From radiative transfer to radiotherapy

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Motivations

□ Radiative transfer

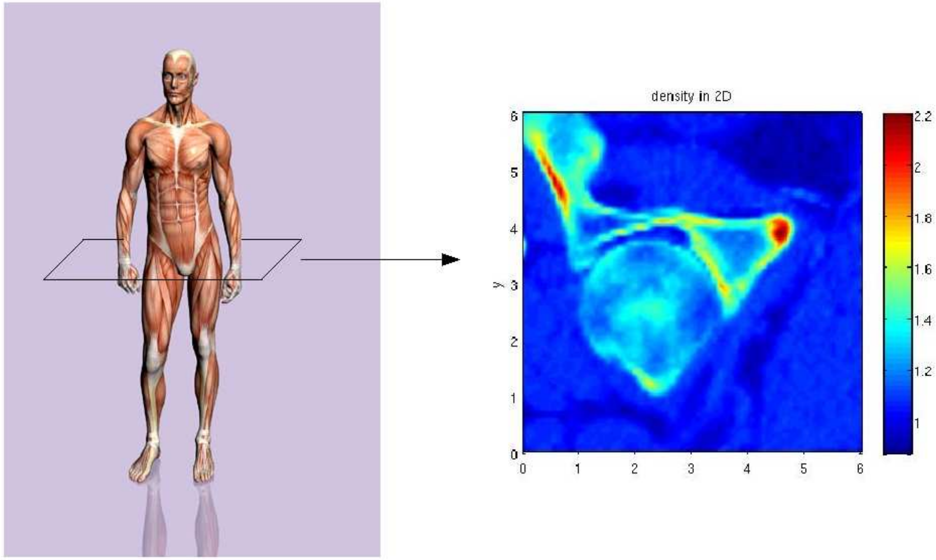
↪ Atmospheric entry

↪ Plasma flows

↪ Fluid photons interactions

□ Radiotherapy

↪ Matter electrons interactions



☐ Radiative Transfer Equation (Kinetic)

$$\frac{1}{c}\partial_t I + \Omega \partial_x I = \sigma(B(T) - I)$$

- I : photon distribution
- c : speed of light
- σ : opacity
- $B(T)$: Planck's function

Prohibitive Numerical cost for our applications

Outline

1. Moment model for the RTE and main properties
2. Numerical approximation → Specific procedure for source term
3. Electron extension → Radiotherapy

The M1 model

□ The M1-model for radiative transfer (Dubroca-Feugeas 99)

$$\begin{cases} \partial_t E + \partial_x F = c\sigma^e aT^4 - c\sigma^a E \\ \partial_t F + \partial_x c^2 P = -c\sigma^f F \\ \partial_t \rho C_v T = c\sigma^a E - c\sigma^e aT^4 \end{cases} \qquad \mathbf{W} = \begin{pmatrix} E \\ F \\ T \end{pmatrix} \qquad \mathcal{F}(\mathbf{W}) \begin{pmatrix} F \\ c^2 P = c^2 E \chi(\frac{F}{cE}) \\ 0 \end{pmatrix}$$

E	radiative energy
F	radiative flux
P	radiative pressure
T	radiative temperature

and the opacities

$$\begin{cases} \sigma^a := \sigma^a(x, \mathbf{W}) \\ \sigma^e := \sigma^e(x, \mathbf{W}) \\ \sigma^f := \sigma^f(x, \mathbf{W}) \end{cases}$$

□ Numerical approximation

↪ Robustness $E > 0$ and $|F/cE| < 1$

↪ Relevant asymptotic behaviors

☐ Diffusion regime

Large opacities $\longrightarrow \epsilon$ rescaling factor

$$\begin{cases} \epsilon\partial_t E + \partial_x F = \dfrac{1}{\epsilon}(c\sigma^e aT^4 - c\sigma^a E) \\ \epsilon\partial_t F + \partial_x c^2 P = -\dfrac{1}{\epsilon}c\sigma^f F \\ \epsilon\partial_t \rho C_v T = \dfrac{1}{\epsilon}(c\sigma^a E - c\sigma^e aT^4) \end{cases}$$

Limit $\epsilon \longrightarrow 0$

- $E \longrightarrow aT^4$ and $F \longrightarrow 0$
- T given by a diffusion equation $\partial_t \left(\rho C_v T + aT^4\right) - \partial_x \left(\dfrac{4c}{3\sigma^f}\partial_x T\right) = 0$

☐ Main numerical difficulties

To preserve the relevant diffusive regimes

Turpault 02, Buet-Cordier 04, Buet-Després 06, CB-Dubroca-Charrier 07, Bouchut et al. 08, Coquel et al.

Objective: Asymptotic preserving methods

An hyperbolic model with source terms

$$\partial_t \mathbf{W} + \partial_x \mathcal{F}(\mathbf{W}) = \sigma \left(\mathbf{R}(\mathbf{W}) - \mathbf{W} \right) \qquad \mathbf{W} \in \Omega \text{ convex}$$

$$\sigma := \sigma(x, \mathbf{W}) > 0 \quad \mathbf{R}(\mathbf{W}) := \mathbf{R}(x, \mathbf{W}) \in \Omega$$

☐ M1-model: Source term given by

$$\mathbf{S}(\mathbf{W}) = c \begin{pmatrix} \sigma^e a T^4 - \sigma^a E \\ -\sigma^f F \\ \frac{1}{\rho C_v}(\sigma^a E - \sigma^e a T^4) \end{pmatrix} \equiv \sigma \left(\mathbf{R}(\mathbf{W}) - \mathbf{W} \right)$$

Assumptions

$$\begin{cases} \sigma^f = \sigma^a + \hat{\sigma} \quad \hat{\sigma} > 0 \quad \Leftrightarrow \quad \sigma^f > \sigma^a \\ \sigma^f = \dfrac{\sigma^e a T^3}{\rho C_v} + \check{\sigma} \quad \check{\sigma} > 0 \quad \Leftrightarrow \quad \sigma^f > \dfrac{\sigma^e a T^3}{\rho C_v} \end{cases}$$

To write

$$\begin{cases} \sigma^e a T^4 - \sigma^a E = \sigma^f \left(\dfrac{\sigma^e a T^4 + \hat{\sigma} E}{\sigma^f} - E \right) \\ \dfrac{1}{\rho C_v}(\sigma^a E - \sigma^e a T^4) = \sigma^f \left(\dfrac{\frac{\sigma^a}{\rho C_v} E + \check{\sigma} T}{\sigma^f} - T \right) \end{cases}$$

An hyperbolic model with source terms

$$\partial_t \mathbf{W} + \partial_x \mathcal{F}(\mathbf{W}) = \sigma\left(\mathbf{R}(\mathbf{W}) - \mathbf{W}\right) \qquad \mathbf{W} \in \Omega \text{ convex}$$

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□ M1-model: Source term given by

$$\mathbf{S}(\mathbf{W}) = c \begin{pmatrix} \sigma^e a T^4 - \sigma^a E \\ -\sigma^f F \\ \frac{1}{\rho C_v}(\sigma^a E - \sigma^e a T^4) \end{pmatrix} \equiv \sigma\left(\mathbf{R}(\mathbf{W}) - \mathbf{W}\right) = c\sigma^f \begin{pmatrix} \frac{\sigma^e a T^4 + \hat{\sigma} E}{\sigma^f} \\ 0 \\ \frac{\frac{\sigma^a}{\rho C_v} E + \check{\sigma} T}{\sigma^f} \end{pmatrix}$$

<u>Assumptions</u>

$$\begin{cases} \sigma^f = \sigma^a + \hat{\sigma} \quad \hat{\sigma} > 0 \quad \Leftrightarrow \quad \sigma^f > \sigma^a \\ \sigma^f = \dfrac{\sigma^e a T^3}{\rho C_v} + \check{\sigma} \quad \check{\sigma} > 0 \quad \Leftrightarrow \quad \sigma^f > \dfrac{\sigma^e a T^3}{\rho C_v} \end{cases}$$

