



Uncertainty Quantification in Simulations of Reactive Flows

Part 2: Applications

Gianluca Iaccarino

ME & iCME

Stanford University

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UQ in Reacting Flows

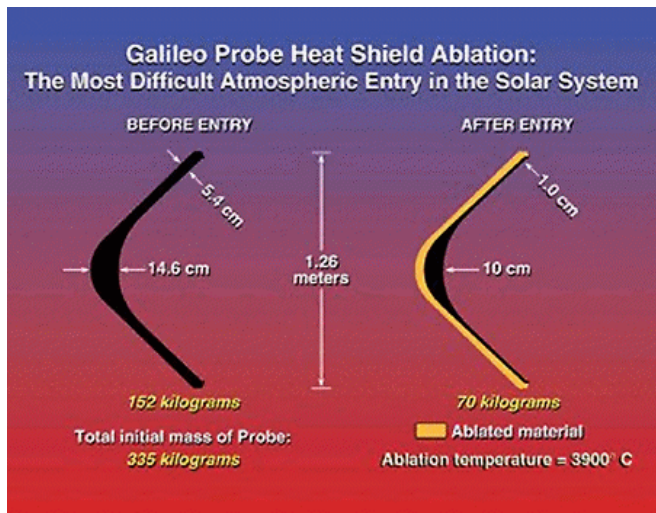
Planetary Entry Simulations

High-Temperature Reactive Flow

- ▶ During descent in the atmosphere vehicles experience **extreme** heating loads
- ▶ The design of the thermal protection system (TPS) is the most critical component of every planetary entry mission
- ▶ TPS design is fundamentally **computation-based** because no ground-test can reproduce all the aspects of flight
- ▶ Safety (and reliability) requires **rigorous** evaluation of the uncertainties present



Jupiter Entry Probe - Galileo



Source: NASA

Titan Entry Simulations

High-Temperature Reactive Flow

Predictions of TPS heating loads re-entry are **challenging**

- ▶ Physics Components
 - Chemistry
 - Radiation
 - Turbulence
 - etc.
- ▶ Computational issues
 - Strong shocks
 - Thin boundary layers
 - Flow separation
 - etc.

We focus on the uncertainties in the chemical kinetics, and their impact on the heat transfer at the stagnation point...

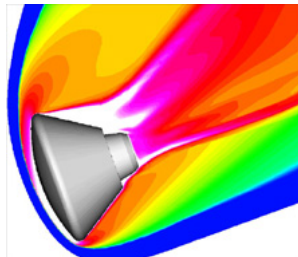
Titan Entry Simulations

Aero-thermodynamic model

We consider *nominal* conditions for the Titan entry:

Table: Freestream conditions

N_2	CH_4	ρ_∞ (kg/m ³)	V_∞ (km/s)	T_∞ (K)
95%	5%	1.49×10^{-4}	5.76	152.7



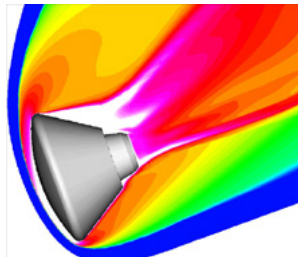
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The strong bow shock induced strong non-equilibrium effects and dissociation/ionization

- ▶ We assumed a 13-species mechanism:
- ▶ C , H , N , C_2 , CH_4 , CH_3 , CH_2 , CH , CN , H_2 , HCN , N_2 , NH
- ▶ 26 reactions: 12 dissociation & 14 exchange

