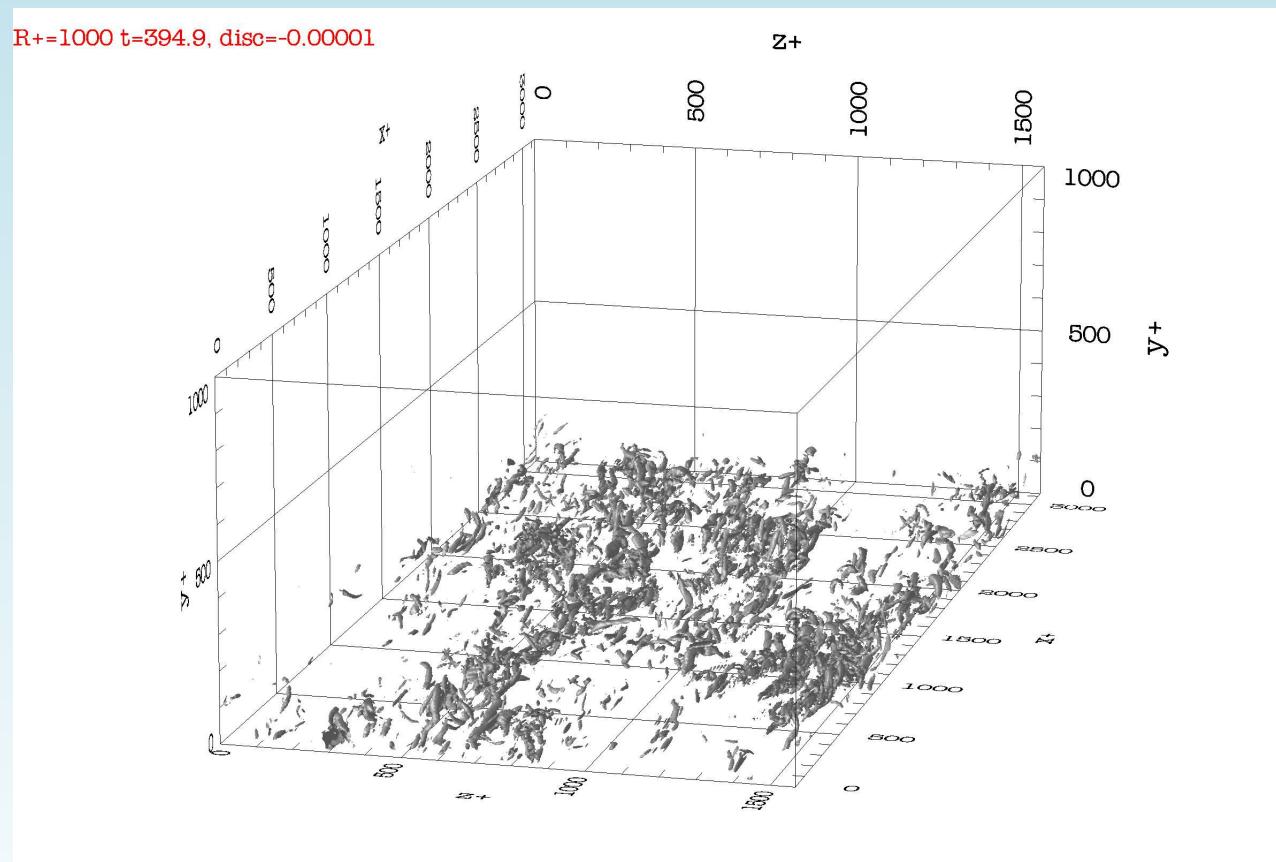
- Critère de **pression non linéaire**

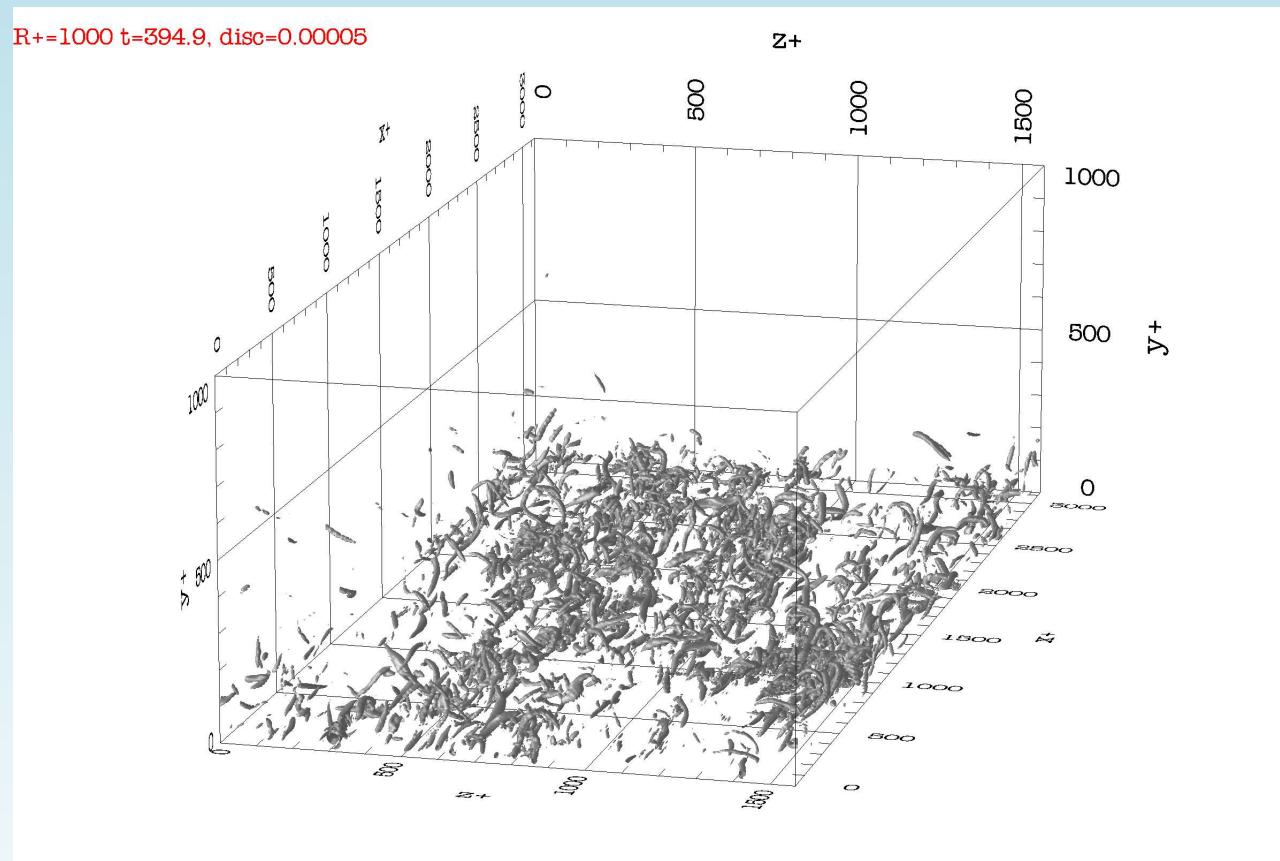


$R_+ = 1000$

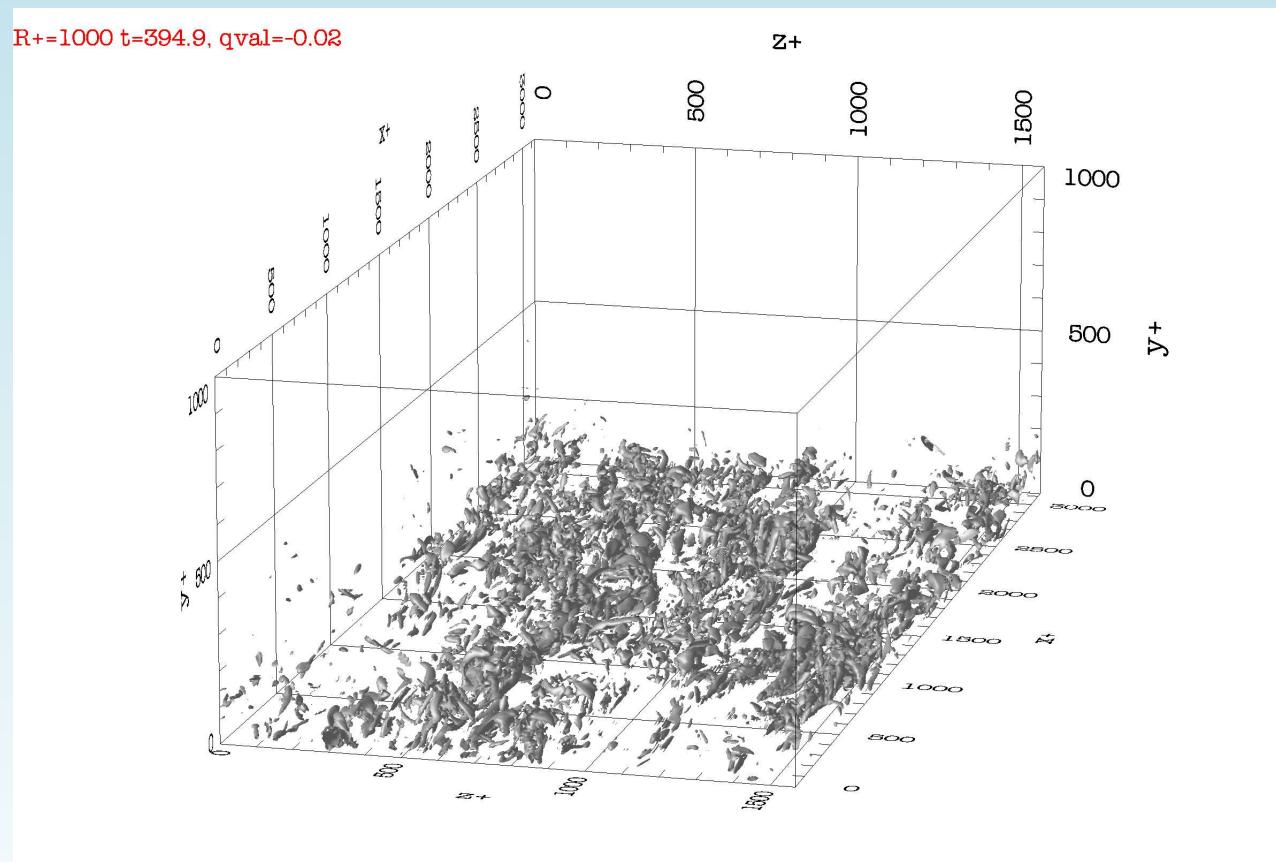


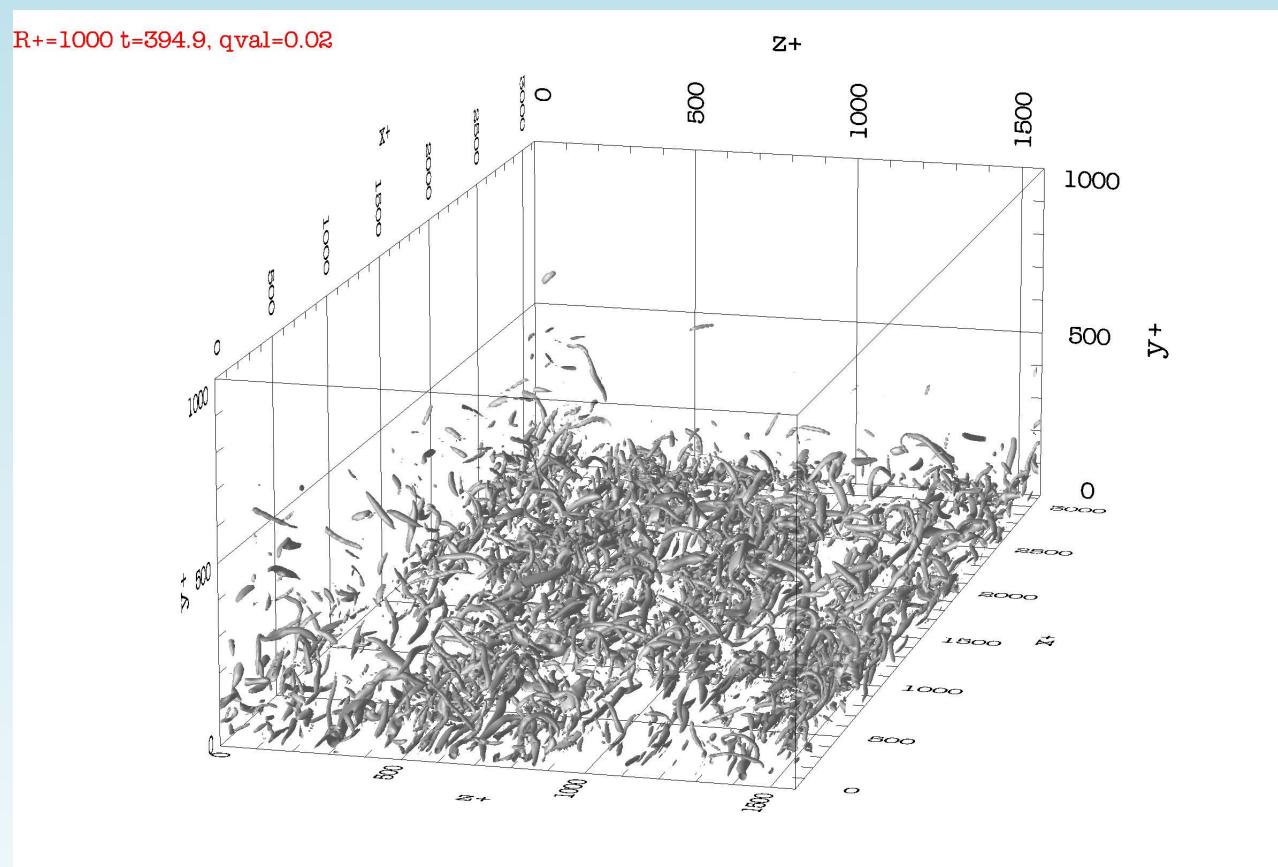
- Critère du **discriminant** [-0.00001;0.00005]