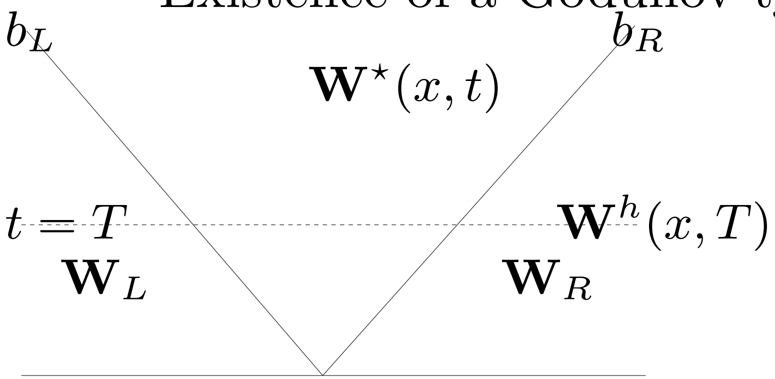
To write

$$\begin{cases} \sigma^e a T^4 - \sigma^a E = \sigma^f \left(\dfrac{\sigma^e a T^4 + \hat{\sigma} E}{\sigma^f} - E \right) \\ \dfrac{1}{\rho C_v}(\sigma^a E - \sigma^e a T^4) = \sigma^f \left(\dfrac{\frac{\sigma^a}{\rho C_v} E + \check{\sigma} T}{\sigma^f} - T \right) \end{cases}$$

A Riemann solver with stiff source terms

☐ Assumptions

Existence of a Godunov type Riemann solver to approximate



$$\partial_t \mathbf{W} + \partial_x \mathcal{F}(\mathbf{W}) = 0$$

b_L and b_R: velocities min and max of the approximate Riemann solver

$\mathbf{W}^\star$ approximate solution in the cone

↪ Formalism of Harten-Lax-van Leer (84)

$$\mathbf{W}_i^{n+1} = \mathbf{W}_i^n - \frac{\Delta t}{\Delta x}(\mathcal{F}_{i+\frac{1}{2}} - \mathcal{F}_{i-\frac{1}{2}})$$

$$\mathcal{F}_{i+\frac{1}{2}} = \mathcal{F}(\mathbf{W}_i^n) - \frac{\Delta x}{2\Delta t}\mathbf{W}_{i+1}^n + \frac{1}{\Delta t}\int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \mathbf{W}^h(x, t^n + \Delta t)\partial_x$$

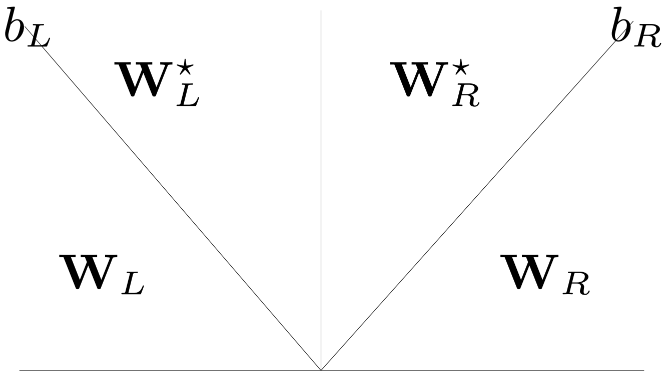
↪ CFL restriction $\quad \frac{\Delta t}{\Delta x} \max\left(|b^R_{i-\frac{1}{2}}|, |b^L_{i+\frac{1}{2}}|\right) \le \frac{1}{2}$

Lemma

Assume $\mathbf{W}^\star \in \Omega$. As a consequence, as soon as $\mathbf{W}_i^n \in \Omega$ for all $i \in \mathbb{Z}$ then $\mathbf{W}_i^{n+1} \in \Omega$ for all $i \in \mathbb{Z}$

□ Stiff source terms

Modify the Riemann solver to consider the source terms



$$\mathbf{W}_L^\star = \alpha \mathbf{W}^\star + (1-\alpha)\mathbf{R}^-(\mathbf{W}_L)$$

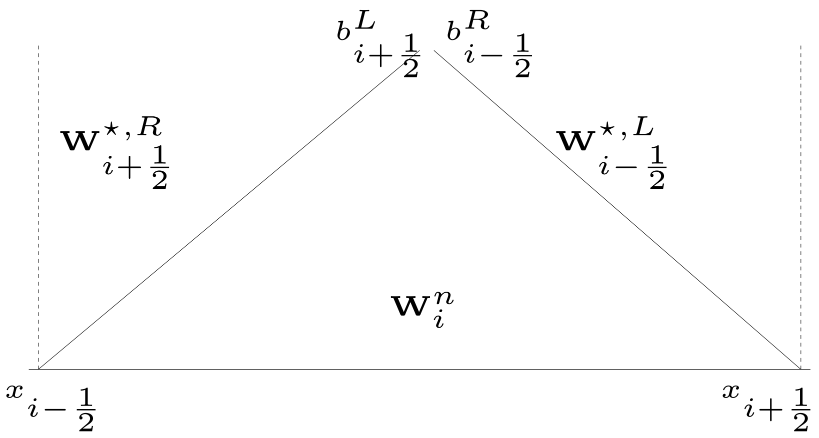
$$\mathbf{W}_R^\star = \alpha \mathbf{W}^\star + (1-\alpha)\mathbf{R}^+(\mathbf{W}_R)$$

$$\alpha = \frac{b^R - b^L}{b^R - b^L + \sigma \Delta x}$$

We note $\tilde{\mathbf{W}}^h(x,t)$ the approximate solution

$\hookrightarrow$ $\mathbf{R}^\pm(\mathbf{W})$ consistent with $\mathbf{R}(\mathbf{W})$

$\hookrightarrow$ Unchanged CFL restriction $\quad \frac{\Delta t}{\Delta x} \max\left(|b^R_{i-\frac{1}{2}}|, |b^L_{i+\frac{1}{2}}|\right) \leq \frac{1}{2}$



$$\mathbf{W}_i^{n+1} = \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathbf{W}^h(x, t^{n+1})dx$$

$$\mathbf{W}^{\star,L}_{i-\frac{1}{2}} = \alpha_{i-\frac{1}{2}} \mathbf{W}^\star_{i-\frac{1}{2}} + (1-\alpha_{i-\frac{1}{2}})\mathbf{R}^+_{i-\frac{1}{2}}(\mathbf{W}_i^n)$$

$$\mathbf{W}^{\star,R}_{i+\frac{1}{2}} = \alpha_{i+\frac{1}{2}} \mathbf{W}^\star_{i+\frac{1}{2}} + (1-\alpha_{i+\frac{1}{2}})\mathbf{R}^+_{i+\frac{1}{2}}(\mathbf{W}_i^n)$$

To obtain

$$\mathbf{W}_i^{n+1} = \mathbf{W}_i^n - \frac{\Delta t}{\Delta x}\left(\alpha_{i+\frac{1}{2}}\mathcal{F}_{i+\frac{1}{2}} - \alpha_{i-\frac{1}{2}}\mathcal{F}_{i-\frac{1}{2}}\right) + \Delta t\left(\frac{1}{\Delta x}(1-\alpha_{i-\frac{1}{2}})\mathbf{S}^+_{i-\frac{1}{2}} + \frac{1}{\Delta x}(1-\alpha_{i+\frac{1}{2}})\mathbf{S}^-_{i+\frac{1}{2}}\right)$$

$$\mathbf{S}^+_{i-\frac{1}{2}} = \max(0, b^R_{i-\frac{1}{2}})(\mathbf{R}^+_{i-\frac{1}{2}}(\mathbf{W}_i^n) - \mathbf{W}_i^n) - \max(0, b^L_{i-\frac{1}{2}})(\mathbf{R}^+_{i-\frac{1}{2}}(\mathbf{W}_i^n) - \mathbf{W}_{i-1}^n) + \mathcal{F}(\mathbf{W}_i^n)$$

$$\mathbf{S}^-_{i+\frac{1}{2}} = \min(0, b^R_{i+\frac{1}{2}})(\mathbf{R}^-_{i+\frac{1}{2}}(\mathbf{W}_i^n) - \mathbf{W}_{i+1}^n) - \min(0, b^L_{i+\frac{1}{2}})(\mathbf{R}^-_{i+\frac{1}{2}}(\mathbf{W}_i^n) - \mathbf{W}_{i-1}^n) - \mathcal{F}(\mathbf{W}_i^n)$$

Lemma Robustness

Assume the initial Godunov type scheme is robust: Ω stays invariant by the scheme

For all $\mathbf{W}_L$ and $\mathbf{W}_R$ in Ω, assume

$$\alpha \mathbf{W}^\star + (1-\alpha)\mathbf{R}^-(\mathbf{W}_L) \in \Omega$$

$$\alpha \mathbf{W}^\star + (1-\alpha)\mathbf{R}^+(\mathbf{W}_R) \in \Omega$$

Then, whith the same CFL restriction, the full scheme with stiff source term preserve the robustness property

$$\mathbf{W}_i^n \in \Omega \quad \text{for all} \quad i \in \mathbb{Z} \Rightarrow \mathbf{W}_i^{n+1} \in \Omega \quad \text{for all} \quad i \in \mathbb{Z}$$