Probabilistic approach

Reaction rates in ionization/dissociation models

- Uncertainty in the reactions rates, gathered from

- theory
- experiments
- engineering judgment

- Uncertainty in the reaction rates is described using independent u.r.v.s

	Dissociation reactions $k_f = A_f T^b \exp(-C_f/T)$	A_f (cc/mol-s)	b_f	C_f (K)	95% conf. limit [Ref.]
1	$N_2 + M \rightleftharpoons 2N + M$ M=N,C,H	7.00×10^{21} 3.00×10^{22}	-1.60 -1.60	113200 113200	See Table 2
2	$CH_4 + M \rightleftharpoons CH_3 + H + M$	4.70×10^{17}	-8.20	59200	± 0.30 [22]
3	$CH_3 + M \rightleftharpoons CH_3 + H + M$	1.02×10^{16}	0.00	45600	± 0.35 [22]
4	$CH_3 + M \rightleftharpoons CH + H_2 + M$	5.00×10^{15}	0.00	42800	± 0.30 [23]
5	$CH_2 + M \rightleftharpoons CH + H + M$	4.00×10^{15}	0.00	41800	± 0.30 [23]
6	$CH_2 + M \rightleftharpoons C + H_2 + M$	1.30×10^{14}	0.00	29700	± 0.30 [23]
7	$CH + M \rightleftharpoons C + H + M$	1.90×10^{14}	0.00	33700	± 0.30 [23]
8	$C_2 + M \rightleftharpoons 2C + M$	1.50×10^{16}	0.00	71600	± 0.30 [24]
9	$H_2 + M \rightleftharpoons 2H + M$	2.23×10^{14}	0.00	48350	± 0.30 [22,25]
10	$CN + M \rightleftharpoons C + N + M$	2.53×10^{14}	0.00	71000	± 0.30 [26,27]
11	$NH + M \rightleftharpoons N + H + M$	1.80×10^{14}	0.00	37600	± 0.30 [28]
12	$HCN + M \rightleftharpoons CN + H + M$	3.57×10^{20}	-2.60	62845	± 0.30 [29]
Exchange reactions					
13	$CH_3 + H \rightleftharpoons CH_2 + H_2$	6.03×10^{13}	0.00	7600	± 1.00 [25]
14	$CH_3 + N_2 \rightleftharpoons HCN + NH$	4.82×10^{12}	0.00	18000	± 1.00 [28]
15	$CH_2 + N \rightleftharpoons HCN + H$	5.00×10^{13}	0.00	0	± 1.00 [30]
16	$CH_2 + H \rightleftharpoons CH + H_2$	6.03×10^{12}	0.00	-900	± 0.87 [25,28]
17	$CH + N_2 \rightleftharpoons HCN + N$	4.40×10^{12}	0.00	11060	± 0.35 [30]
18	$CH + C \rightleftharpoons C_2 + H$	2.00×10^{14}	0.00	0	± 1.00 [23]
19	$C_2 + N_2 \rightleftharpoons 2CN$	1.50×10^{13}	0.00	21000	± 0.30 [31]
20	$CN + H_2 \rightleftharpoons HCN + H$	2.95×10^5	0.00	1130	± 0.60 [32]
21	$CN + C \rightleftharpoons C_2 + N$	5.00×10^{13}	0.00	13000	± 0.54 [18]
22	$N + H_2 \rightleftharpoons NH + H$	1.60×10^{14}	0.00	12650	± 0.30 [33]
23	$C + N_2 \rightleftharpoons CN + N$	5.24×10^{13}	0.00	22600	± 0.50 [T]
24	$C + H_2 \rightleftharpoons CH + H$	4.00×10^{14}	0.00	11700	± 0.30 [34]
25	$H + N_2 \rightleftharpoons NH + N$	3.00×10^{12}	0.50	71400	± 0.50 [T]
26	$CH_4 + H \rightleftharpoons CH_3 + H_2$	1.32×10^4	3.00	4045	± 0.30 [22,25]

Another uncertainty source

Radiation modeling

NASA has identified the *heating from shock layer radiation due to the CN radical formed in the N_2/CH_4 atmosphere as a primary uncertainty*

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Can we predict the CN radical?

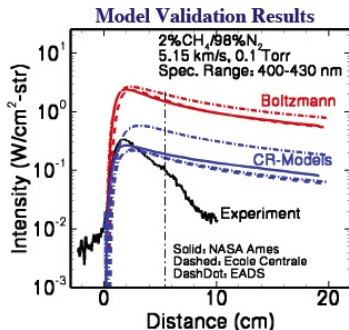
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Can we predict the CN radical?