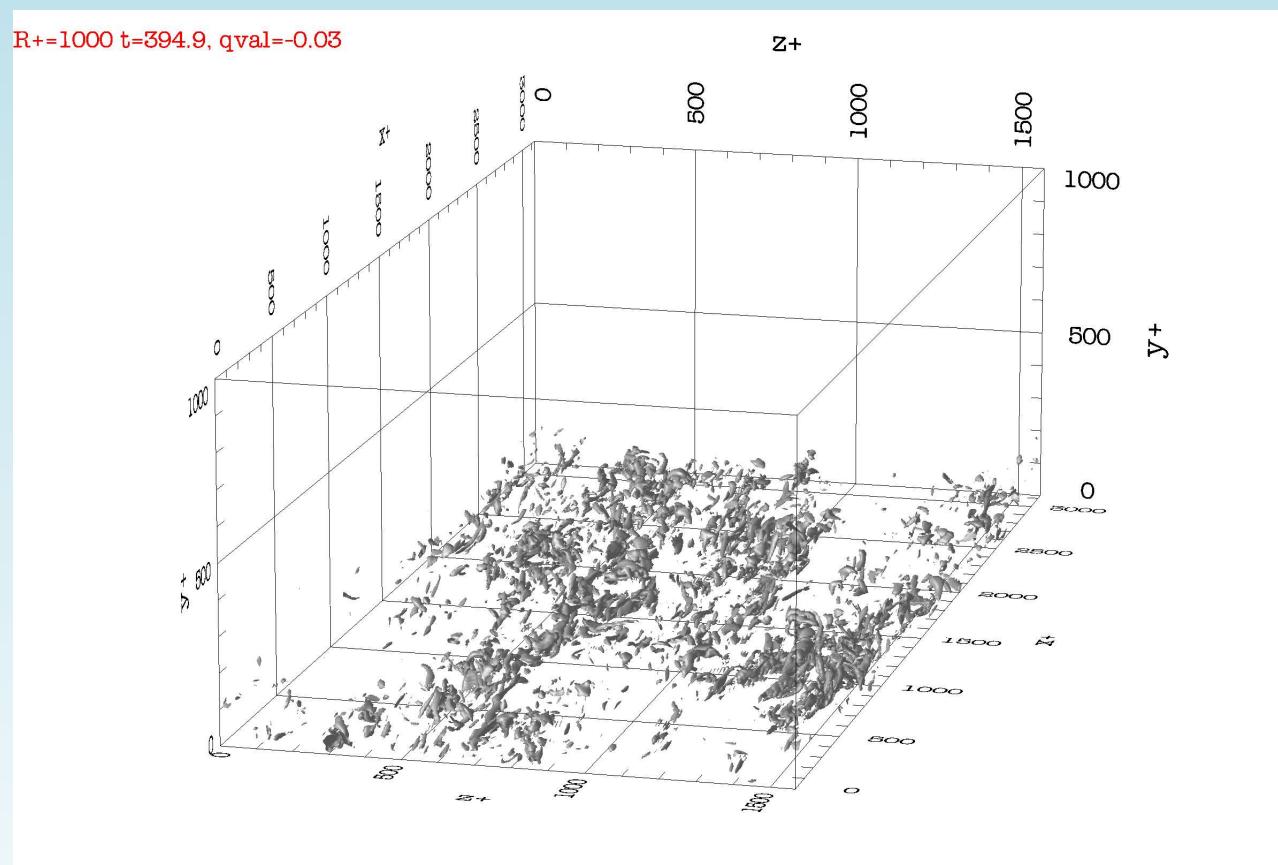


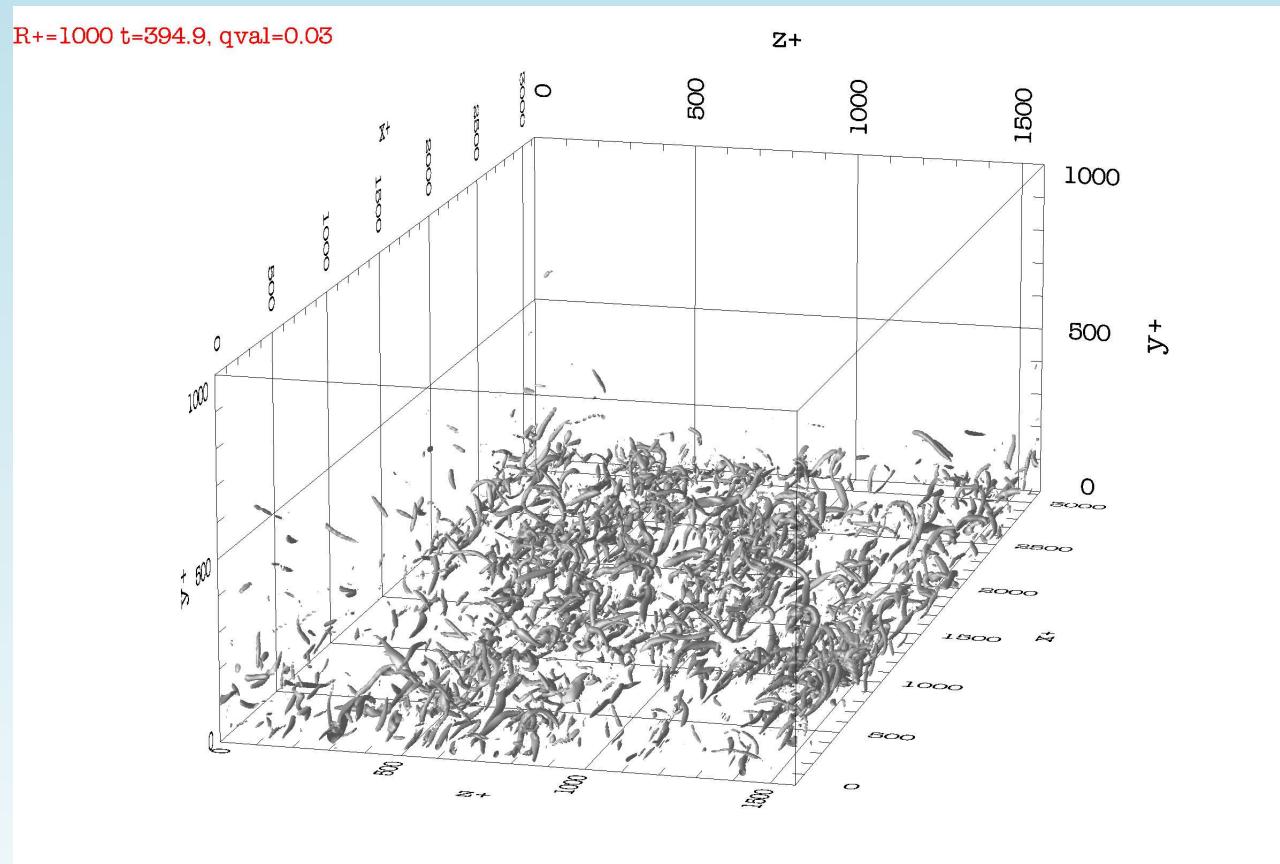


- Critère q [-0.02;0.02;-0.03;0.03]

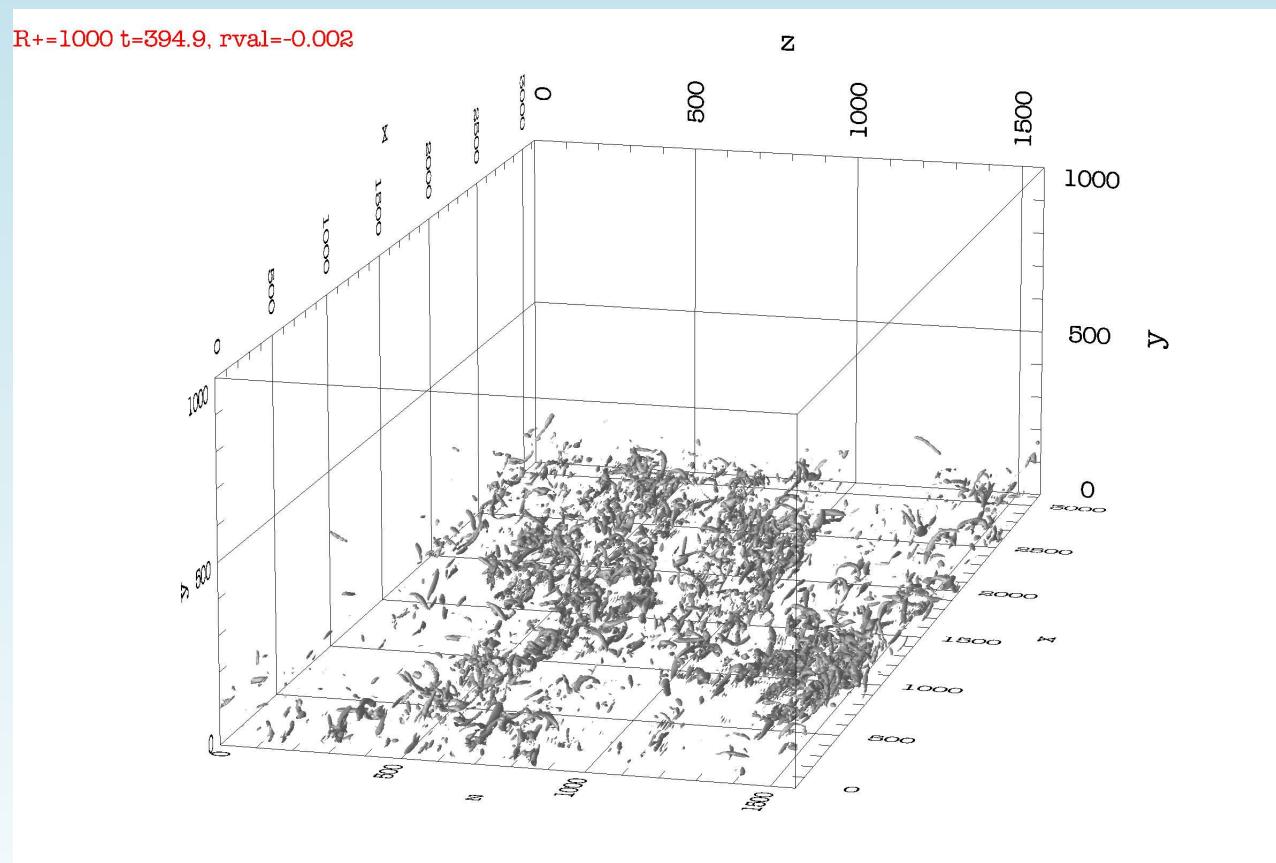


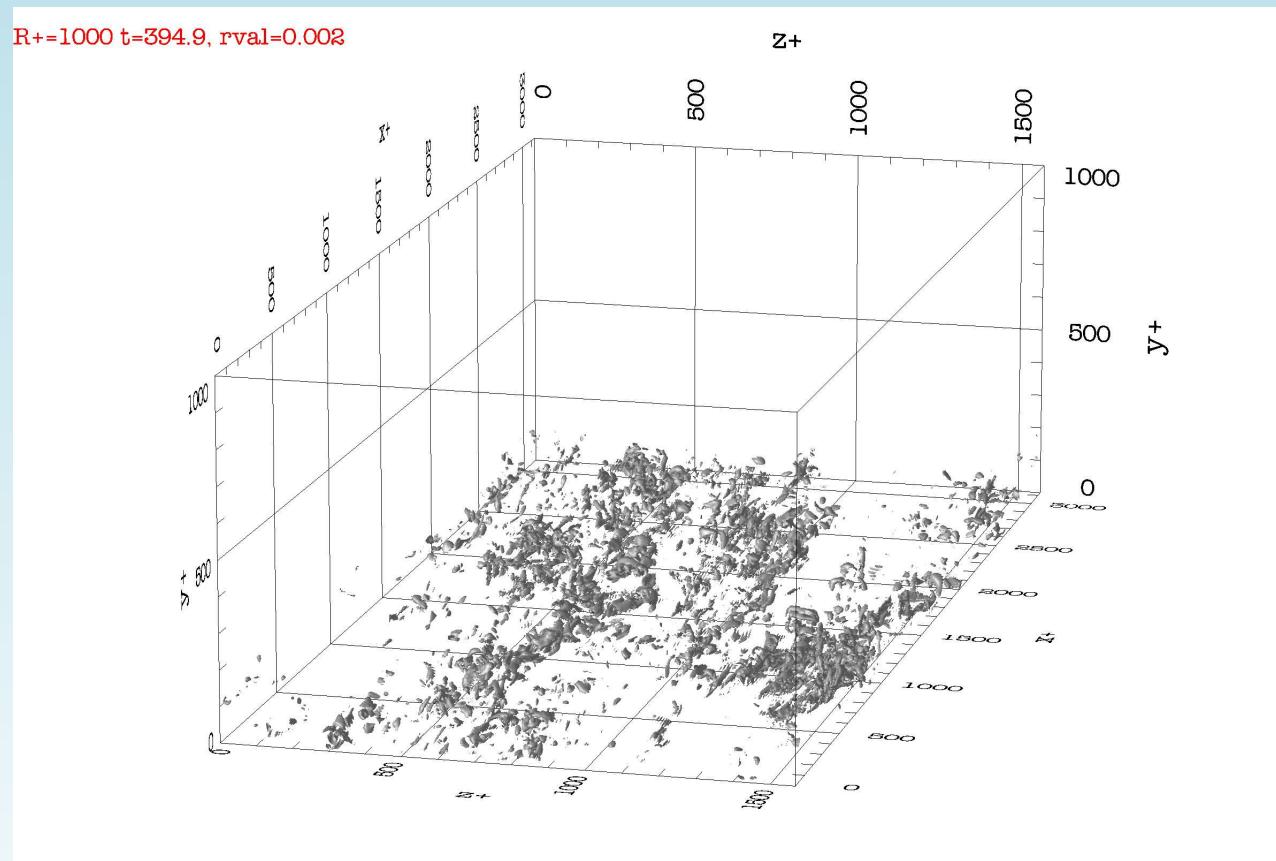




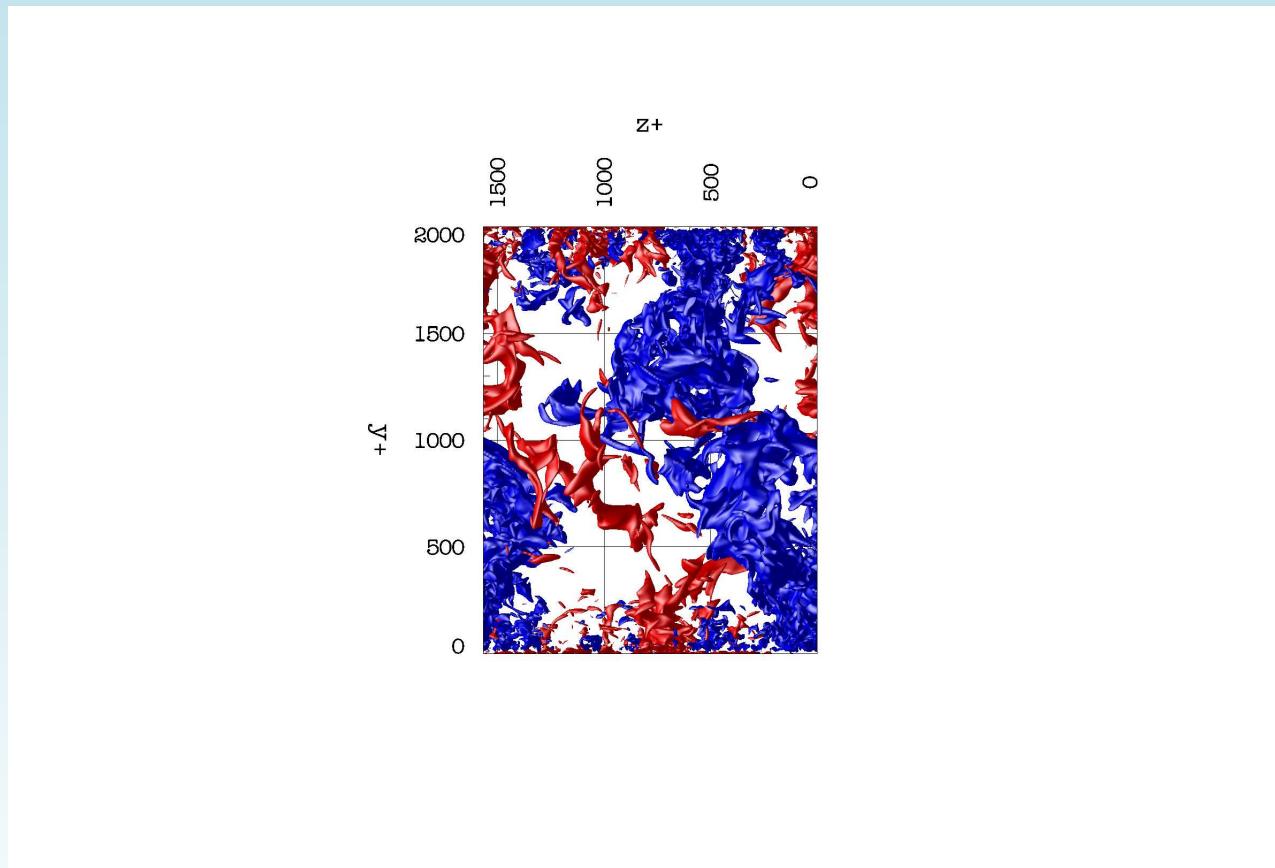


- Critère r [-0.002;0.002]