□ Numerical asymptotic regime

$$\partial_t \mathbf{W} + \partial_x \mathcal{F}(\mathbf{W}) = \sigma \left(\mathbf{R}(\mathbf{W}) - \mathbf{W}\right)$$

Godunov type scheme + Source term $\Leftrightarrow$ Asymptotic regime reached

□ Introduction of a correction

$\bar{\sigma} > 0$ arbitrary

$$\begin{aligned}
\partial_t \mathbf{W} + \partial_x \mathcal{F}(\mathbf{W}) &= (\sigma \mathbf{R}(\mathbf{W}) + \bar{\sigma}\mathbf{W}) - (\bar{\sigma}\mathbf{W} + \sigma \mathbf{W}) \\
&= (\sigma + \bar{\sigma}) \left(\left(\frac{\sigma}{\sigma + \bar{\sigma}} \mathbf{R}(\mathbf{W}) + \frac{\bar{\sigma}}{\sigma + \bar{\sigma}} \mathbf{W} \right) - \mathbf{W} \right) \\
&= (\sigma + \bar{\sigma}) \left(\bar{\mathbf{R}}(\mathbf{W}) - \mathbf{W} \right)
\end{aligned}$$

The same formalism but for

$$\bar{\mathbf{R}}(\mathbf{W}) = \frac{\sigma}{\sigma + \bar{\sigma}} \mathbf{R}(\mathbf{W}) + \frac{\bar{\sigma}}{\sigma + \bar{\sigma}} \mathbf{W} \quad \in \Omega$$

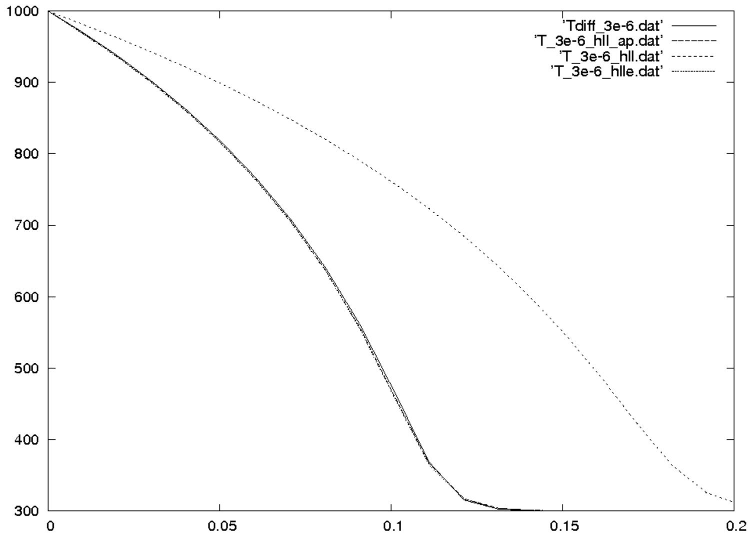
$\bar{\sigma}$ will be a relevant correction to satisfy the diffusive regime

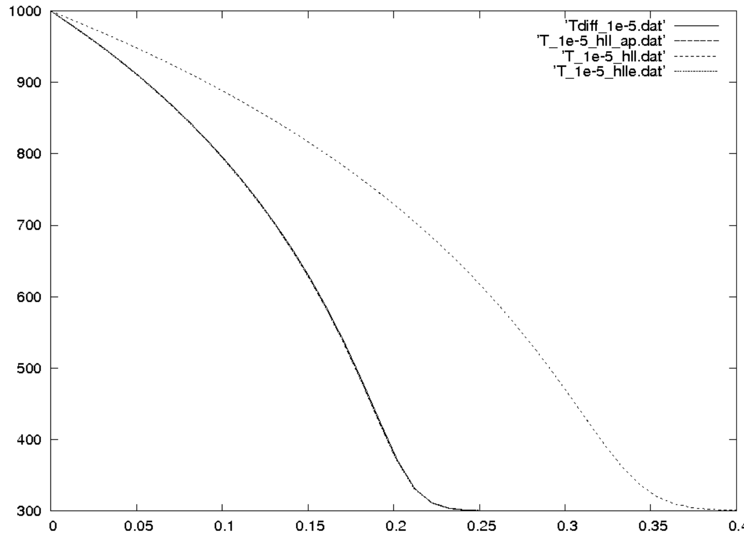
Radiative transfer: Marshak wave

Temperature obainted with

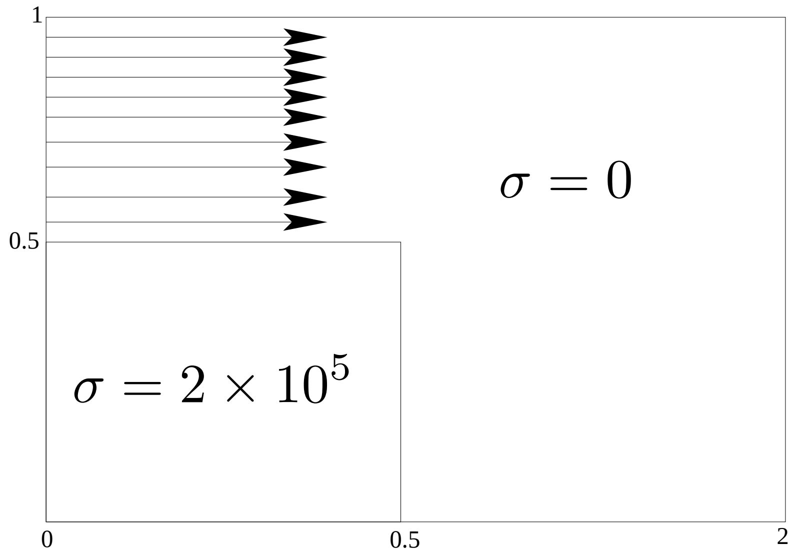
- diffusion equation
- HLL without correction
- HLL with the asymptotic preserving correction $\bar{\sigma}$
- HLL with b^L and b^R fixed to satisfy the asymptotic regime

time $t = 3.\ 10^{-6}$ and $t = 1.\ 10^{-5}$





2D case: shadow cone



Temperature on the left side of the transparent region $T = 5.8\ 10^6$

Initial temperature in the dense region $T = 1$

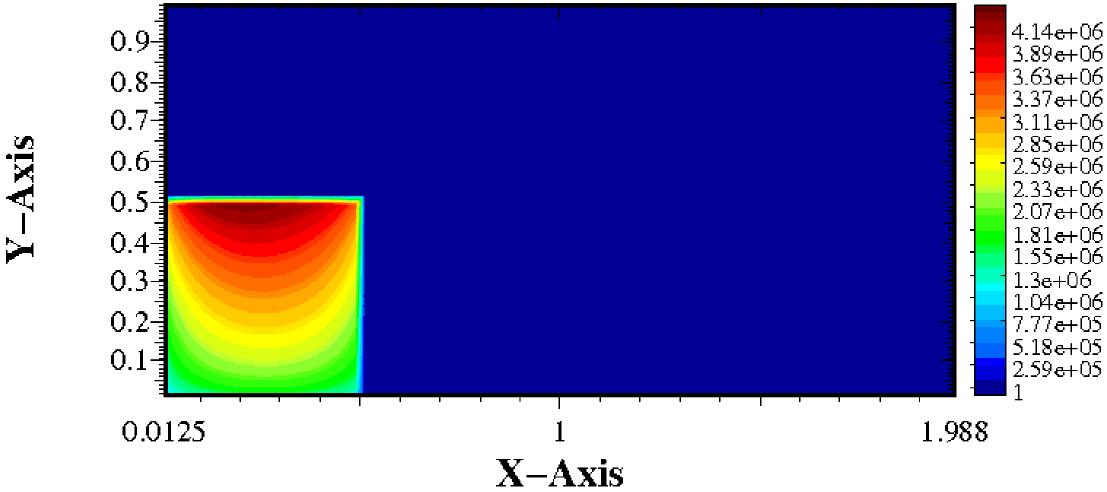
Initial temperature in the transparent region $T = 300$