- ▶ State-of-the art knowledge during the design of the Huygen's probe was the Boltzmann model
- ▶ This led to **overprediction** of the heating rates - **conservative design**
- ▶ Recent work has lead to **collisional-radiative** (CR) models



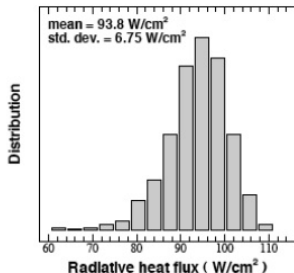
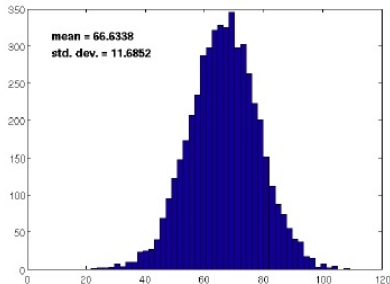
TPS Heating load uncertainty

- ▶ We used Monte Carlo sampling (10,000 runs) to study the effect of the kinetics uncertainties
- ▶ We employed the **CR** model but compared to NASA earlier work (with Boltzmann model) in an attempt to characterize the **epistemic uncertainty**

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Stagnation point heat flux (W/cm^2)



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Correlation: based on cross-plots of output (amount of CN) vs. input (uncertainty in the reaction rates)

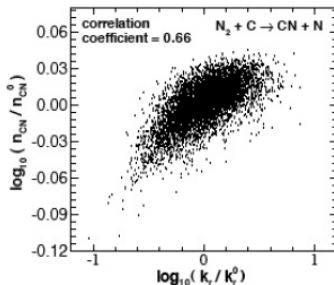
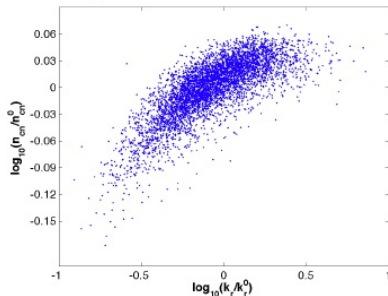
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Correlation plot for $N_2 + C \rightarrow CN + N_2$

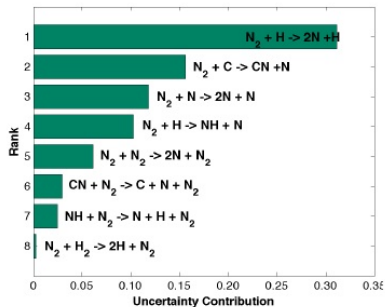
$N_2 + C \rightarrow CN + N$, correlation coeff. = 0.76415



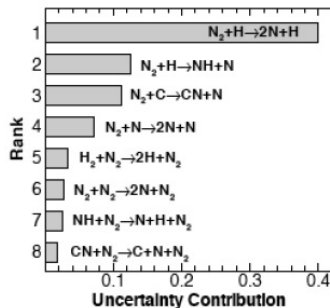
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ANOVA ANalysis Of Variance: **separate** the variance in factors contributed by each input uncertainty

8 major contributors to uncertainty



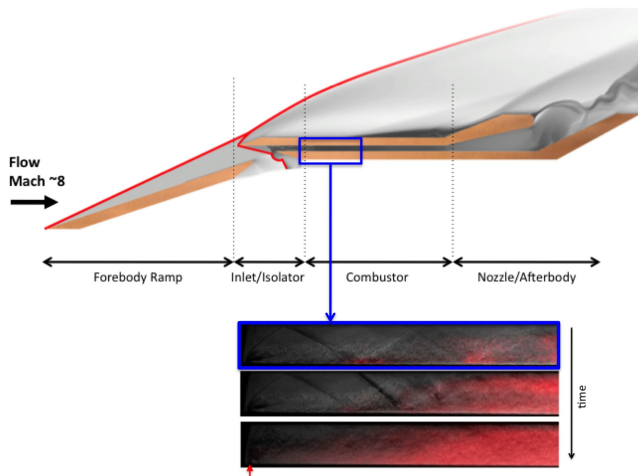
CR Model: Ghaffari, Iaccarino, Magin, 2009



Boltzmann model: Bose & Wright, 2004

Uncertainty in Ignition Delay Time

Determination of Ignition Delay Time is an important design consideration, for example in air-breathing hypersonic propulsion systems



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 - ▶ **What is the uncertainty?**

Hydrogen Chemistry

Reaction Rate Uncertainties

- ▶ Rate (and their uncertainties) are available in the literature
- ▶ Modified Arrhenius form
 $k = AT^n \exp(-E/RT)$
- ▶ The uncertainty factor UF is such that
 $[k/UF : k \times UF]$ provide *probable* bounds!
- ▶ Assume that the reaction rate are **independent, lognormally distributed** r.v.