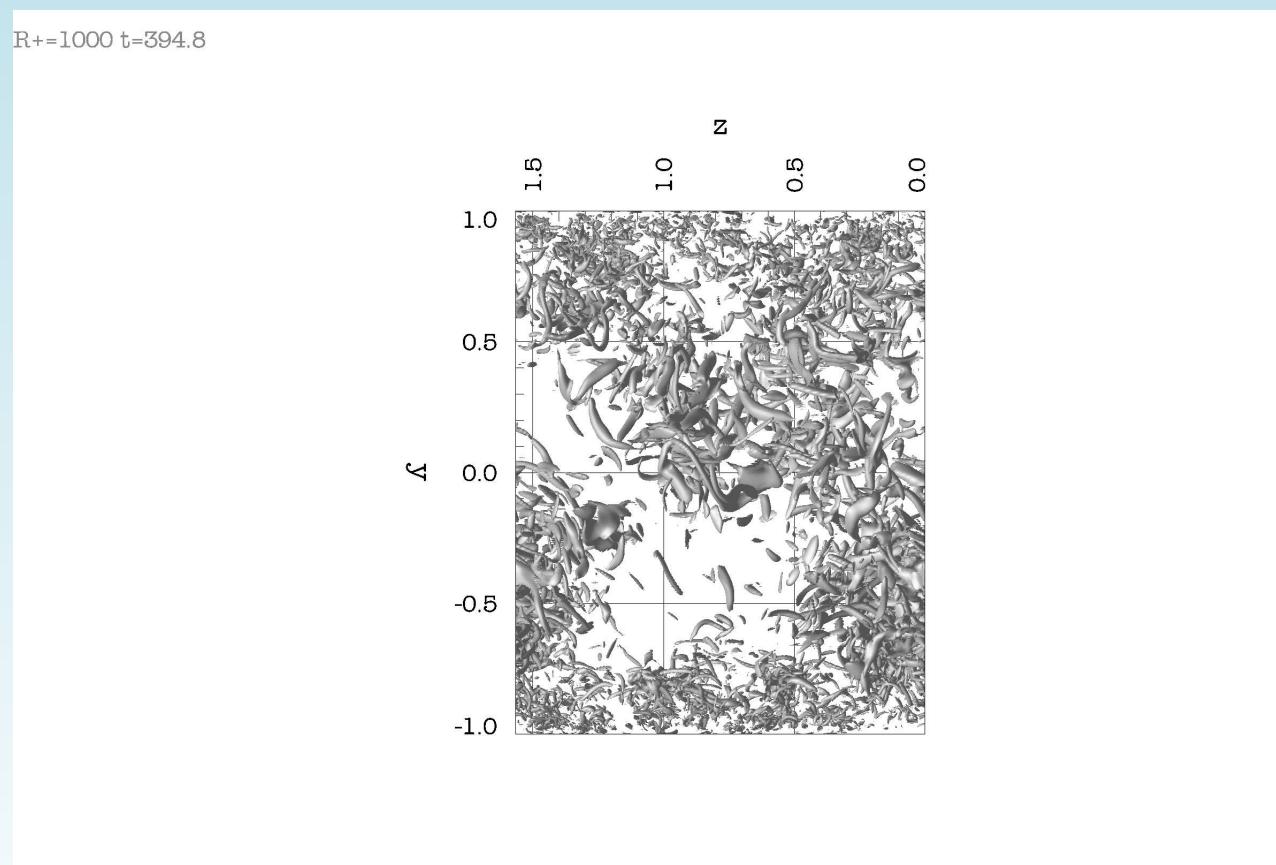




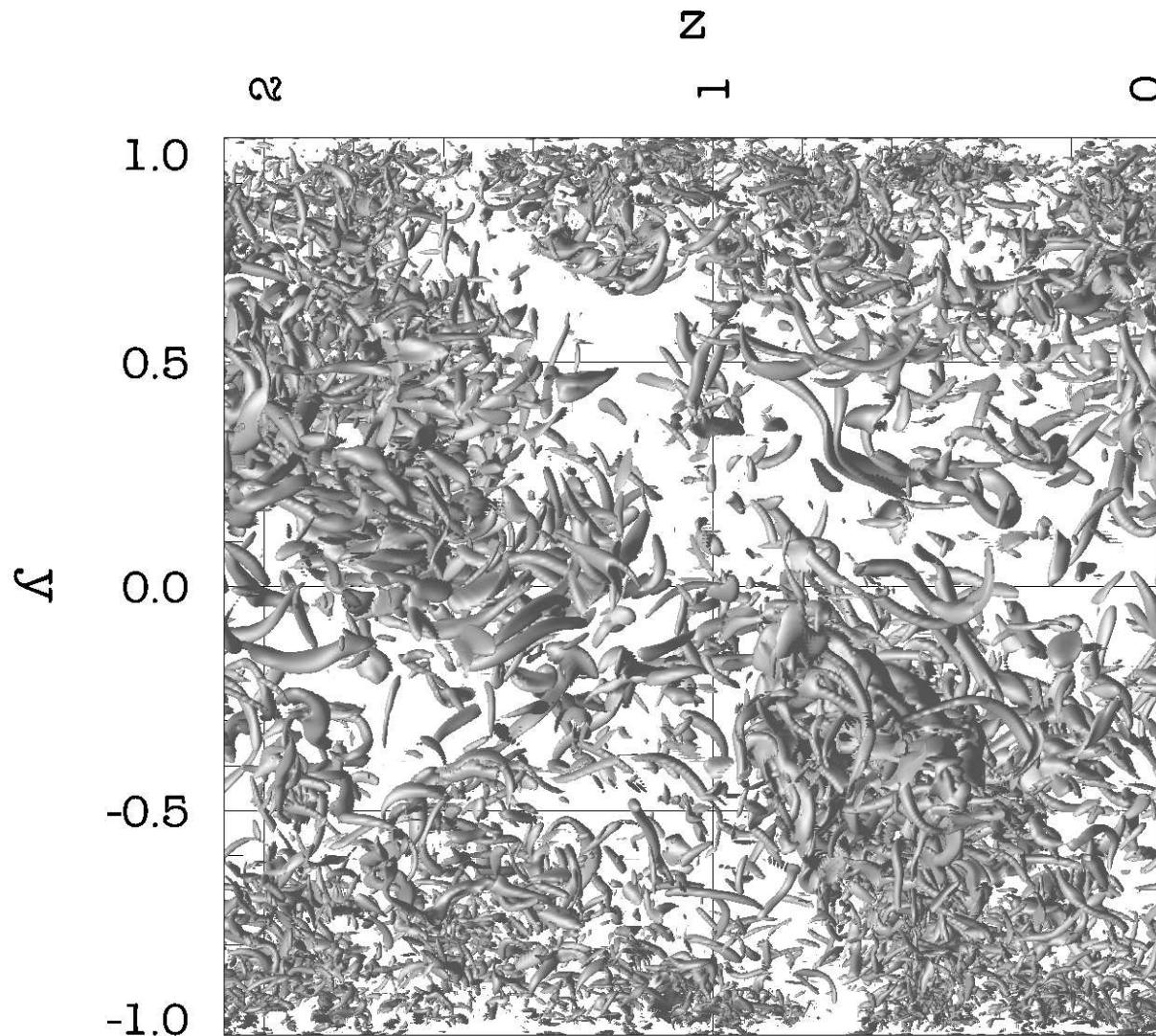
- Vitesse longitudinale fluctuante

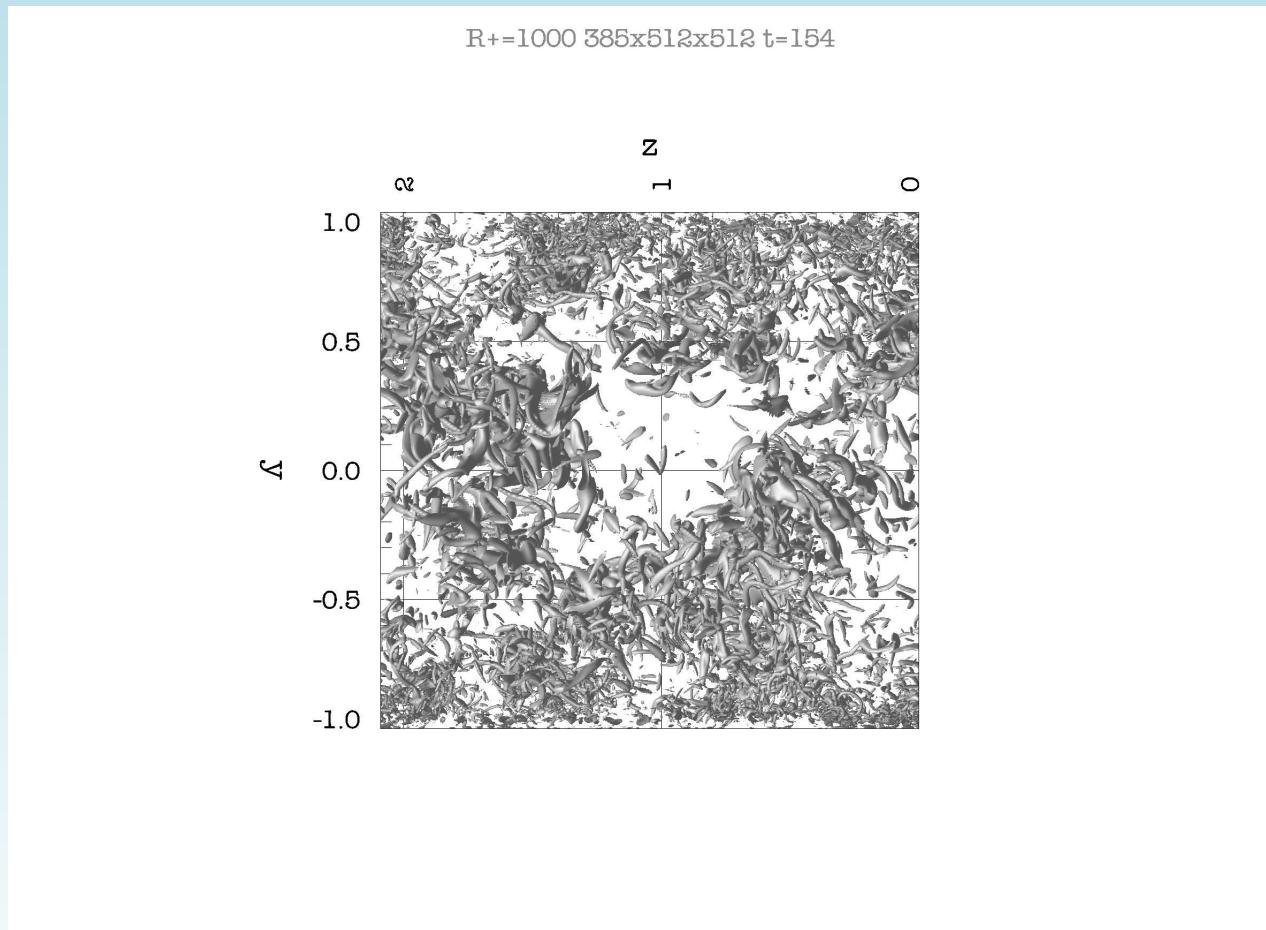


- Enstrophy

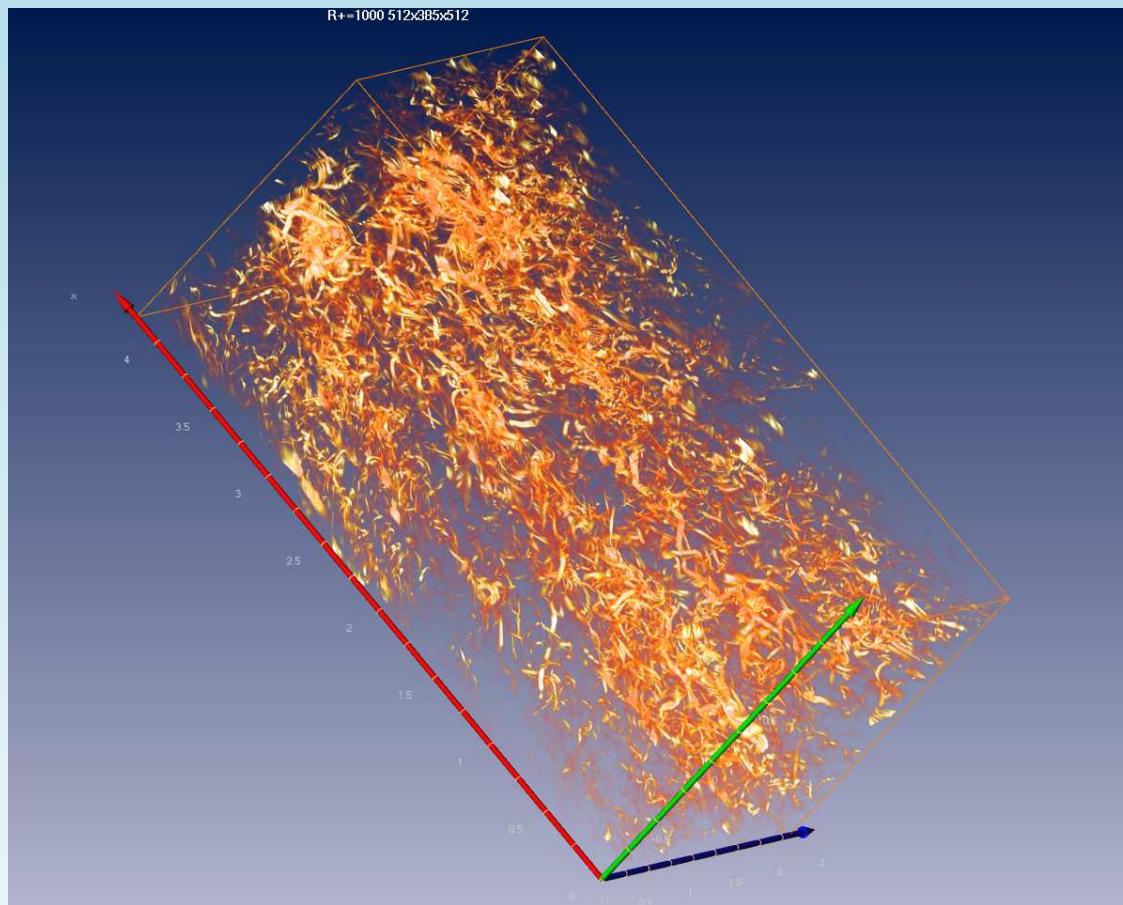


R+=1000 385x512x512 t=143





- Visualisation “volumique” (Amira)