Expected solution

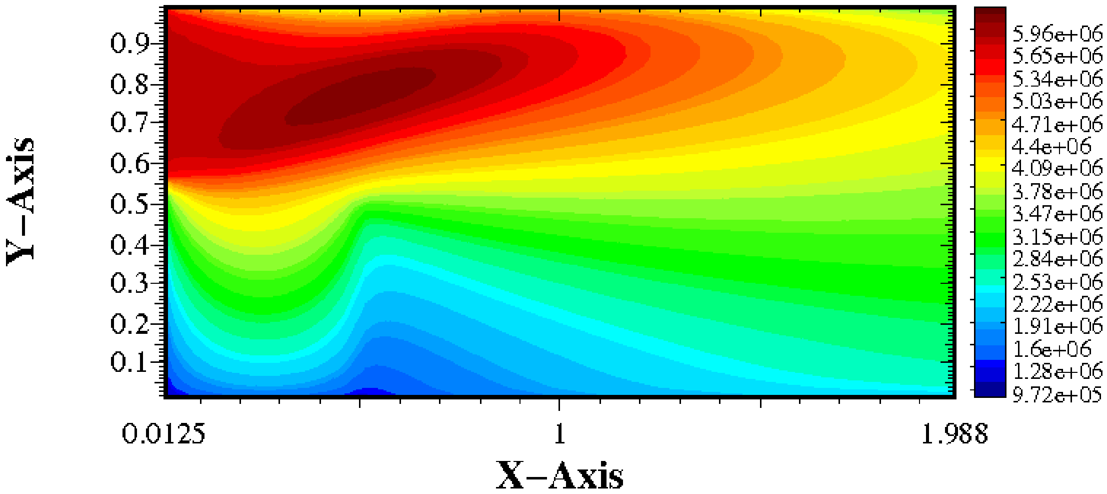
- Upper part: free steaming
- Lower part: constant solution, no photon enters this area
- $y = 0.5$: stationary contact discontinuity
- Temperature in the dense region $T = 1$

Shadow cone with a standard HLL scheme

Matter temperature

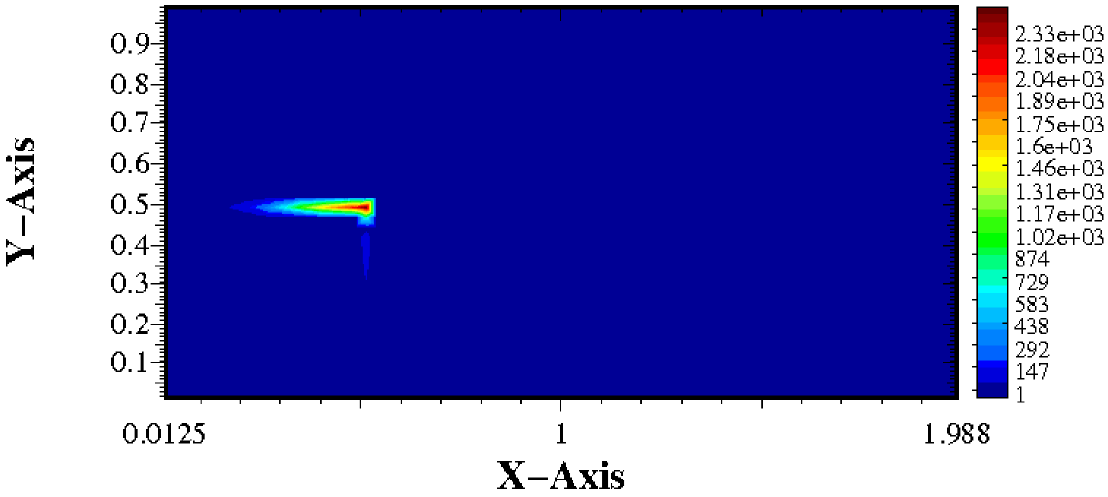


Radiative temperature

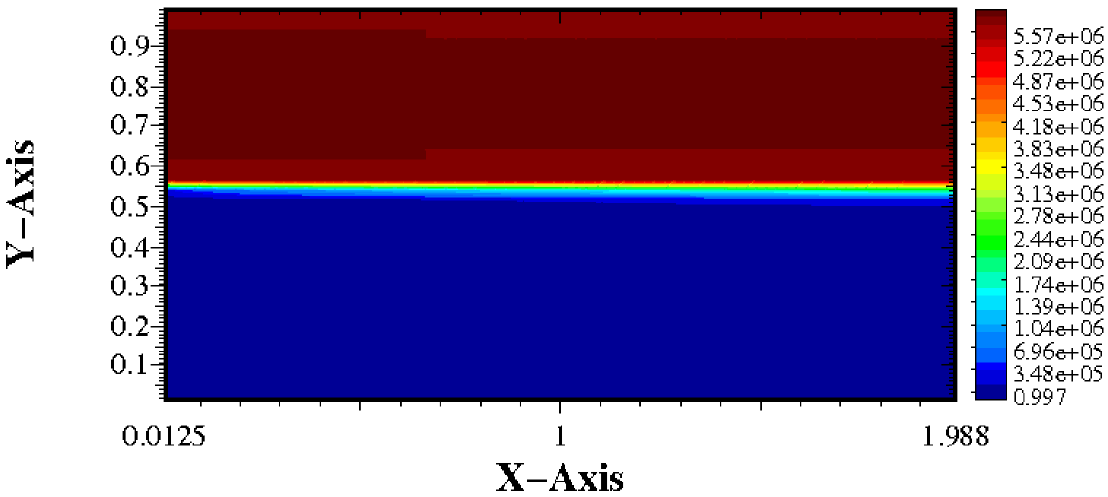


Shadow cone with the HLLC scheme

Matter temperature



Radiative temperature



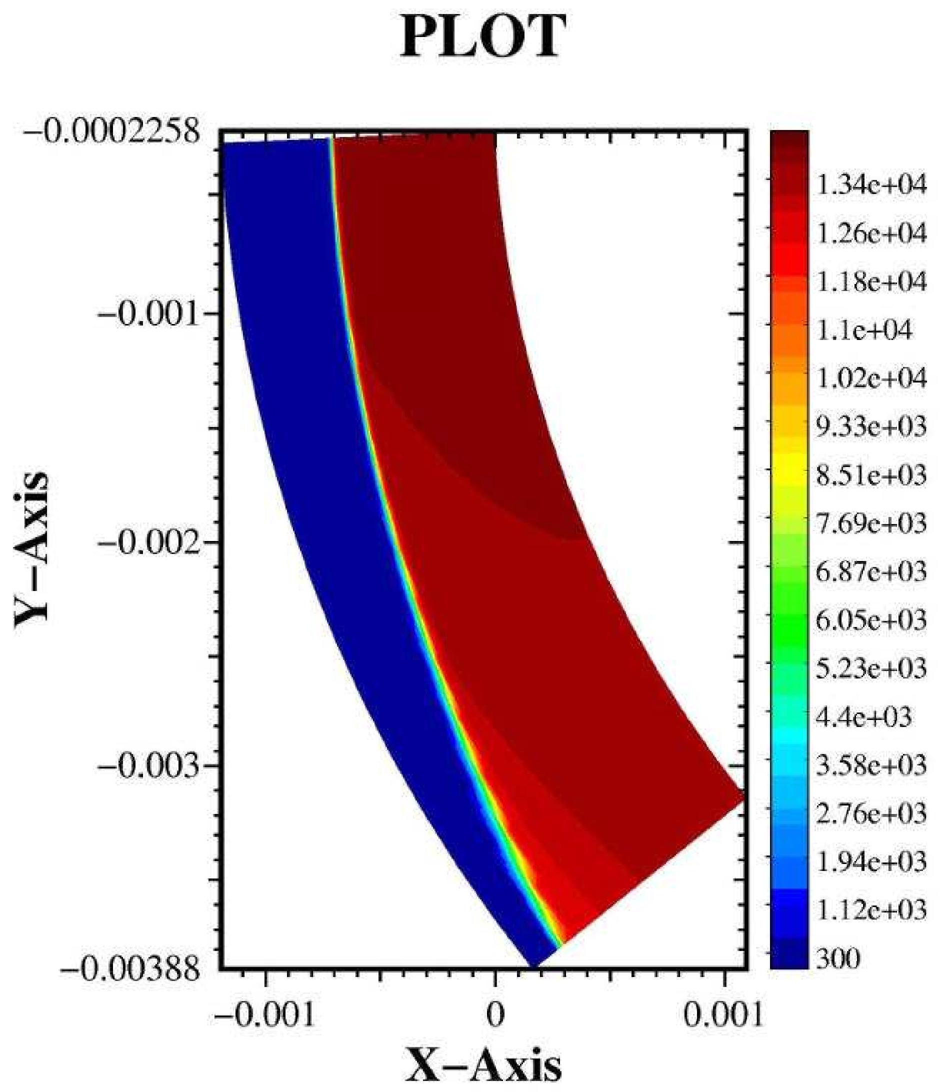
Material temperature in the dense region