Reaction	A	n	E	UF
$H + O_2 \leftrightarrow O + OH$	2.64e16	-0.67	71.30	1.5
$O + H_2 \leftrightarrow H + OH$	4.59e4	2.70	26.19	1.3
$OH + H_2 \leftrightarrow H + H_2O$	1.73e8	1.51	14.35	2.0
$OH + OH \leftrightarrow O + H_2O$	3.97e4	2.40	-8.83	1.5
$H + H + M \leftrightarrow H_2 + M$	1.78e18	-1.00	0.00	2.0
$H + H + H_2 \leftrightarrow H_2 + H_2$	9.00e16	-0.60	0.00	2.5
$H + H + H_2O \leftrightarrow H_2 + H_2O$	5.62e19	-1.25	0.00	2.0
$H + OH + M \leftrightarrow H_2O + M$	4.40e22	-2.00	0.00	2.0
$H + O + M \leftrightarrow OH + M$	9.43e18	-1.00	0.00	3.0
$O + O + M \leftrightarrow O_2 + M$	1.20e17	-1.00	0.00	2.0
$H + O_2 + M \leftrightarrow HO_2 + M$	6.33e19	-1.40	0.00	1.2
$H_2 + O_2 \leftrightarrow HO_2 + H$	5.92e5	2.43	223.85	2.0
$OH + OH + M \leftrightarrow H_2O_2 + M$	2.01e17	-0.58	-9.59	2.5
$HO_2 + H \leftrightarrow O + H_2O$	3.97e12	0.00	2.81	3.0
$HO_2 + H \leftrightarrow OH + OH$	7.49e13	0.00	2.66	2.0
$HO_2 + O \leftrightarrow OH + O_2$	4.00e13	0.00	0.00	1.2
$HO_2 + OH \leftrightarrow H_2O + O_2$	2.38e13	0.00	-2.09	3.0
	1.00e16	0.00	72.51	3.0
$HO_2 + HO_2 \leftrightarrow O_2 + H_2O_2$	1.30e11	0.00	-6.82	1.4
	3.66e14	0.00	50.21	2.5
$H_2O_2 + H \leftrightarrow HO_2 + H_2$	6.05e6	2.00	21.76	3.0
$H_2O_2 + H \leftrightarrow H_2O + OH$	2.41e13	0.00	16.61	2.0
$H_2O_2 + O \leftrightarrow HO_2 + OH$	9.63e6	2.00	16.61	3.0
$H_2O_2 + OH \leftrightarrow HO_2 + H_2O$	2.00e12	0.00	1.79	2.0
	2.67e41	-7.00	157.32	2.0

Ignition Delay Time

Uncertainty Propagation

- ▶ Conditions: Stoichiometric Hydrogen-Air Mixture (29.6% H₂; 14.8% O₂); Temperature: 1000 K; Pressure: 1 atm;
- ▶ Non-intrusive LHS Sampling (25 uncertain variables)