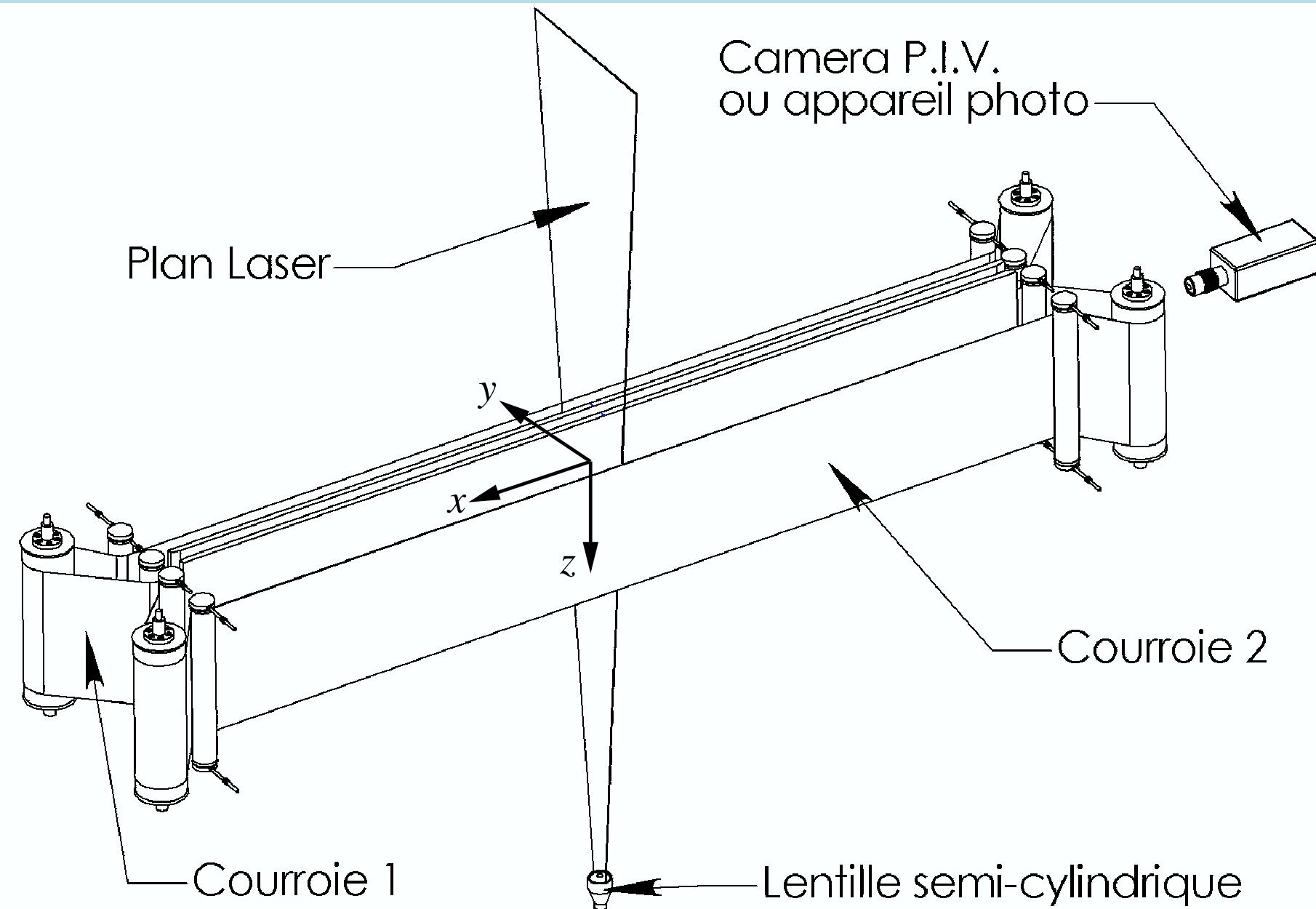


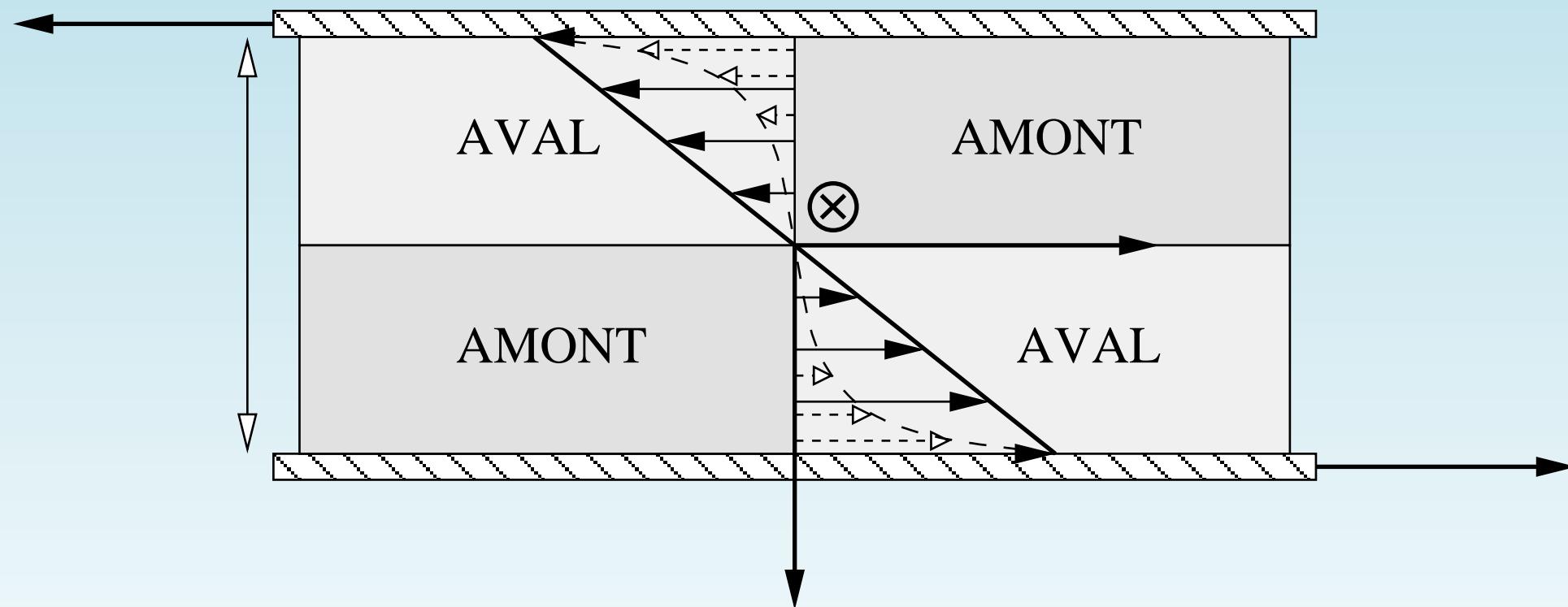
- Animation : `animvorth+=1000b.miff` canal plan turbulent : vorticité longitudinale [perspective]
 - Animation : `animvorth+=1000_end.miff` [vue de bout arrière]

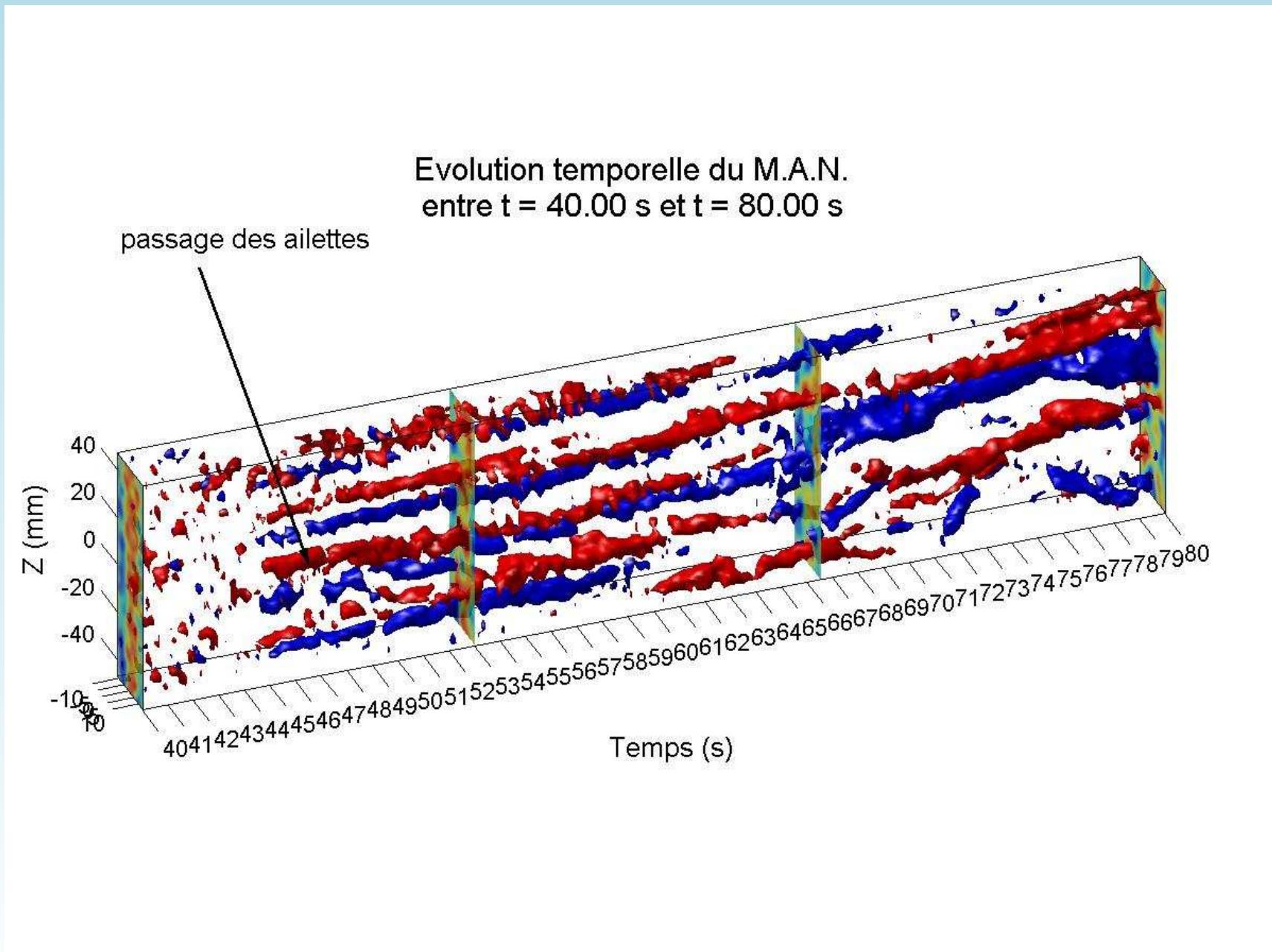
- Animation : `animworth+=1000front.miff` [vue de bout avant]
- Animation : `coutourb_3_sim_6pi_nam_v3d_u_neg_0_159.miff` canal plan $L = 6\pi$ “transitionnel” : fluctuations < 0 de vitesse longitudinale + NAM longitudinal
- Animation : `coutourb_3_sim_6pi_nam_v3d_u_pos_0_159.miff` canal plan $L = 6\pi$ “transitionnel” : fluctuations > 0 de vit. longitudinale + NAM longitudinal
- Animation : `coutourb_3_sym_3pi_nam_u_neg_v3d.miff` canal plan $L = 3\pi$ “transitionnel” : fluctuations négatives de vit. longitudinale + NAM longitudinal

Écoulement de Couette

Étude expérimentale par Stéphane Amalfi et Faouzi Laadhari (LMFA), 2002–2005.







Étude et modélisation de l'écoulement sur les aubages

de la roue mobile du compresseur dans le concept MTFD3

Fabien Godefert et Pascale Kulisa (2004)

Géométrie périodique. Maillage structuré.

1 Maillage

La géométrie d'un canal est représentée sur la figure 20, l'axe x étant l'axe de rotation de la roue mobile du compresseur. La périodicité permet de reconstituer la roue complète, qui comporte 34 aubages. Une portion de la roue à quatre aubages est représentée sur la figure 21.

Le maillage est de type multiblocs HOH, à points coïncidants. Le premier domaine de calcul h1, à gauche sur la figure 20, contient un maillage structuré classique en H (figure 23). Puis, en se déplaçant vers l'aval, le domaine o2 autour de l'aubage lui-même est maillé en O (figure 22), et enfin le domaine de sortie h5 est maillé en H (figure 24). La seule partie mobile est le domaine autour de l'aubage, les zones d'entrée et de sortie représentant respectivement la sortie du stator de l'étage compresseur, et la sortie de l'étage complet vers l'aval de la turbomachine.

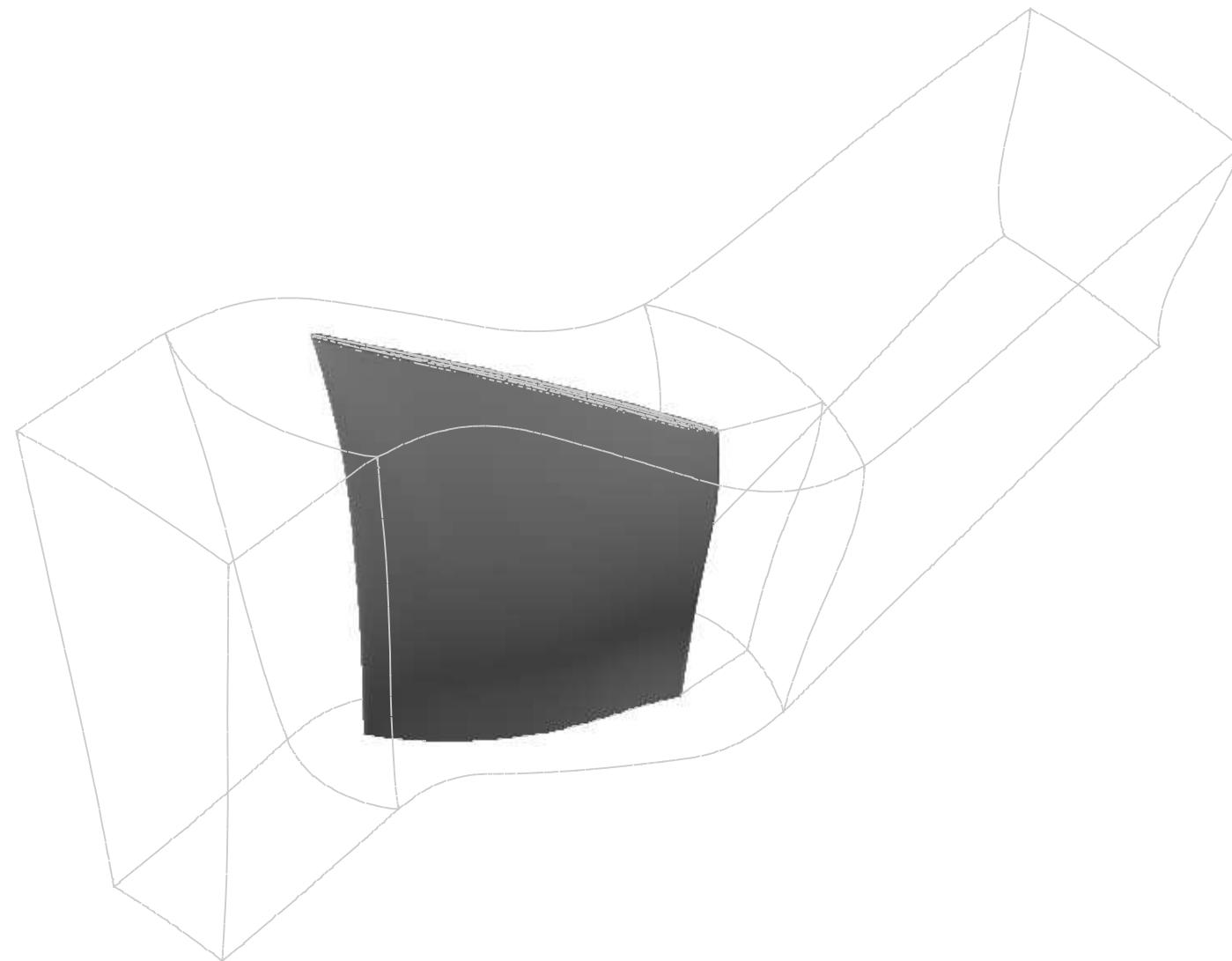
Outre ces deux domaines, des contraintes de calcul rendent nécessaires l'introduction d'un maillage en H dans un domaine en O pour la zone de jeu (figures 25 et 26 respectivement).