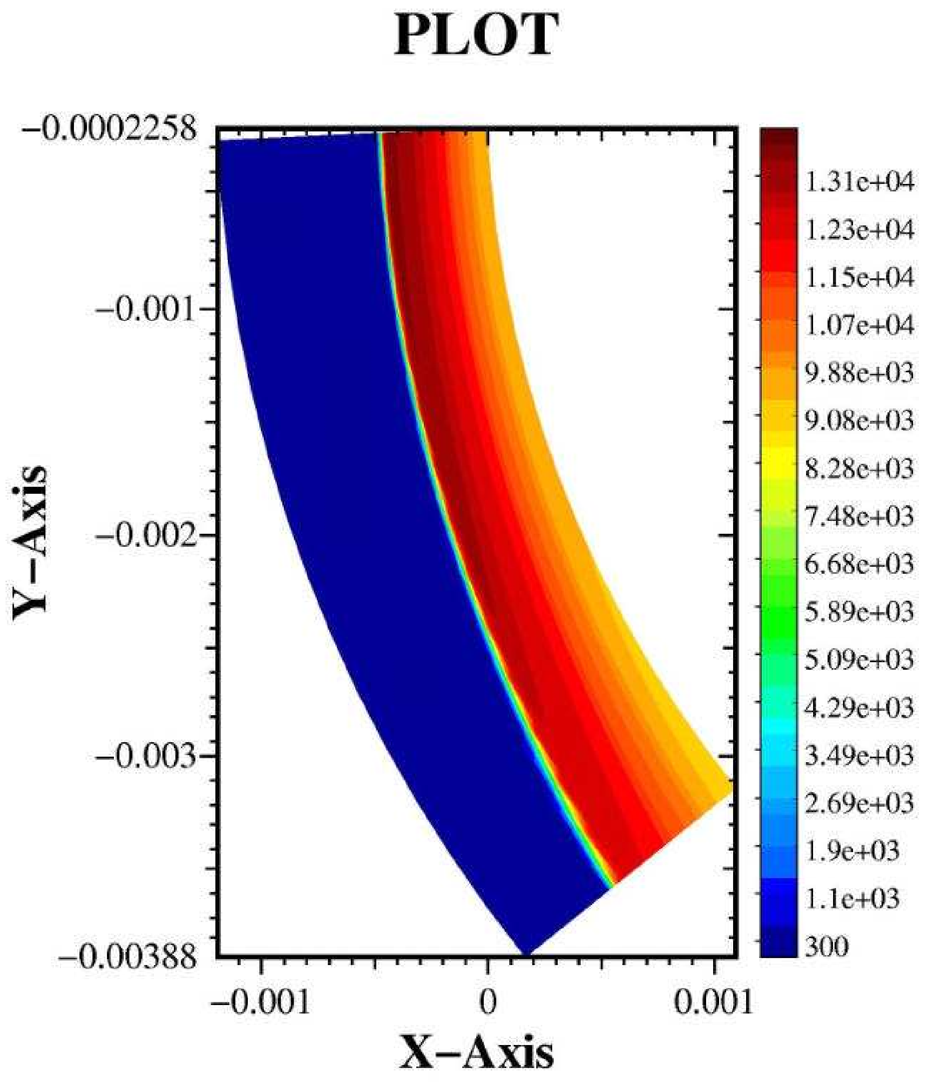
Exact solution is $T = 1$

Schemes	max material T		average material T	
	HLL	HLLC	HLL	HLLC
b_R, b_L csts	4300000	17000	350000	840
b_R, b_L variables	3700000	170000	290000	11000
b_R, b_L csts + AP	3600000	9000	43000	40
b_R, b_L variables + AP	740000	110000	5200	690
b_R, b_L csts + Minmod	3600000	16000	240000	340
b_R, b_L csts + Minmod + AP	2700000	6400	34000	26
b_R, b_L csts + Superbee + AP	1600000	2400	11000	6.1