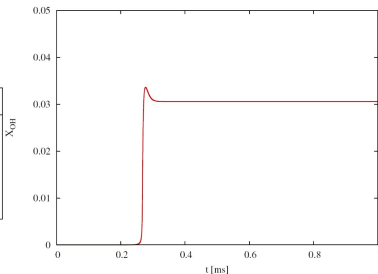
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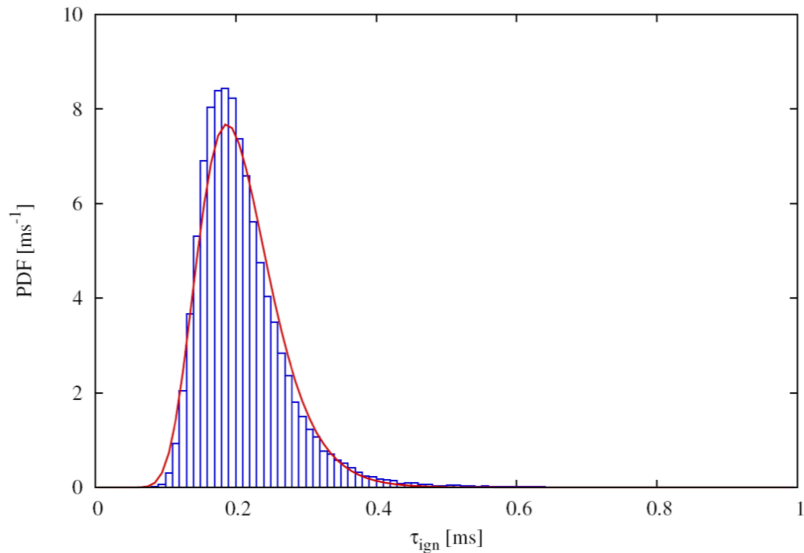
Number of Samples	Mean [ms]	UF
100	0.202060	1.7362
1000	0.201304	1.7054
10000	0.201256	1.7039
100000	0.201263	1.7075

One Sample Showing Ignition at ~0.25 ms



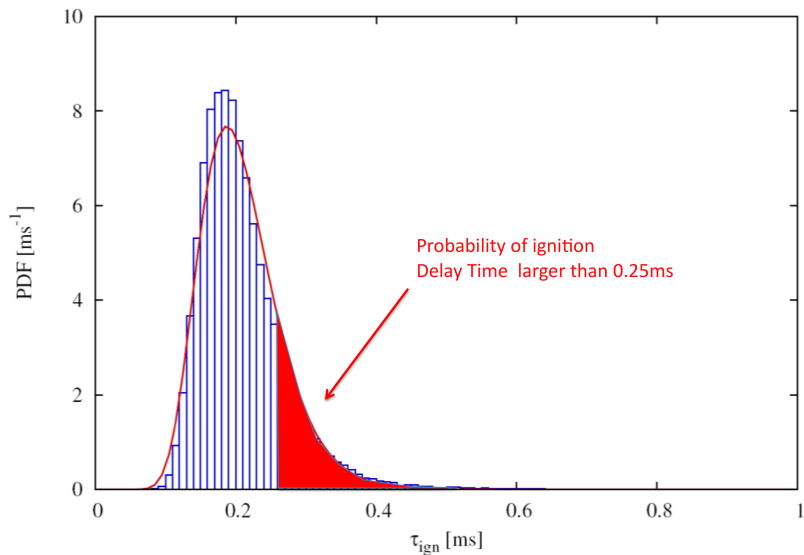
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 - ▶ Good to know, so what?
 - ▶ Can we do this faster?

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Pose the UQ quest as an Inverse Problem

- ▶ What uncertainty in the reaction rates can we **tolerate** to ensure that the probability of ignition delay time exceeding 0.25 ms is less than 10%?

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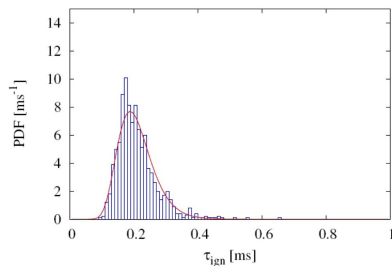
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Quantity	Nominal	Optimal
UF Branching Reaction	1.5	1.29
Mean τ_{ign} [ms]	0.201304	0.198947
UF τ_{ign}	1.7054	1.4023
Probability of Failure	0.191	0.100

Ignition Delay Time

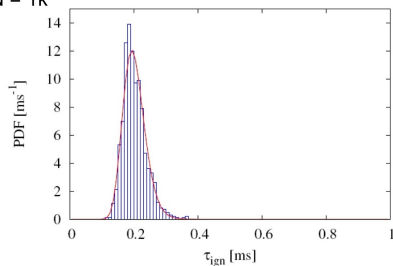
Uncertainty Propagation

Nomimal



Optimal

N = 1k



- ▶ Overall uncertainty in the IDT is reduced
- ▶ Failure probability below critical requirement