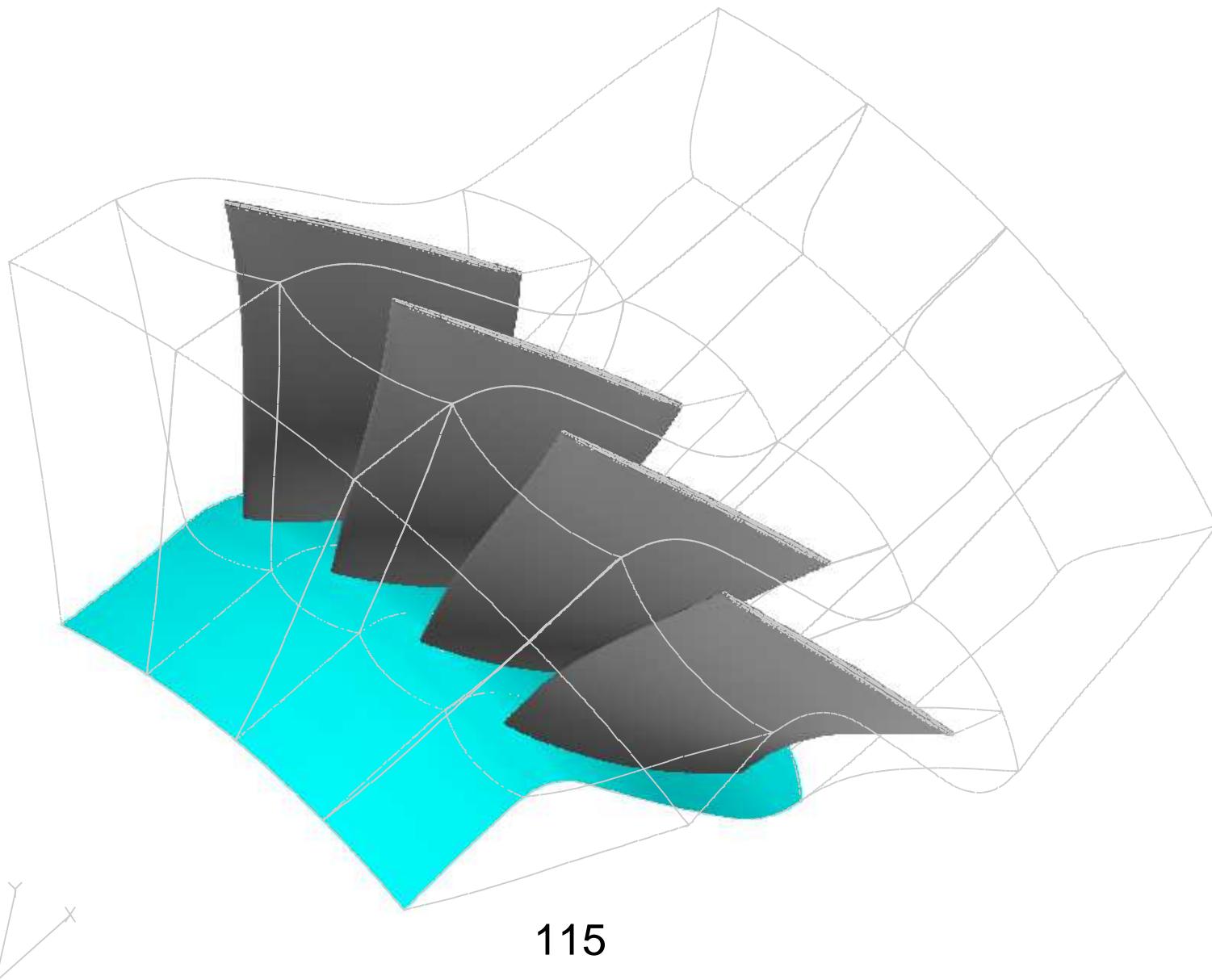
Le maillage complet est constitué de cinq domaines qui totalisent 1 250 682 noeuds. Le détail des résolutions pour chacun des domaines est donné dans le tableau 1.

Domaine	N_i	N_j	N_k	Zone
h1	49	37	69	H amont
o2	201	57	69	O aubage
o3	201	9	15	jeu
h4	76	26	15	jeu
h5	109	37	69	H aval

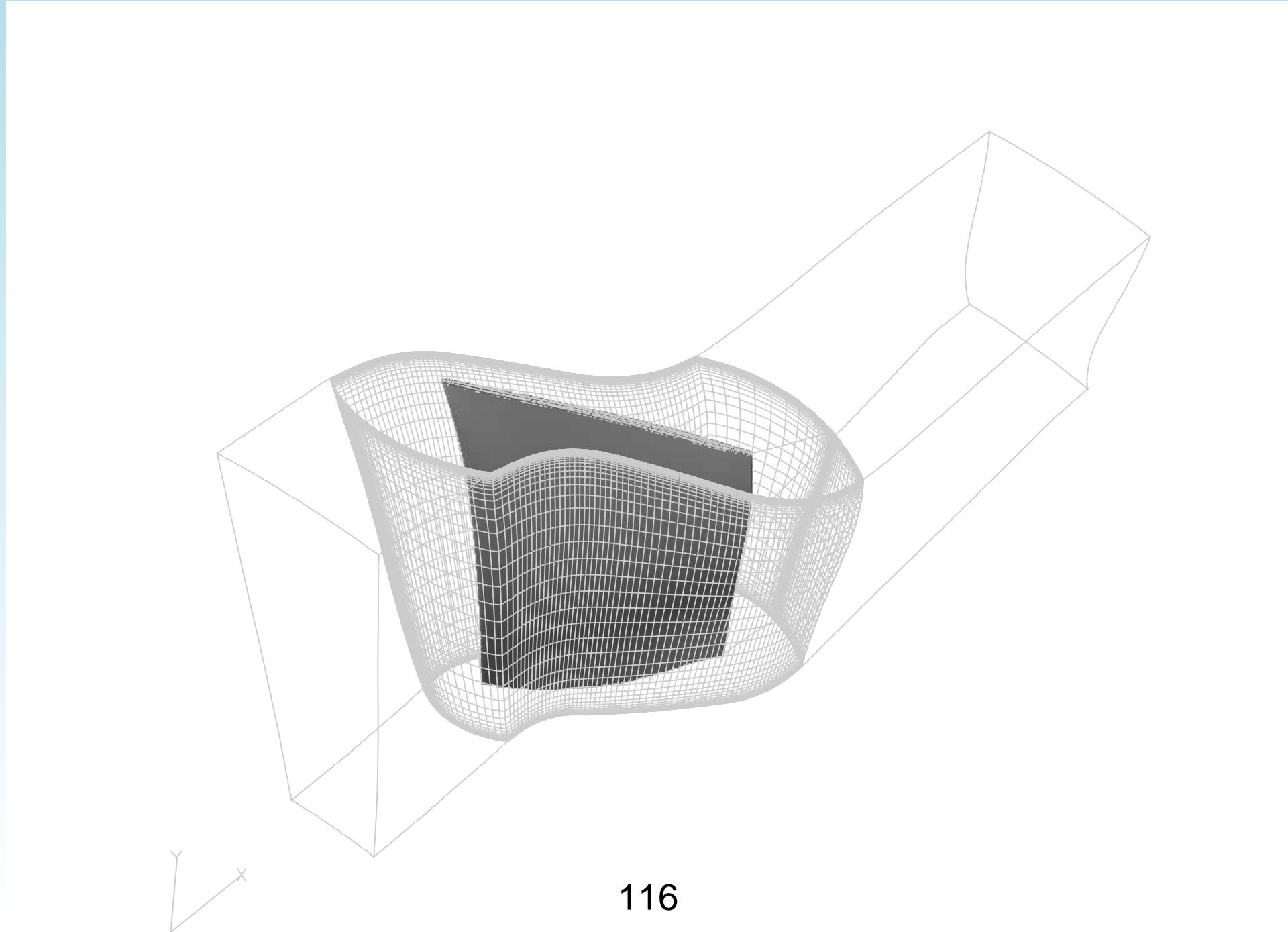
Table 1: Nombre de points pour chacun des domaines du maillage.