Bullet test

velocity = $14\ \mathrm{km.s}^{-1}$

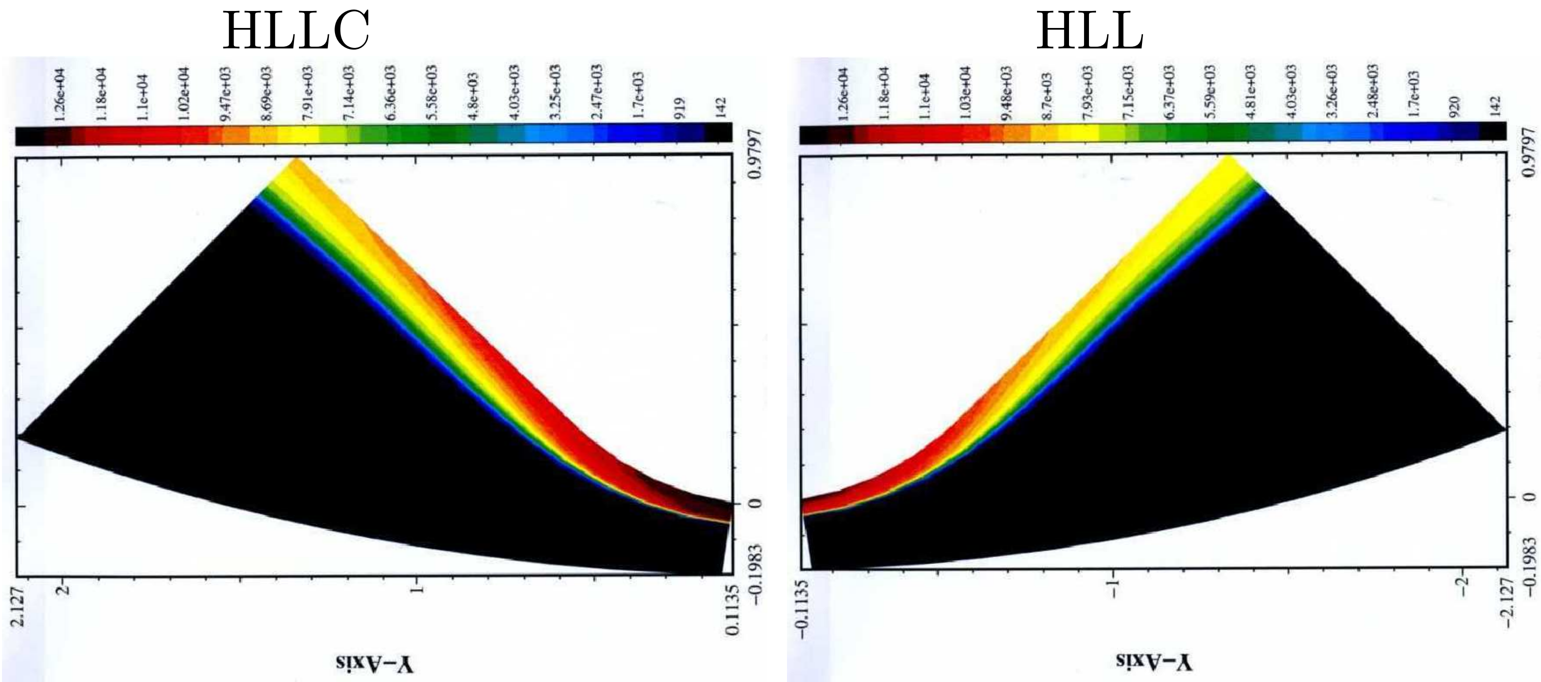




Venus entry

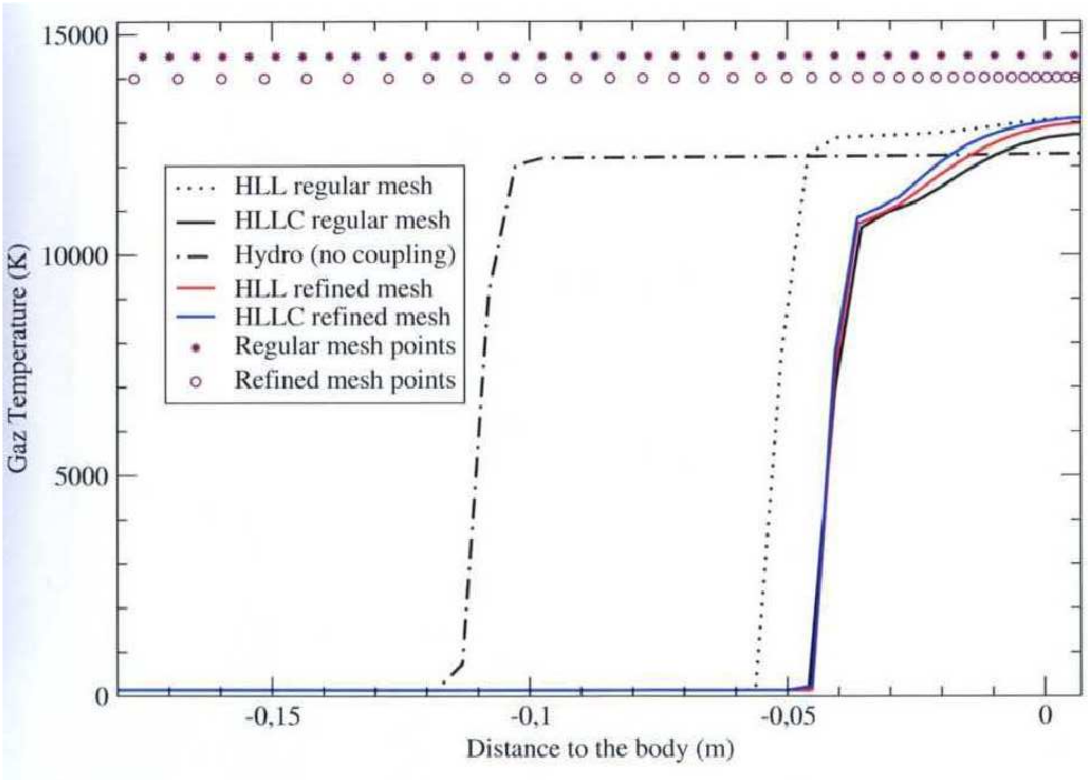
Pioner probe

Alt = 80 km, V = 11 km/s, T =142 K, P = 300 Pa



$T_{max} \sim 12000$ K

Venus entry



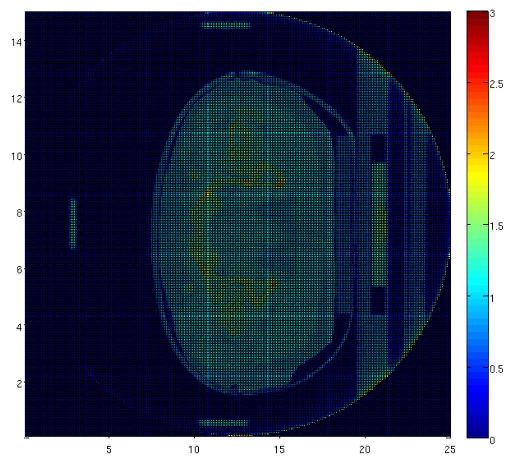
Refined mesh $\sim$ 20 hours to converge

Regular mesh $\sim$ 8 hours to converge

HLLC scheme cheaper

From ERT to Radiotherapy

- Modelisation of electron flows
  - ↪ Use of ionizing radiation to treat cancer
  - ↪ Aim: destroy the cancer cells and preserve healthy cells
  - ↪ One or several beams sent into the body of the patient
  - ↪ Linear accelerator: x-rays of very high energy



☐ Extension: the M1 model to electrons

$$\partial_t E + \partial_x F = \partial_\varepsilon \rho(x) S(\varepsilon) E$$

$$\partial_t F + \partial_x E\chi(\frac{F}{E}) = \partial_\varepsilon \rho(x) S(\varepsilon) F - \rho(x) T(\varepsilon) F$$

$\varepsilon > 0$ an energy

Numerical simulations $\Leftrightarrow$ Numerical approximations of the M1 model

☐ Extension: the M1 model to electrons

$$\partial_t \Psi^0 + \partial_x \Psi^1 = \partial_\varepsilon \rho(x) S(\varepsilon) \Psi^0$$

$$\partial_t \Psi^1 + \partial_x \Psi^0 \chi(\frac{\Psi^1}{\Psi^0}) = \partial_\varepsilon \rho(x) S(\varepsilon) \Psi^1 - \rho(x) T(\varepsilon) \Psi^1$$

$\varepsilon > 0$ the energy

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$\varepsilon > 0$ the energy

☐ Numerical evaluation of the dose

$$D = \int_0^{+\infty} S(\varepsilon) \Psi^0 d\varepsilon$$

$\hookrightarrow$ Quadrature $\Rightarrow$ Evaluation of $(\Psi^0(x, t, \varepsilon^p))_p$ with p large

M1 model must be solved p times

□ Extension: the M1 model to electrons

$$\partial_t \Psi^0 + \partial_x \Psi^1 = \partial_\varepsilon \rho(x) S(\varepsilon) \Psi^0$$

$$\partial_t \Psi^1 + \partial_x \Psi^0 \chi(\frac{\Psi^1}{\Psi^0}) = \partial_\varepsilon \rho(x) S(\varepsilon) \Psi^1 - \rho(x) T(\varepsilon) \Psi^1$$

$\varepsilon > 0$ the energy

□ Numerical evaluation of the dose

$$D(x) = \int_0^{+\infty} S(\varepsilon) \Psi^0 d\varepsilon$$

$\hookrightarrow$ Quadrature $\Rightarrow$ Evaluation of $(\Psi^0(x,t,\varepsilon^p))_p$ with p large

M1 model must be solved p times

$\hookrightarrow$ Ψ^0 given by the stationary regime $t \to +\infty$

□ Objective: FAST

$\hookrightarrow$ Robustness

$\hookrightarrow$ $\rho(x)$ is a given stiff function

Reformulation

□ Stationary model

$$\begin{cases} \partial_x \Psi^1 = \partial_\varepsilon \rho S \Psi^0 \\ \partial_x \Psi^0 \chi = \partial_\varepsilon \rho S \Psi^1 - \rho T \Psi^1 \end{cases} \Leftrightarrow \begin{cases} \partial_\varepsilon \rho S \Psi^0 - \partial_x \Psi^1 = 0 \\ \partial_\varepsilon \rho S \Psi^1 - \partial_x \Psi^0 \chi = \rho T \Psi^1 \end{cases}$$

ε as a pseudo time

□ Backwards model

$$\begin{cases} \lim_{\varepsilon \to \infty} \Psi^0 = 0 \\ \lim_{\varepsilon \to \infty} \Psi^1 = 0 \end{cases} \quad \text{with fast decay} \quad D(x) \sim \int_0^{\varepsilon_{\max}} S(\varepsilon) \Psi^0 d\varepsilon$$

$\hookrightarrow$ Approximation of "initial" data

$$\Psi^0(x, \varepsilon_{\max}) = 0 \qquad \Psi^1(x, \varepsilon_{\max}) = 0$$

$\varepsilon_{\max}$ large enough

Evaluation of $\Psi^0(x, \varepsilon)$ and $\Psi^1(x, \varepsilon)$ with $\varepsilon \in (0, \varepsilon_{\max})$

Changes of variables

□ Distortion of the phase space: $\rho(x)$ and $S(\varepsilon)$

$$\begin{cases} \partial_\varepsilon \rho S \Psi^0 - \partial_x \Psi^1 = 0 \\ \partial_\varepsilon \rho S \Psi^1 - \partial_x \Psi^0 \chi(\dfrac{\Psi^1}{\Psi^0}) = \rho T \Psi^1 \end{cases}$$

Changes of variables

☐ Distortion of the phase space: $\rho(x)$ and $S(\varepsilon)$

$$\begin{cases} \partial_\varepsilon S \Psi^0 - \dfrac{1}{\rho}\partial_x \Psi^1 = 0 \\ \partial_\varepsilon S \Psi^1 - \dfrac{1}{\rho}\partial_x \Psi^0 \chi(\dfrac{\Psi^1}{\Psi^0}) = T\Psi^1 \end{cases}$$

$$S\Psi^0 = \hat{\Psi}^0 \qquad S\Psi^1 = \hat{\Psi}^1$$

Changes of variables

□ Distortion of the phase space: $\rho(x)$ and $S(\varepsilon)$

$$\begin{cases} \partial_\varepsilon \hat{\Psi}^0 - \dfrac{1}{\rho}\partial_x \dfrac{\hat{\Psi}^1}{S} = 0 \\ \partial_\varepsilon \hat{\Psi}^1 - \dfrac{1}{\rho}\partial_x \dfrac{\hat{\Psi}^0}{S}\chi(\dfrac{\hat{\Psi}^1}{\hat{\Psi}^0}) = \dfrac{T}{S}\hat{\Psi}^1 \end{cases}$$