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We need to **use/develop** algorithms that **discover** the true dependency of the solution

Low-Rank Approximations

High-Dimensional UQ

Extend the concept of **Separation of variables** to computational methodologies

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High-Dimensional UQ

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- ▶ Assume y_j for $j = 1, \dots, d$ are the input uncertainties
- ▶ Define

$$u(y_1, \dots, y_d) \approx \sum_{k=1}^r u_1^{(k)}(y_1) \times u_2^{(k)}(y_2) \times \dots \times u_d^{(k)}(y_d)$$

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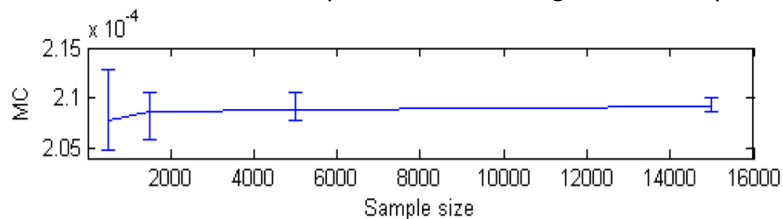
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- ▶ Need to **discover** the functions $u^{(k)}$ and the *rank* r
- ▶ We cast it as an optimization problem: Find the lowest possible r which approximates a set of given function evaluations with $u^{(k)}$ being polynomials of fixed maximum order

Low-Rank Approximations

Mean Ignition Delay Time

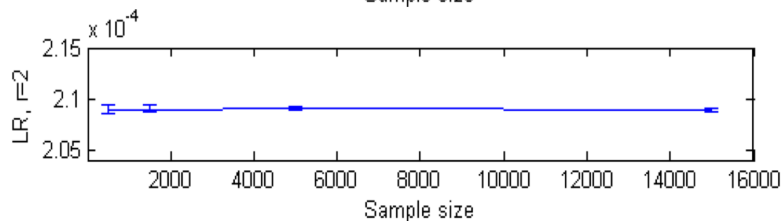
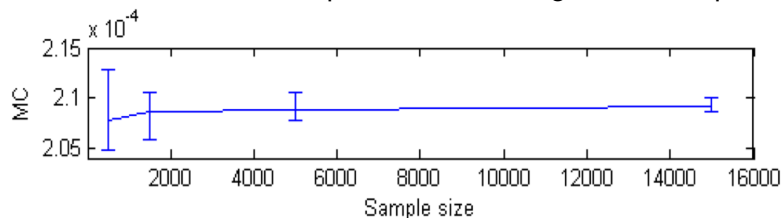
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Low-Rank Approximations

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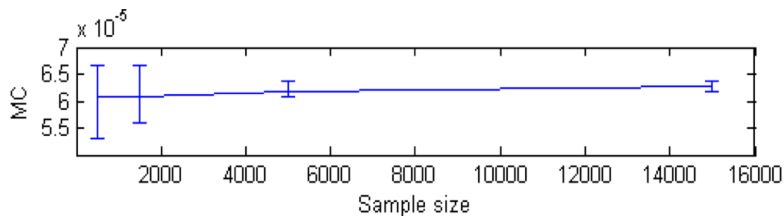


Even with 500 samples the estimate of the mean ignition delay is acceptable (smaller than MC with 14,000)