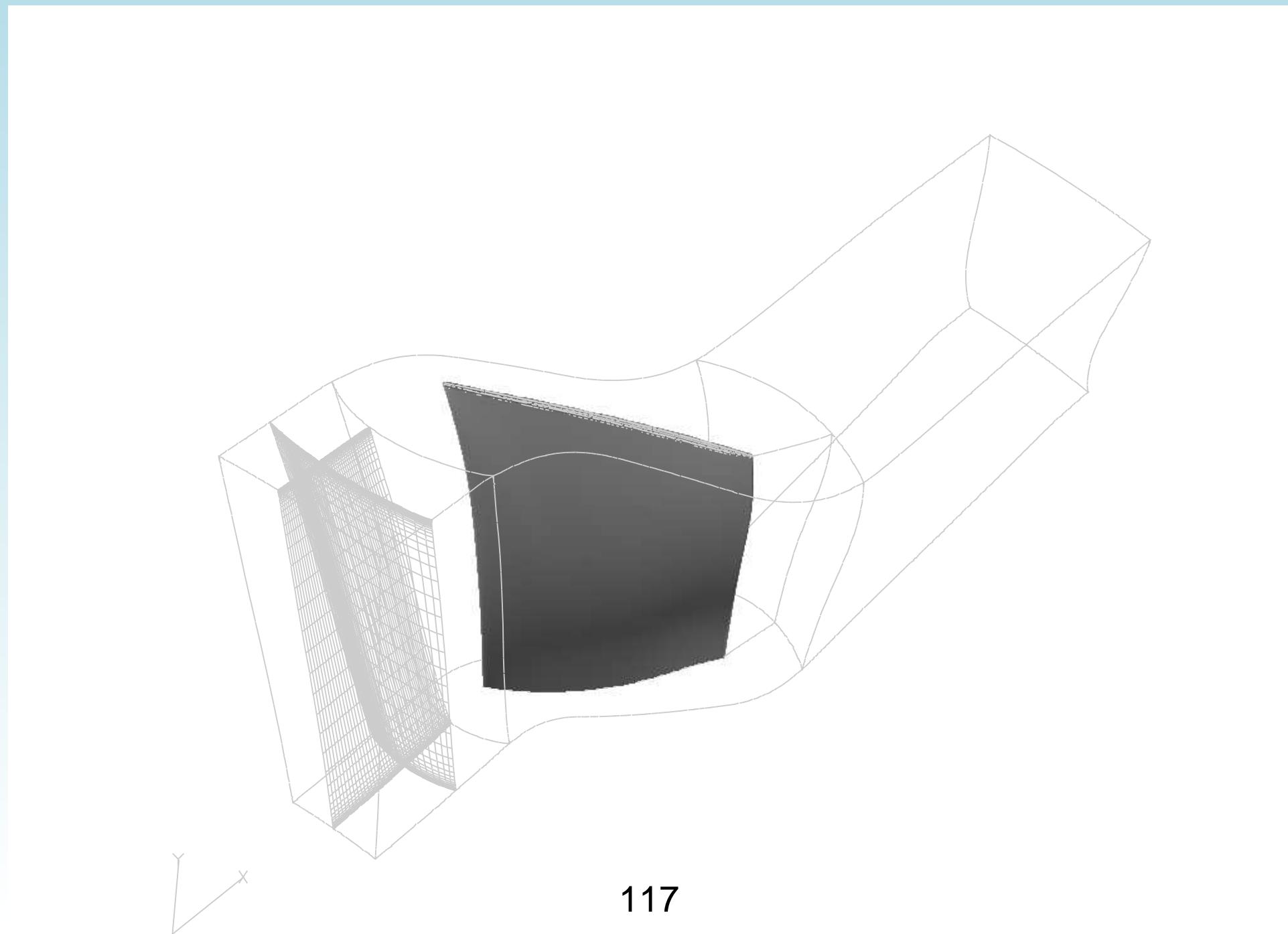
Géométrie et maillage

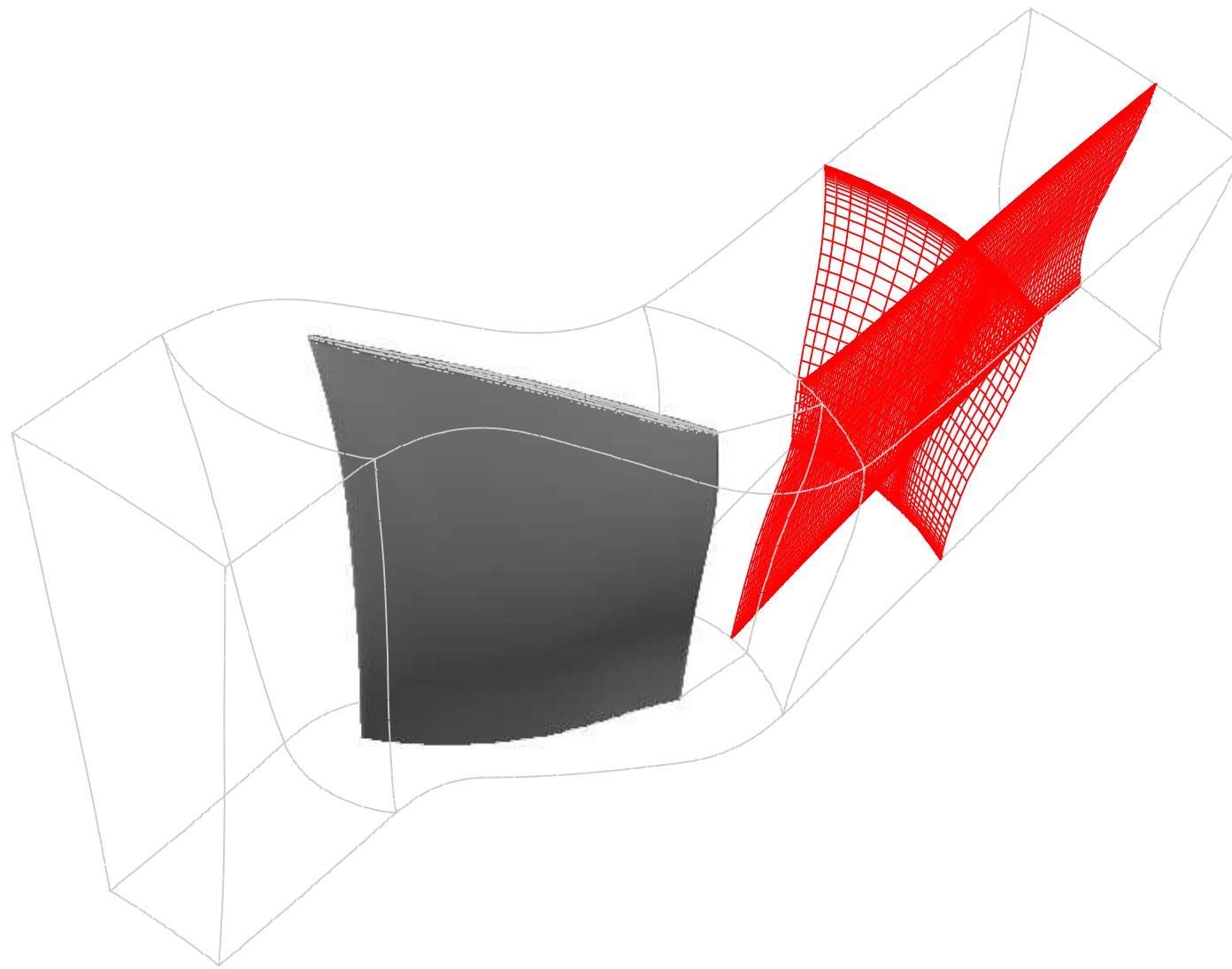


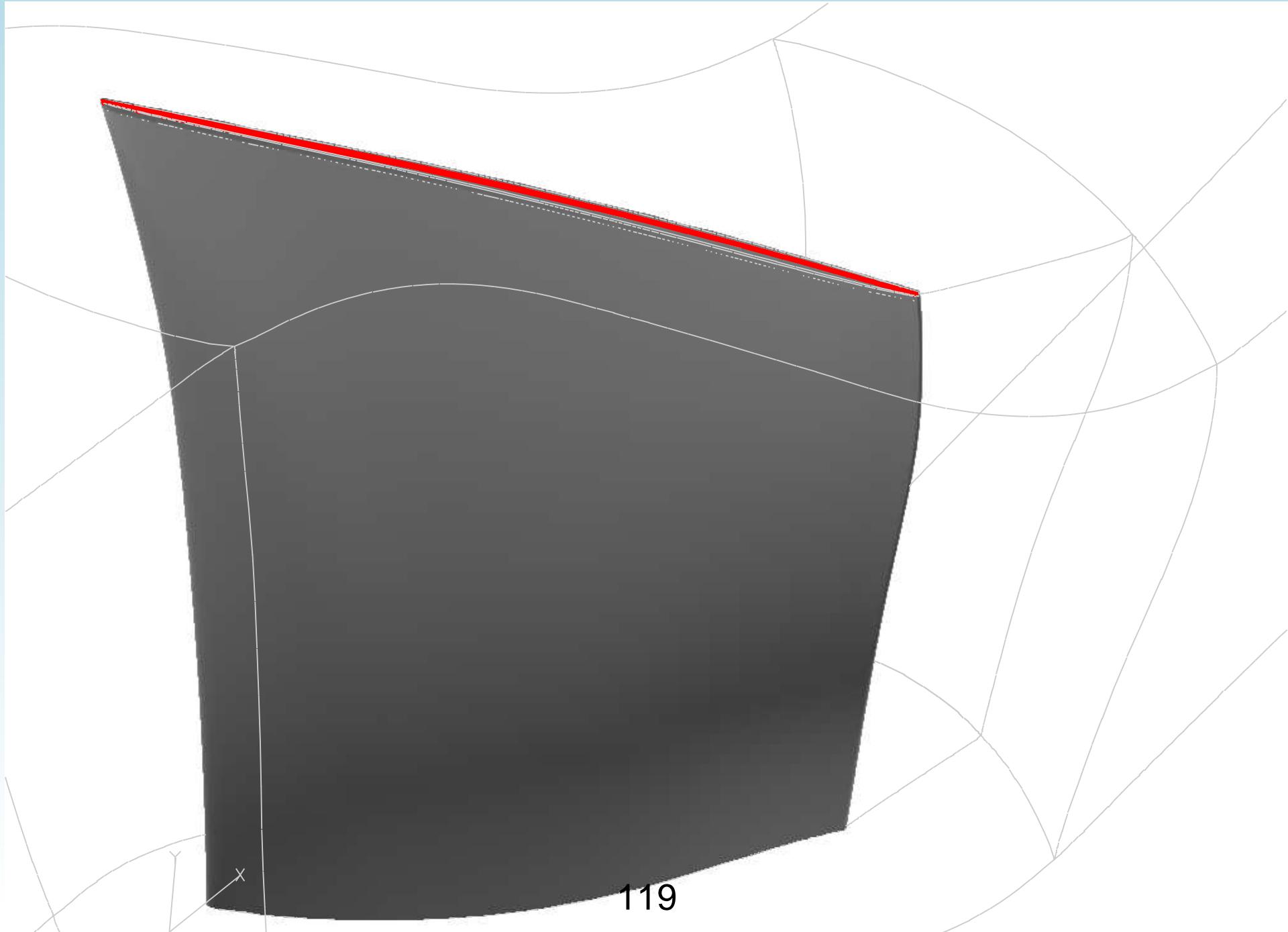


115

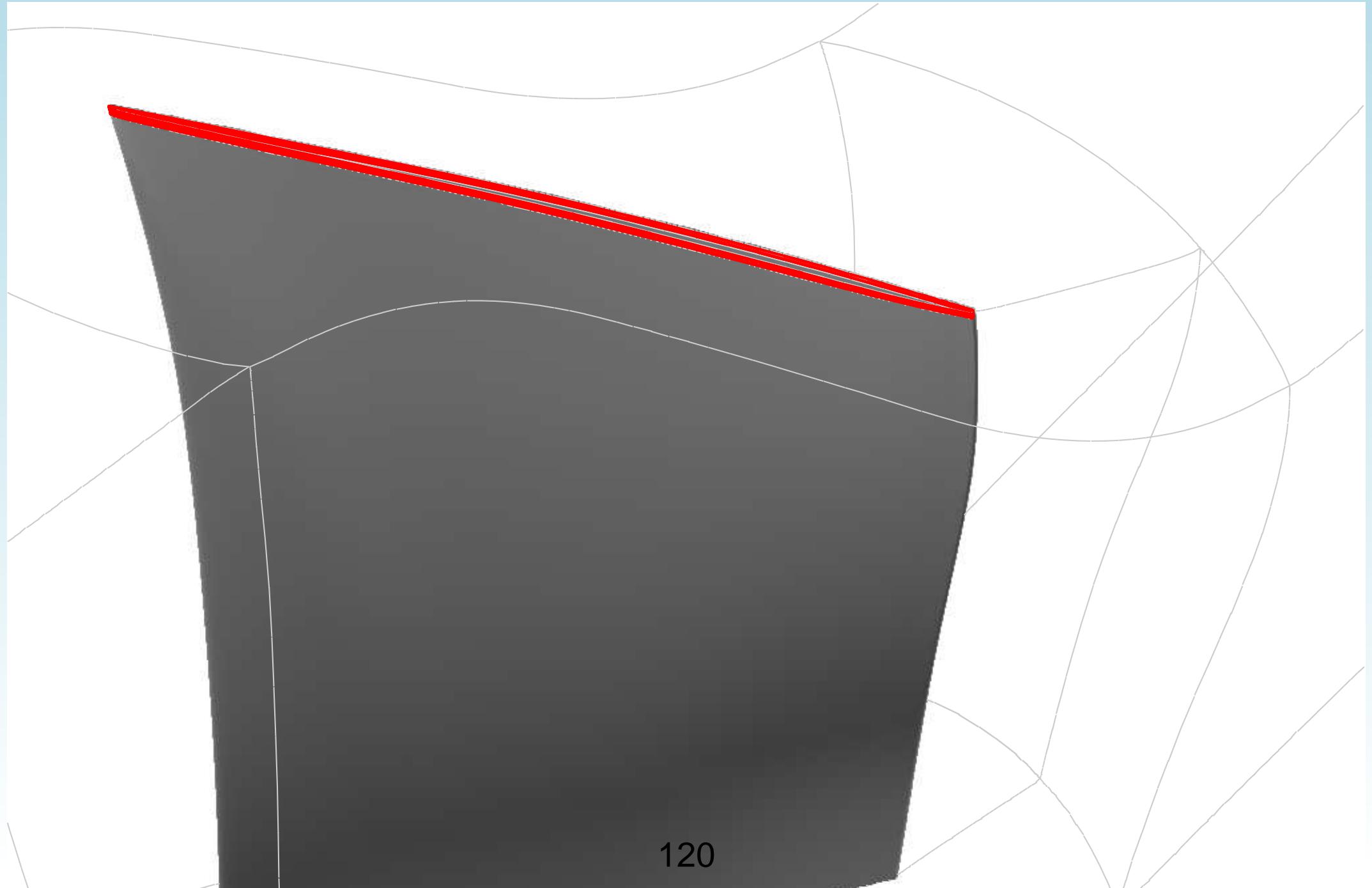








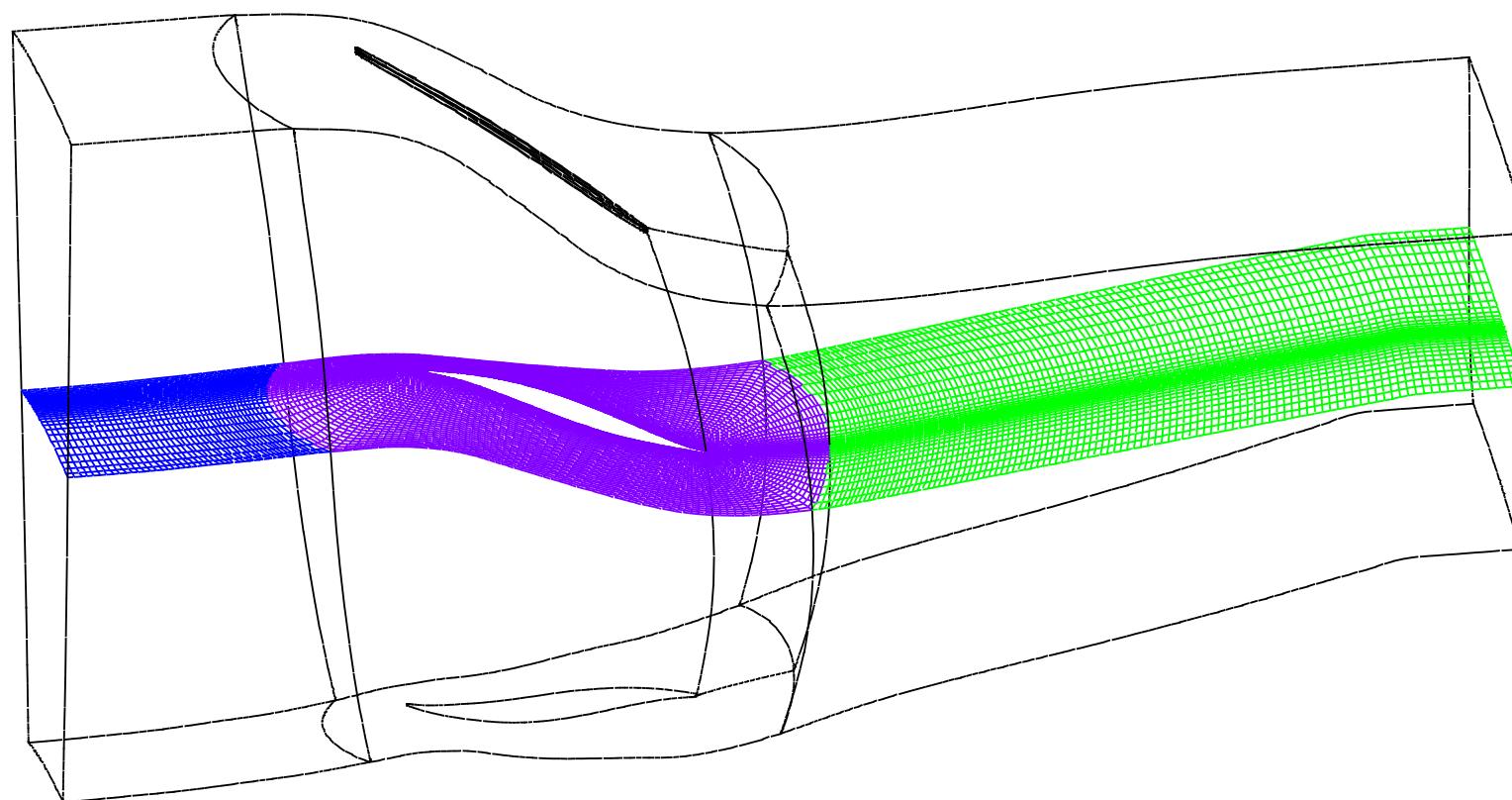
119