$$S\Psi^0 = \hat{\Psi}^0 \qquad S\Psi^1 = \hat{\Psi}^1$$

Changes of variables

□ Distortion of the phase space: $\rho(x)$ and $S(\varepsilon)$

$$\begin{cases} S(\varepsilon)\partial_\varepsilon \hat{\Psi}^0 - \dfrac{1}{\rho(x)}\partial_x \hat{\Psi}^1 = 0 \\ S(\varepsilon)\partial_\varepsilon \hat{\Psi}^1 - \dfrac{1}{\rho(x)}\partial_x \hat{\Psi}^0 \chi(\dfrac{\hat{\Psi}^1}{\hat{\Psi}^0}) = T\hat{\Psi}^1 \end{cases}$$

$$S\Psi^0 = \hat{\Psi}^0 \qquad S\Psi^1 = \hat{\Psi}^1$$

Changes of variables

□ Distortion of the phase space: $\rho(x)$ and $S(\varepsilon)$

$$\begin{cases} \partial_{\tilde{\varepsilon}} \tilde{\Psi}^0 - \partial_{\tilde{x}} \tilde{\Psi}^1 = 0 \\ \partial_{\tilde{\varepsilon}} \tilde{\Psi}^1 - \partial_{\tilde{x}} \tilde{\Psi}^0 \chi(\dfrac{\tilde{\Psi}^1}{\tilde{\Psi}^0}) = \tilde{T}(\tilde{\varepsilon}) \tilde{\Psi}^1 \end{cases}$$

$$S\Psi^0 = \hat{\Psi}^0(x,\varepsilon) = \tilde{\Psi}^0(\tilde{x},\tilde{\varepsilon}) \qquad S\Psi^1 = \hat{\Psi}^1(x,\varepsilon) = \tilde{\Psi}^1(\tilde{x},\tilde{\varepsilon})$$

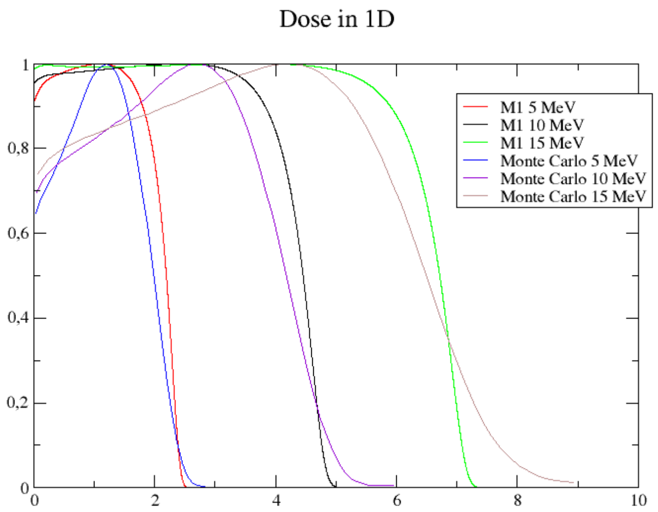
with

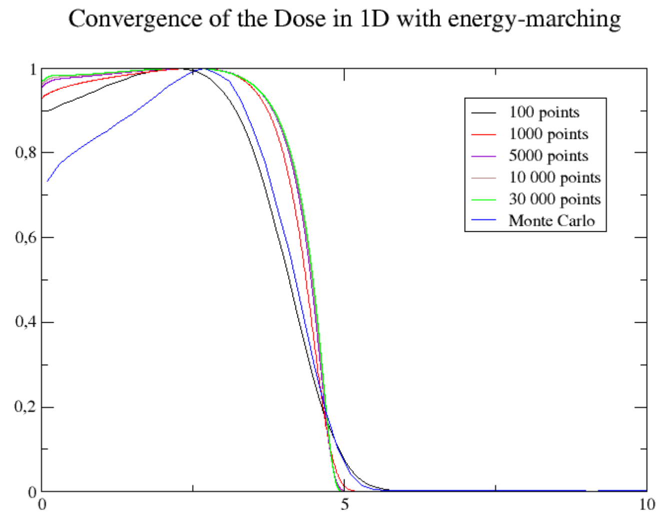
$$\tilde{x}(x) = \int_0^x \rho(t)dt \qquad \tilde{\varepsilon}(\varepsilon) = \int_0^\varepsilon \frac{1}{S(t)} dt$$

□ Numerical approximation

$\hookrightarrow$ HLL scheme

1D valadation tests

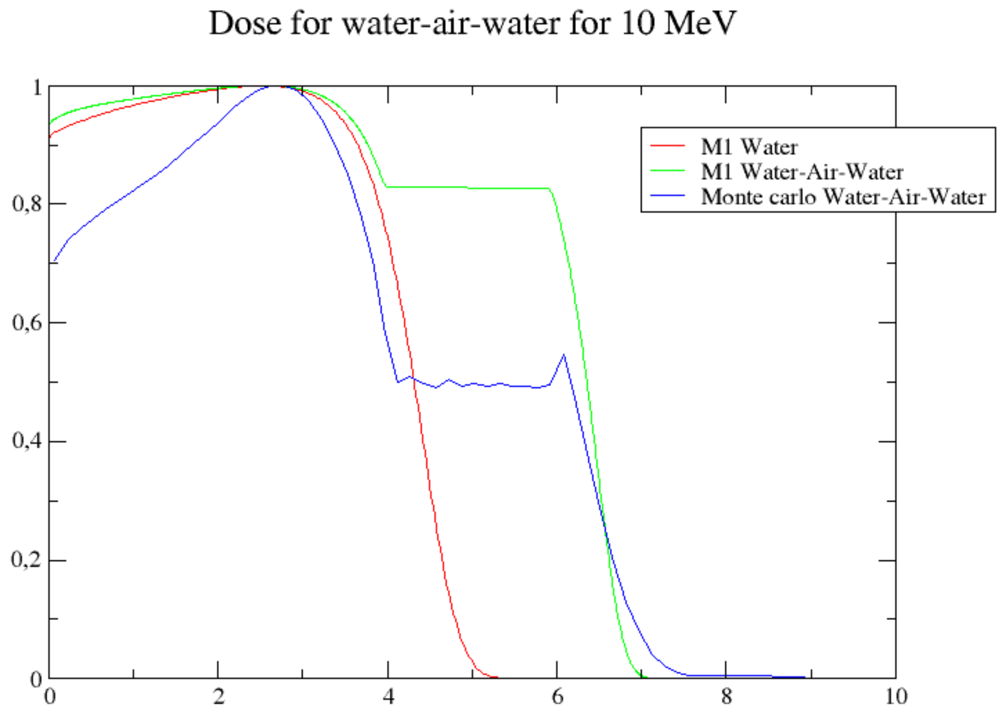




CPU time

- M1 model (5000 nodes) $\simeq$ 30 secondes
- Monte Carlo method $\simeq$ 10 minutes