Low-Rank Approximations

STD of Ignition Delay Time

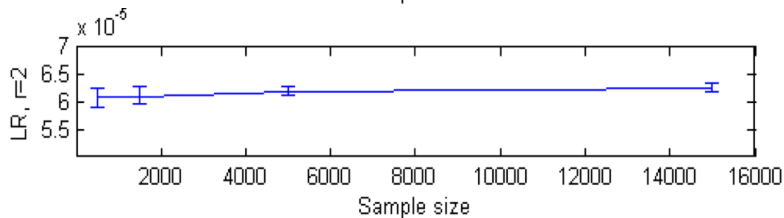
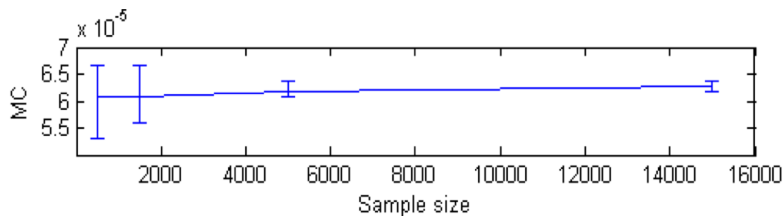
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Summary of the examples

...so far

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Summary of the examples

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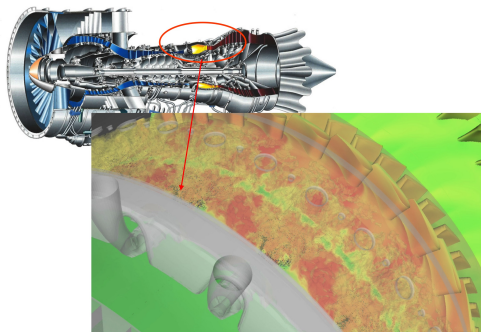
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To finish, I want to give you an example of UQ combined with **realistic** flow simulations...

Turbulent reacting flows

Challenges

- ▶ **Turbulent** flow simulations in **realistic geometries** with **detailed kinetic mechanisms** are still beyond the reach of computational engineering



PW6000 simulations enabled by the **flamelet modeling** approach, but still requiring 1000 CPUs

Turbulent reacting flows

UQ Challenges

- ▶ Uncertainties in kinetic mechanisms might still dominate, especially for the **prediction of pollutants**
- ▶ It is **impossible** to perform more than a handful of simulation ($\mathcal{O}(5)$)
- ▶ Non-intrusive approach (even our fancy LR method) have no hope of success
- ▶ Need to be **intrusive** and connect the physics/mathematics/UQ

Turbulent reacting flows

Non-intrusive framework



- ▶ Perform MANY simulations and sample the outputs

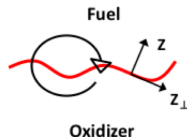
Reacting Flow Modeling - Flamelet

▶ Basic Premise

- ▶ Since scales of chemical reaction are much smaller than the smallest scales of turbulent, a turbulent flame is simply an ensemble of laminar "flamelets" embedded in a turbulent flow field
- ▶ Solve for flame structure independently from the flow field

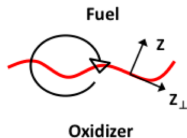
▶ Coordinate Transformation

- ▶ Transform to coordinate system attached to the flame
 $(x, y, z) \rightarrow (Z, Z_{\perp})$
- ▶ Neglect all gradients in tangential directions



Reacting Flow Modeling - Flamelet

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 $(x, y, z) \rightarrow (Z, Z_{\perp})$
 - ▶ Neglect all gradients in tangential directions
 - ▶ Resulting equations are a one-dimensional set of reaction-diffusion equations parameterized by the mixture fraction Z
 - ▶ Solved in advance and tabulated for a given fuel



Reacting Flow Modeling - Governing Equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[2\mu \left(S_{ij} - \frac{1}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right]$$

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$$T = T(\rho, p, Z) \quad \rightarrow \quad \text{tabulated}$$

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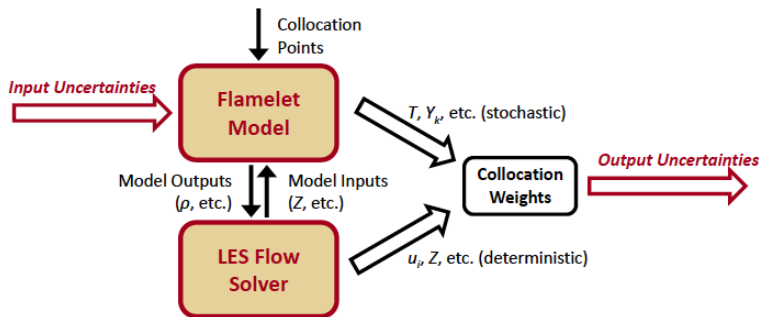
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- ▶ Uncertainty in the kinetic rates appears indirectly through the density
- ▶ Use the flamelet equations to condition the high-dimensional uncertainty
- ▶ Can use efficient UQ methods requiring few full system simulations

Turbulent reacting flows

Intrusive framework



- ▶ Split the flamelet-generation part from the actual flow simulations
- ▶ Propagate the uncertainty through the flamelet
- ▶ Inject uncertainties in the link between flamelets and flow equations (via mixture fraction, density, etc.)