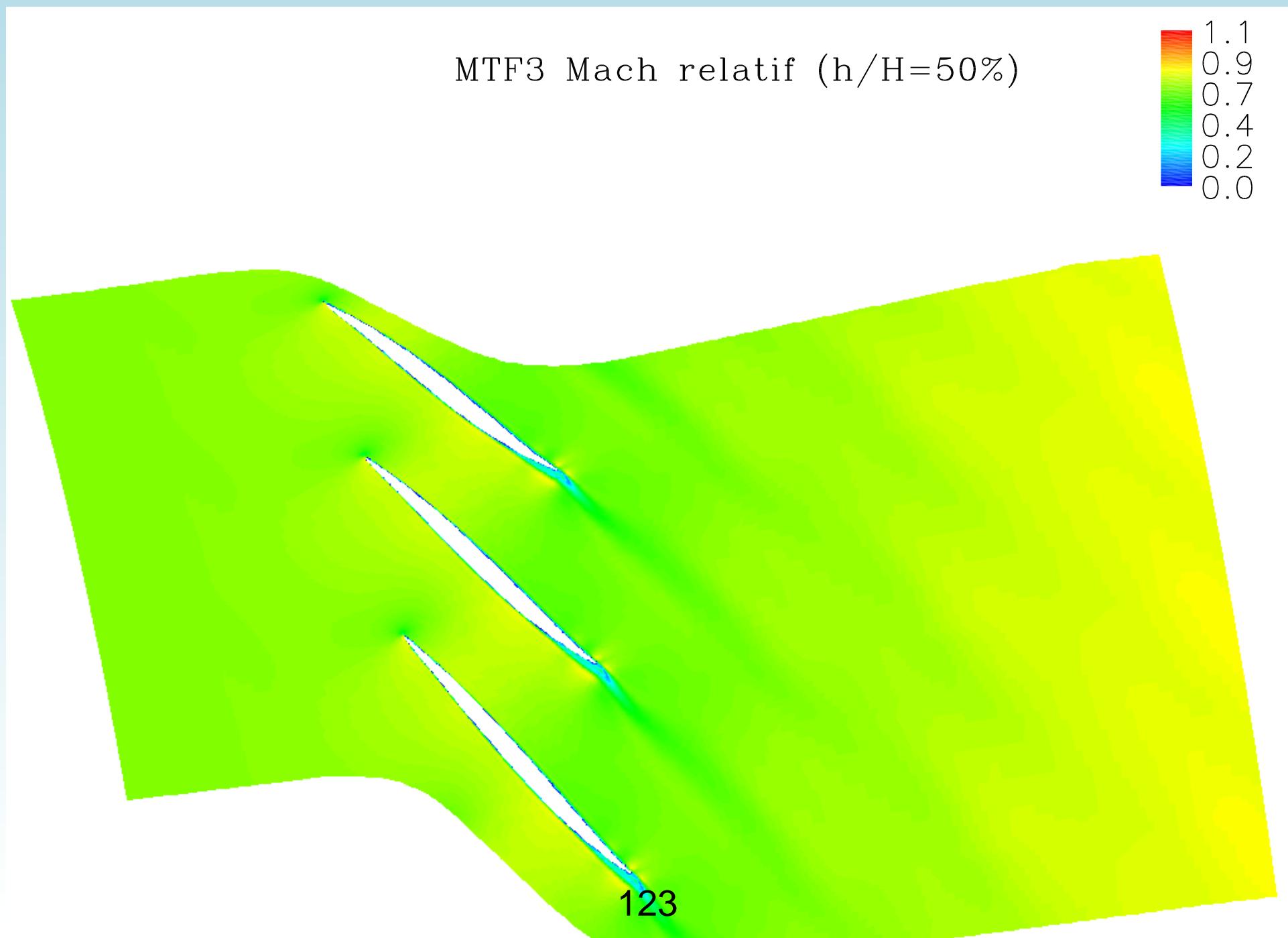
120

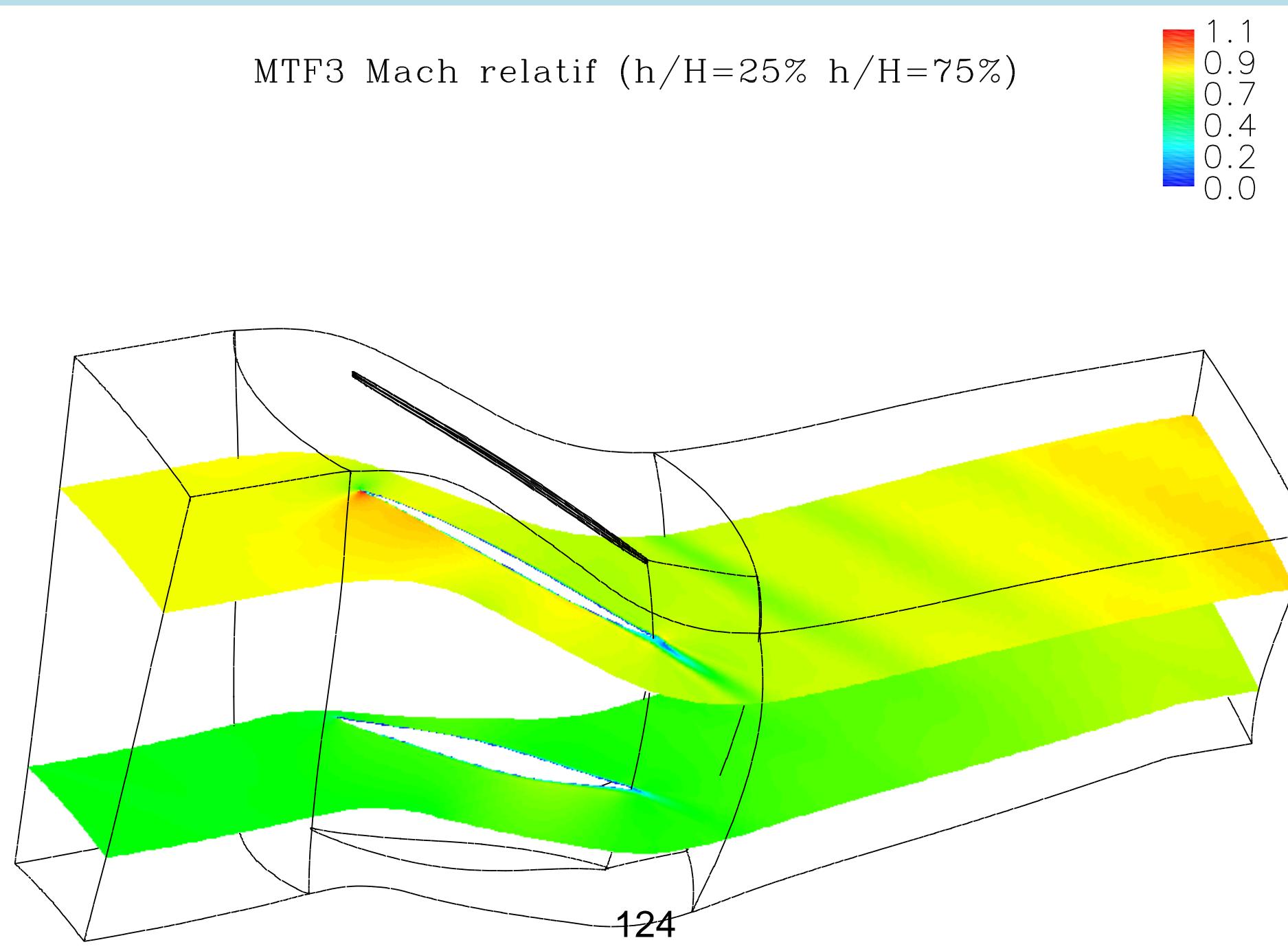
Résultats : Nombre de Mach

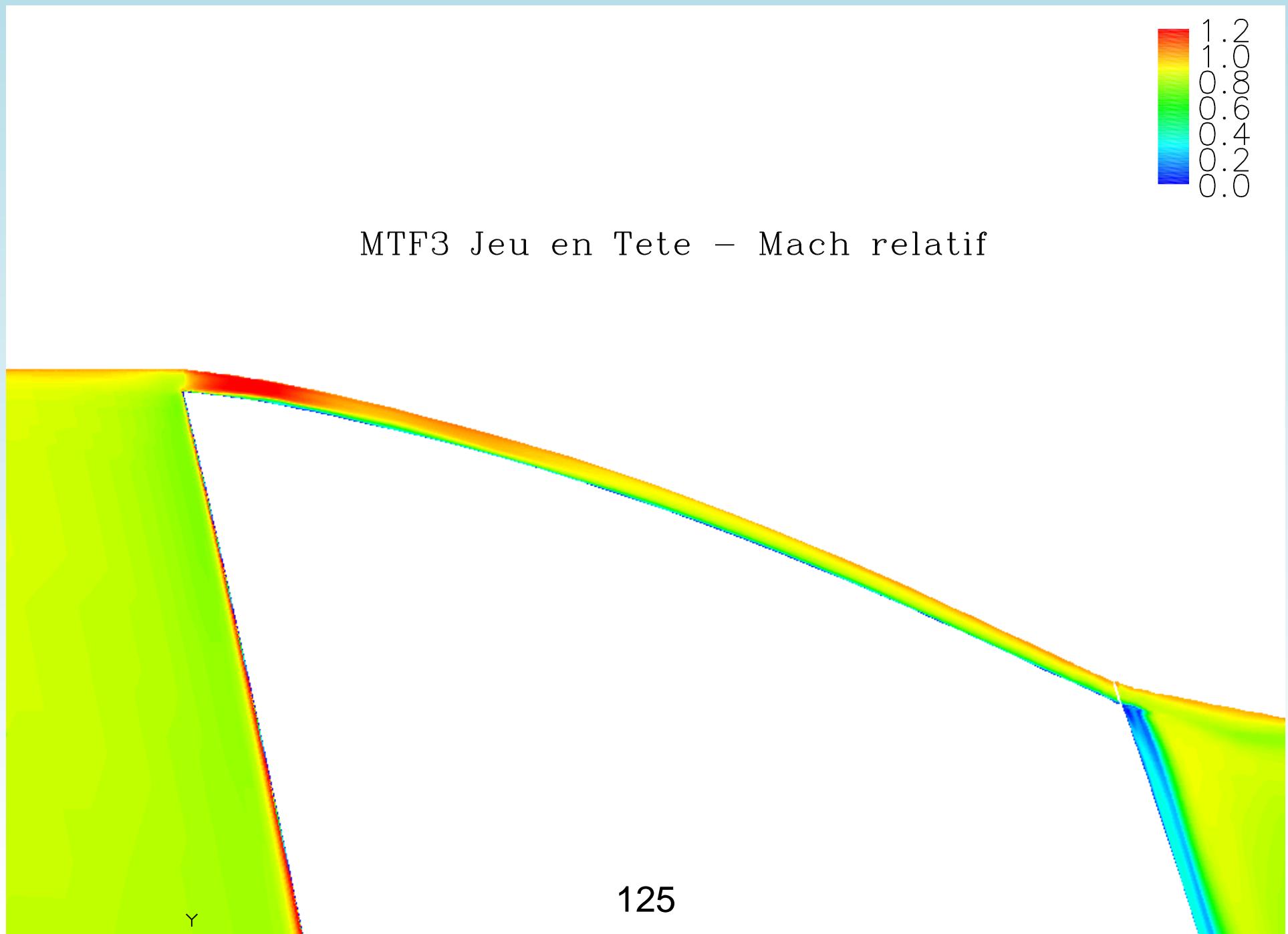
MTF3

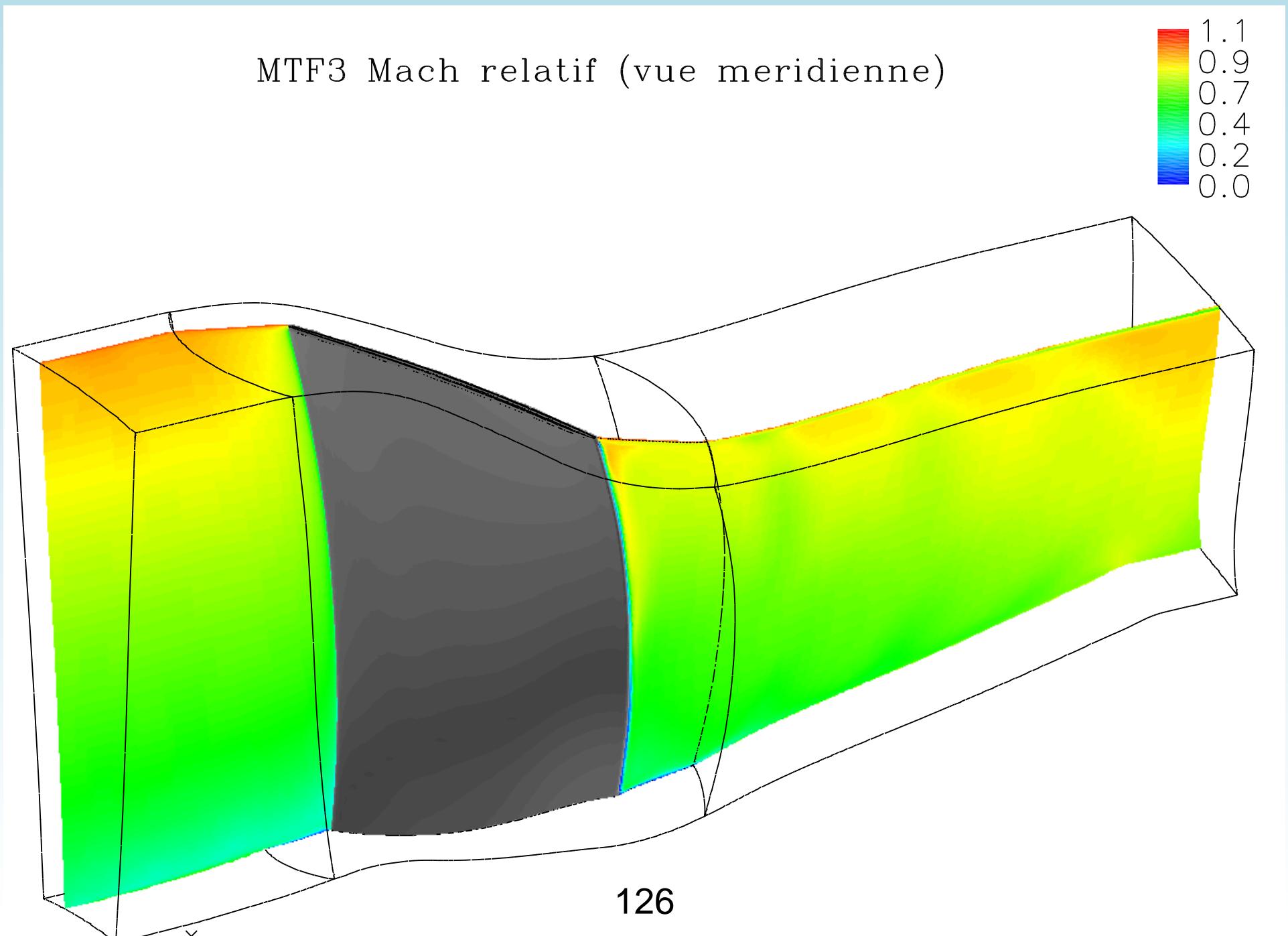


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Vitesse : recirculation ?