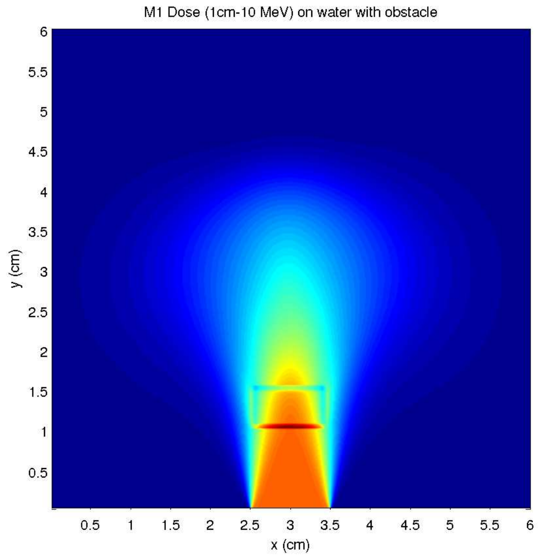
Discontinuous density

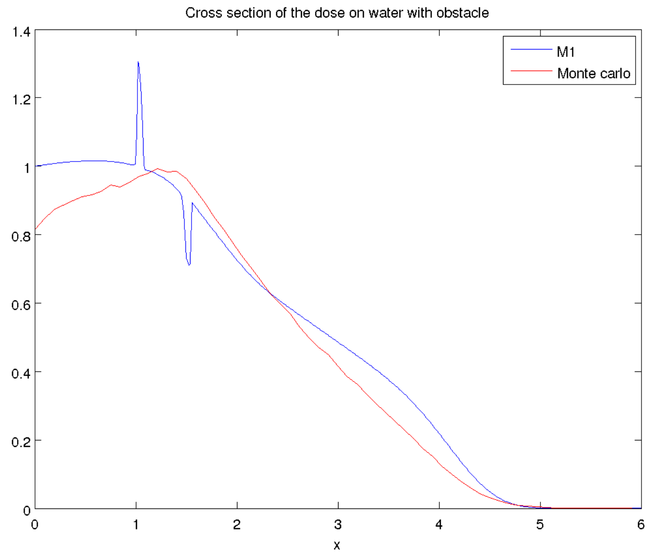


2D extension

It does not exist glogal 2D change of variables $(\tilde{x}, \tilde{y})$

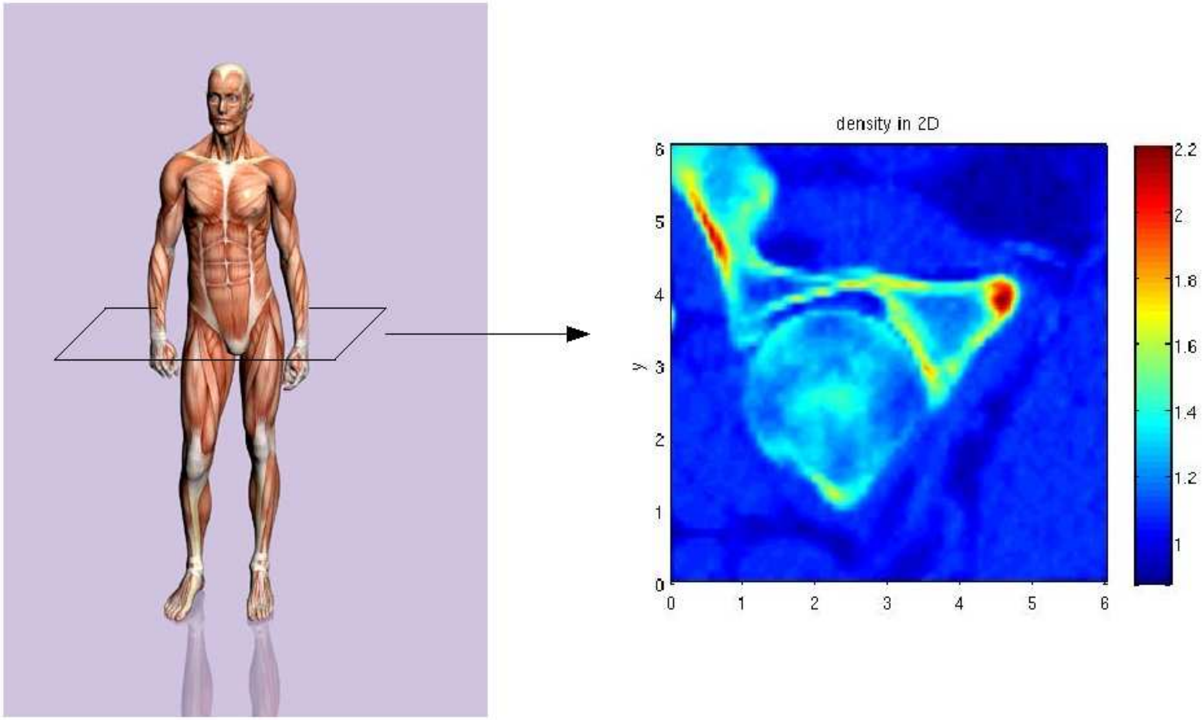
☐ Algorithm based on local space distortions



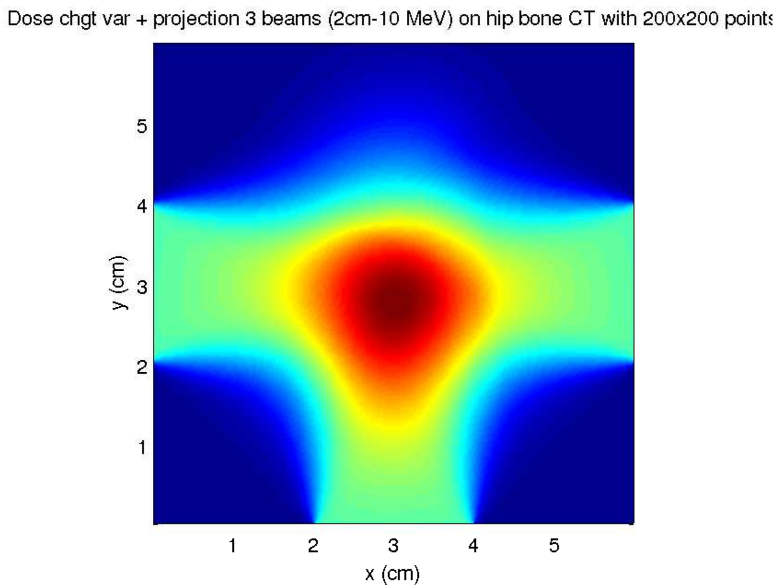


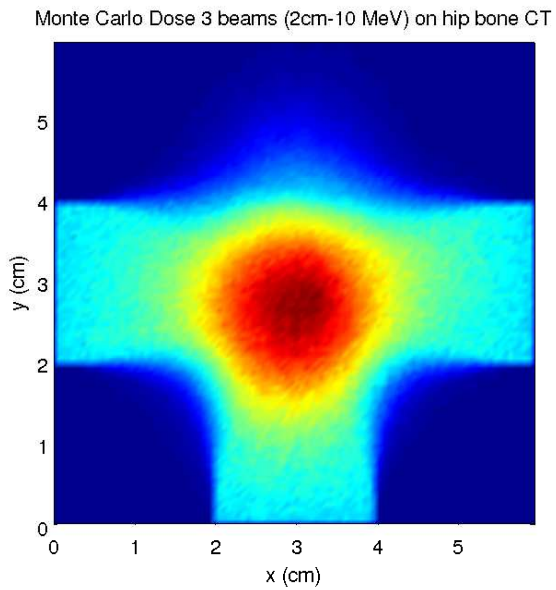
CT scan

Scanner at hip level of a man,

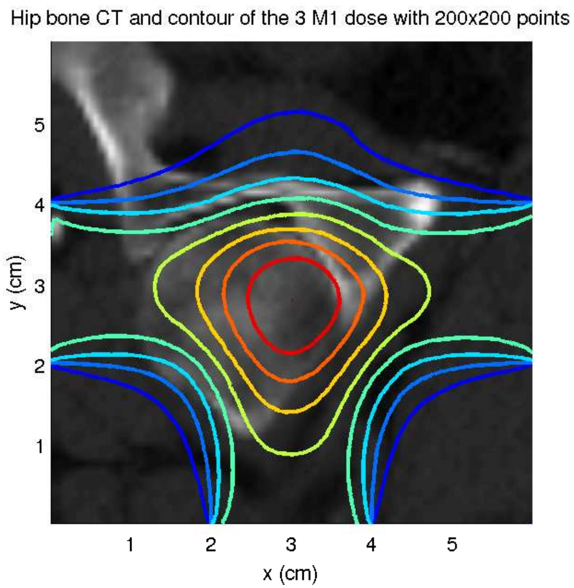


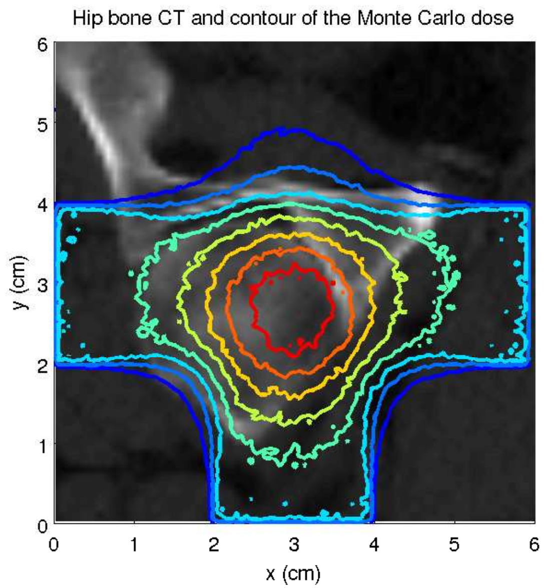
Cut an interesting part with different tissues.





- M1 model (200x200) $\simeq$ 1 hour/ray
- Monte Carlo method $\simeq$ 23 hours/ray


Hip bone CT and contour of the 3 M1 dose with 200x200 points
y (cm)
x (cm)
1
2
3
4
5
1
2
3
4
5


Hip bone CT and contour of the Monte Carlo dose
y (cm)
x (cm)
0
1
2
3
4
5
6
0
1
2
3
4
5
6

Thanks for your attention