Turbulent reacting flows

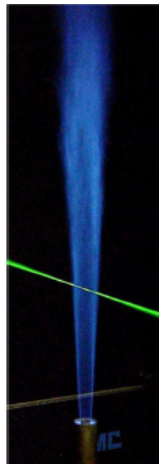
Sandia Flame D

Piloted partially premixed methane/air flame

- ▶ Used NGA (low Mach, structured grid)
- ▶ GRI 3.0 mechanism
- ▶ Uncertainties in rates from Sheen *et al.* 2009

Simulations

1. Used LHS sampling for flamelets (10,000 solutions)
2. Compiled tables with mean and variances of density (other uncertainties, e.g. viscosity ignored for now)
3. Performed 7 LES simulations sampling on the density distribution

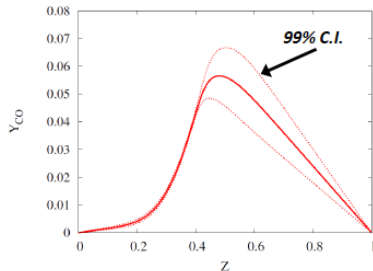
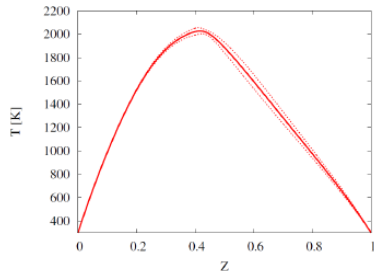


Turbulent reacting flows

Sandia Flame D

Step 1

From kinetic rate uncertainties to flamelet output uncertainties



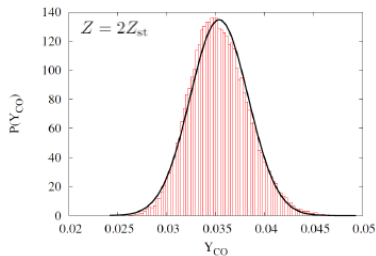
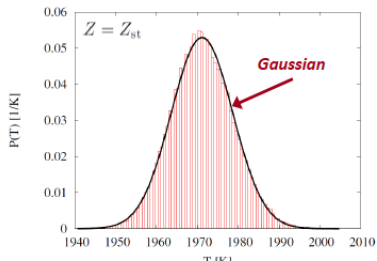
Rich part of the flame more *uncertain*

Turbulent reacting flows

Sandia Flame D

Step 2

Create a **stochastic** flamelet table



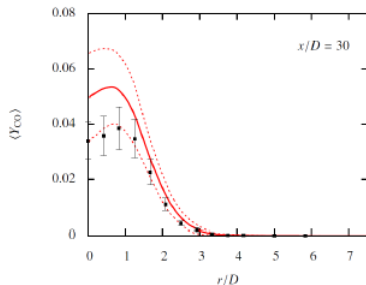
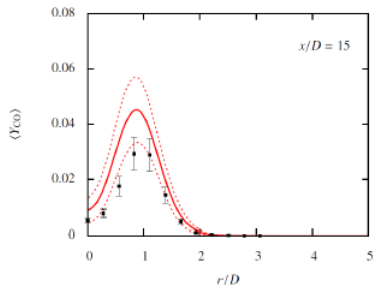
Distributions are Gaussian-like

Turbulent reacting flows

Sandia Flame D

Step 3

LES with **stochastic** flamelet table



Emission uncertainty is quite high, and might be comparable with other uncertainties (steady flamelet assumption, kinetic mechanism, etc.)



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