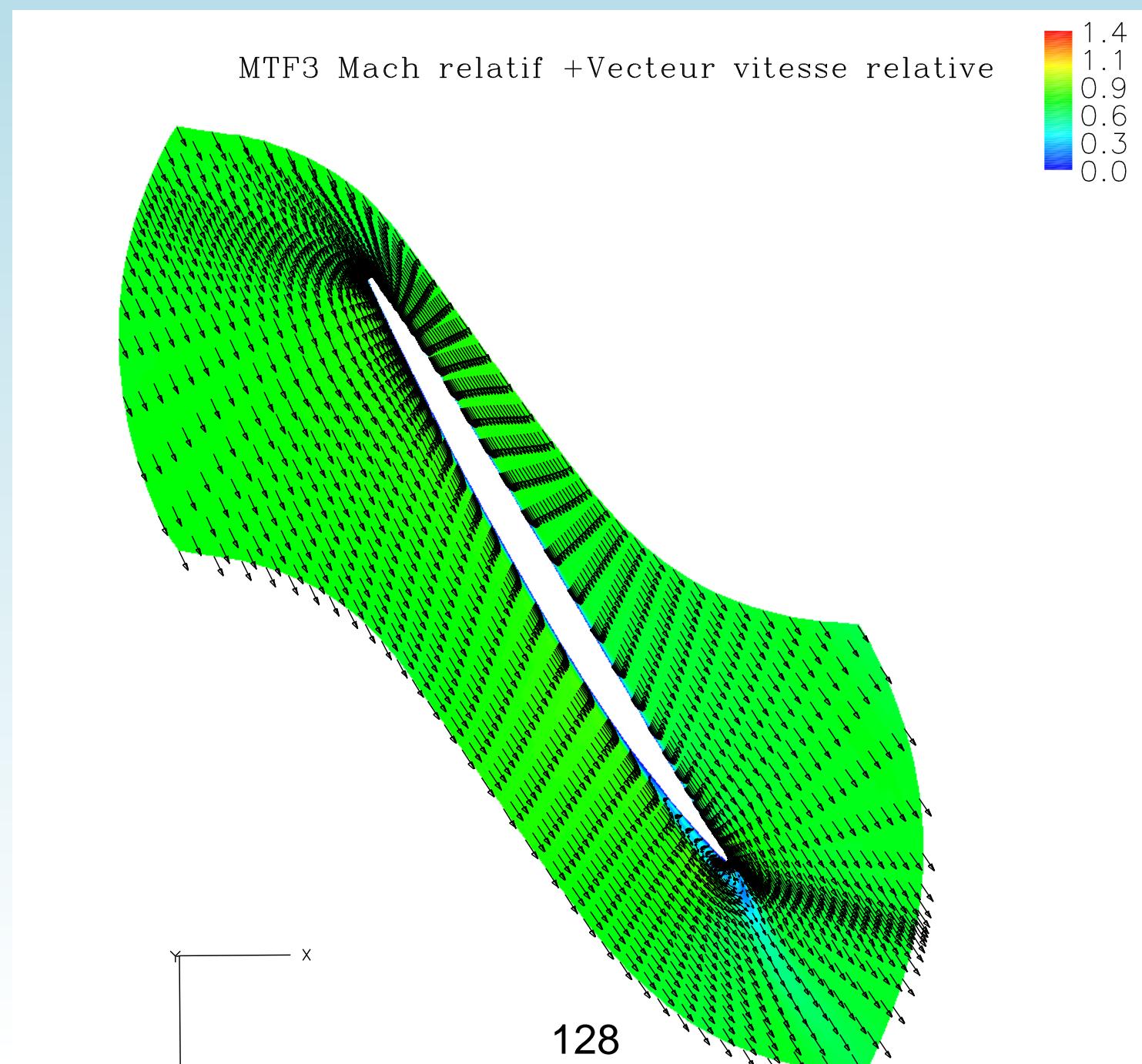
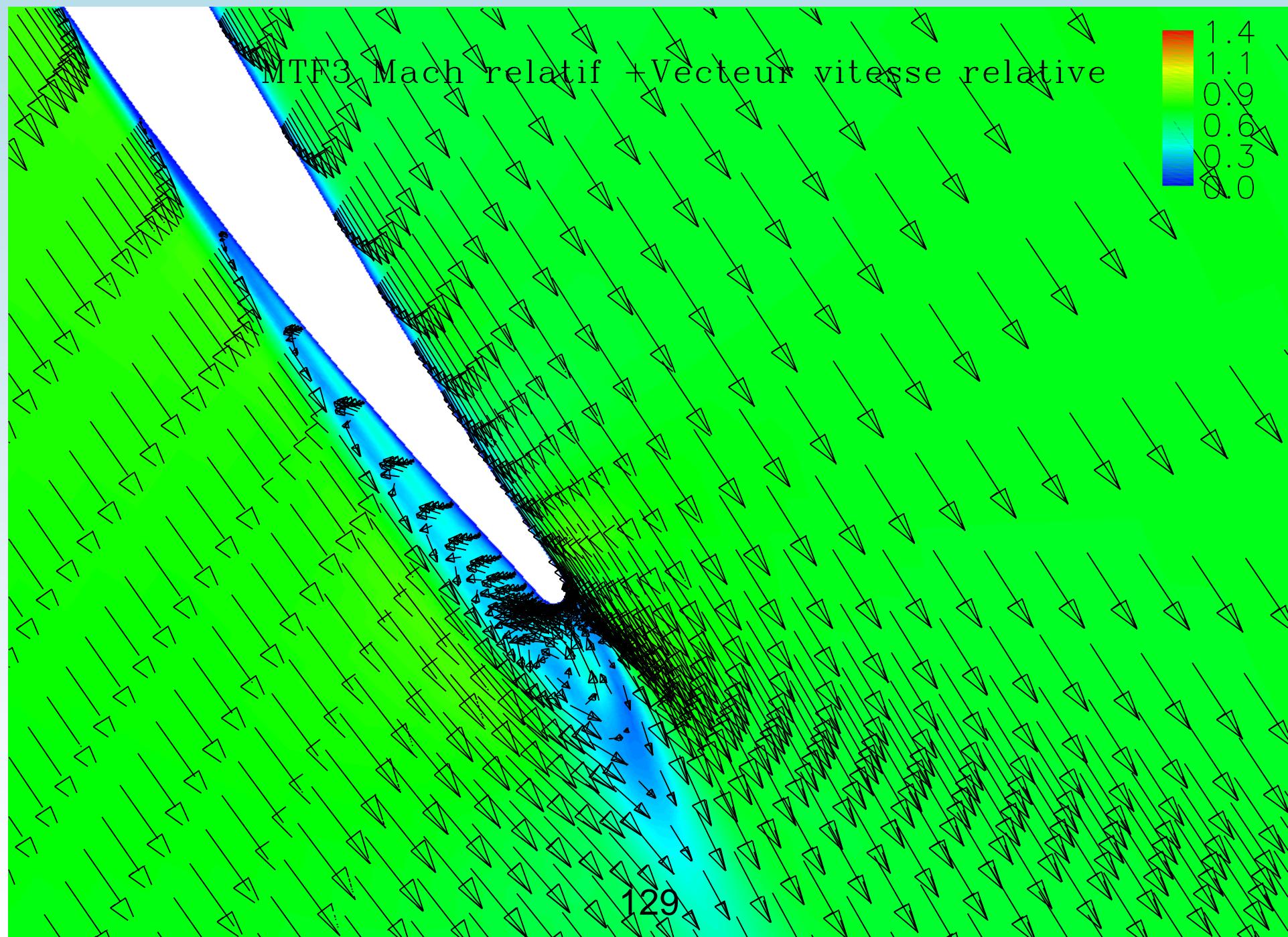


Figure 22: Nombre de Mach relatif et champ de vitesse dans le plan médius à mi hauteur



Wave turbulence modelling:

weakly nonlinear at asymptotically high rotation rate

Resonant surfaces

- Triads which satisfy the *resonance conditions*:

$$\begin{cases} \mathbf{k} + \mathbf{p} + \mathbf{q} = \mathbf{0} \\ F_{\epsilon\epsilon'\epsilon''}(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \epsilon\sigma(\mathbf{k}) + \epsilon'\sigma(\mathbf{p}) + \epsilon''\sigma(\mathbf{q}) = 0 \end{cases}$$

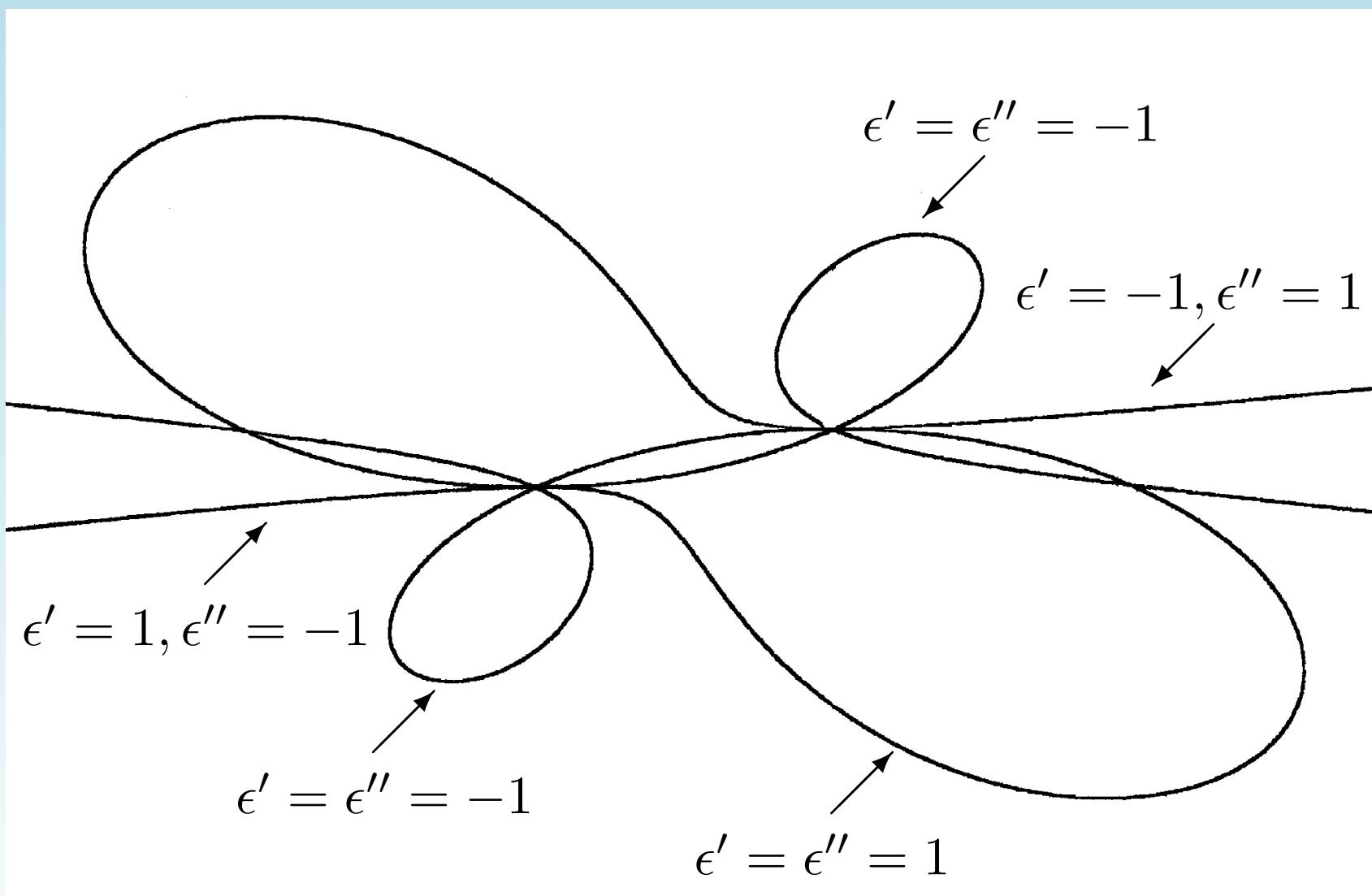
which defines complex *resonant surfaces* $S_{\epsilon'\epsilon''}$

- The remaining triads with $\mathbf{k} + \mathbf{p} + \mathbf{q} = \mathbf{0}$ but $F_{\epsilon\epsilon'\epsilon''}(\mathbf{k}, \mathbf{p}, \mathbf{q}) \neq 0$, in the 3D spectral space.

Energy equation $e(k, \theta)$:

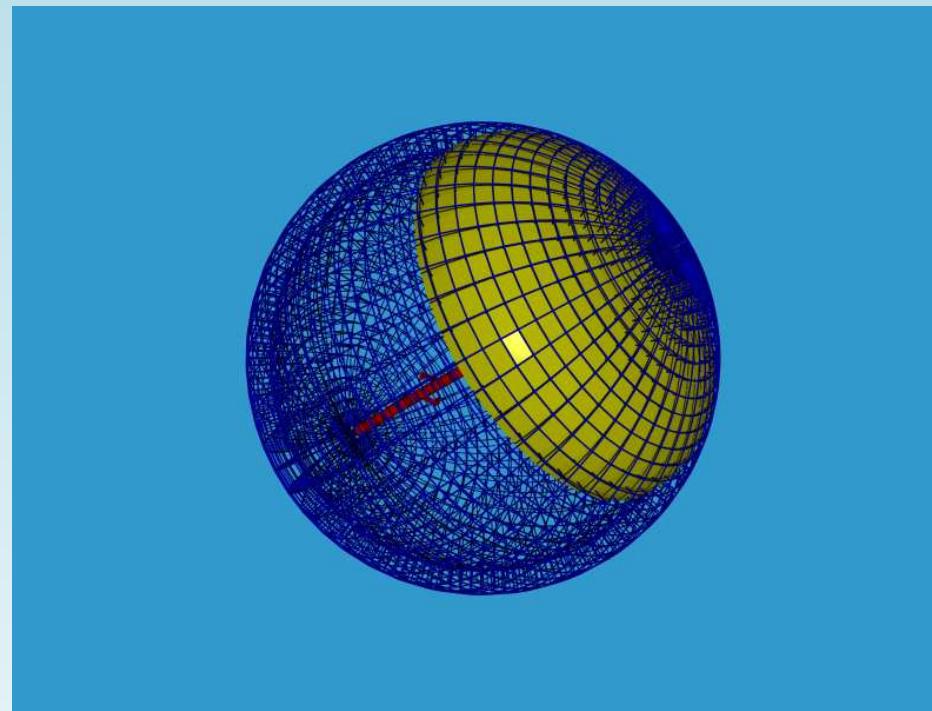
$$\frac{\partial e}{\partial t} + 2\nu k^2 e = \sum_{\epsilon' \epsilon''} \int_{S_{\epsilon' \epsilon''}} \frac{g_{\epsilon' \epsilon''}(k, p, q)}{\alpha_{\epsilon' \epsilon''}(p, q)} e(p, t) [e(q, t) - e(k, t)] d^2 p$$

$$\alpha_{\epsilon' \epsilon''}(p, q) = \frac{1}{\pi} |\epsilon' c_g(p) - \epsilon'' c_g(q)|$$

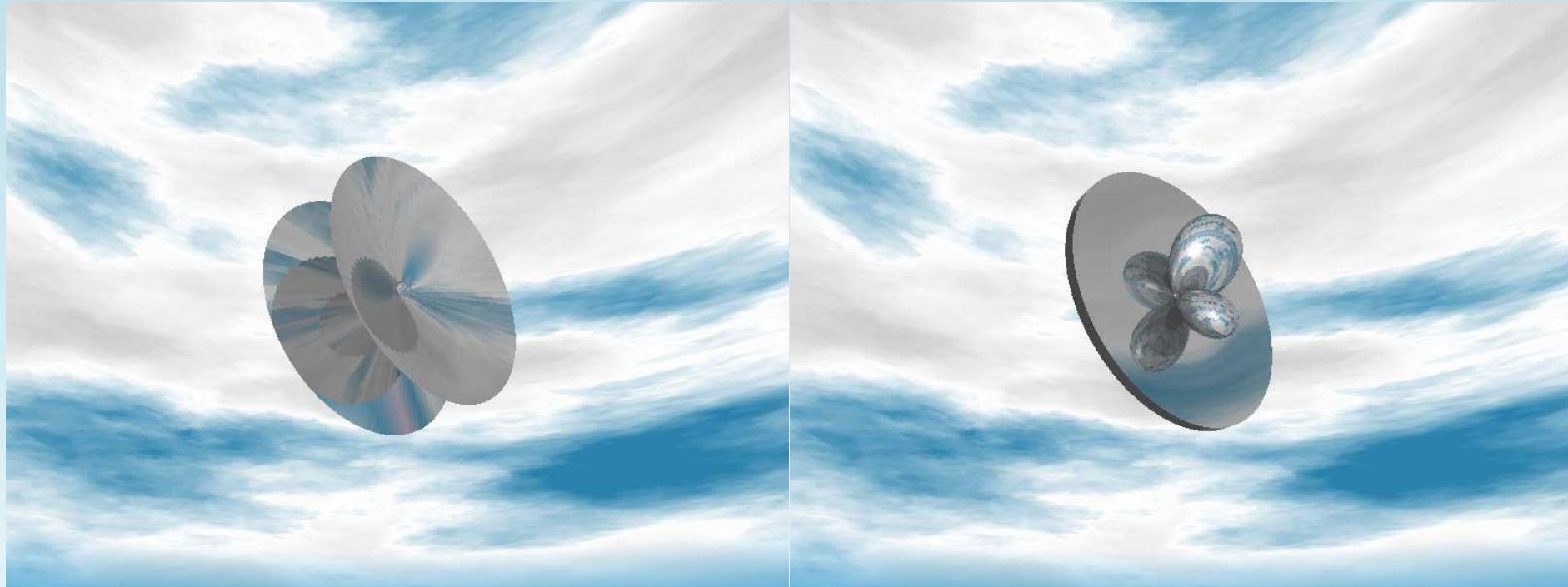


Cut trough the resonant surface for $k = 1$ et $\theta_k = 1.3$

Numerical resolution



- Spherical discretisation of spectral space: Typical resolution $400 \times 400 \times 400$
- Compute intersection of resonant surface with each grid cell \Rightarrow elementary area and integration geometrical coefficients
- 3D interpolation of spectrum for q



$$\widehat{(k, \Omega)} = \theta = 0.85$$

$$\theta = 1.49$$

[Animation1](#) [Animation2](#)