DE LA RECHERCHE À L'INDUSTRIE

Scalable Poisson solver for gyrokinetic simulations

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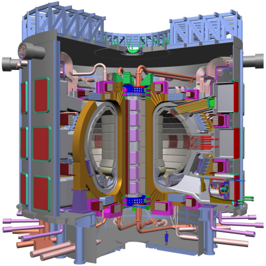

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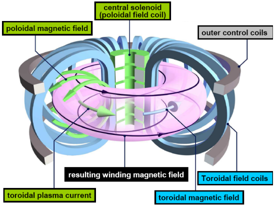
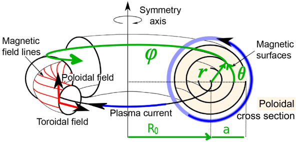


magnetic toroidal geometry (r, θ, φ)



- Scaling law in tokamaks: plasma volume $\times \tau_E \approx$ cte
  with $\tau_E =$ energy confinement time $\sim$ measure of thermal insulation.

➡ Two main possibilities to increase tokamak performances:

❶ increase the size of the machine or/and ❷ increase τ_E

- Turbulence governs τ_E
  - ➡ Generates loss of heat and particles
  - ➡ ↘ Confinement properties of the magnetic configuration

- Understanding, predicting and controlling turbulence for optimizing experiments like ITER and future reactors is a subject of utmost importance.

Gyrokinetic theory:
➠ large phase space reduction 6D to 5D

Kinetic theory: ➠ 6D distribution function of particles
(3D in space and 3D in velocity) $F_s(r, \theta, \varphi, v_\parallel, v_\perp, \alpha)$

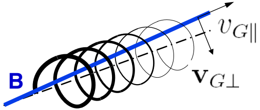
- Fusion plasma turbulence is low frequency:

$$\omega_{\text{turb}} \sim 10^5 s^{-1} \ll \omega_{ci} \sim 10^8 s^{-1}$$

- Phase space reduction: fast gyro-motion is averaged out
  - ➠ Adiabatic invariant: magnetic moment $\mu = m_s v_\perp^2 / (2B)$
  - ➠ Velocity drifts of guiding centers

☺ Large reduction memory/CPU time

☹ Complexity of the system



Gyrokinetic theory: ➠ 5D distribution function of guiding-centers
$\bar{F}_s(r, \theta, \varphi, v_{G\parallel}, \mu)$ where μ parameter

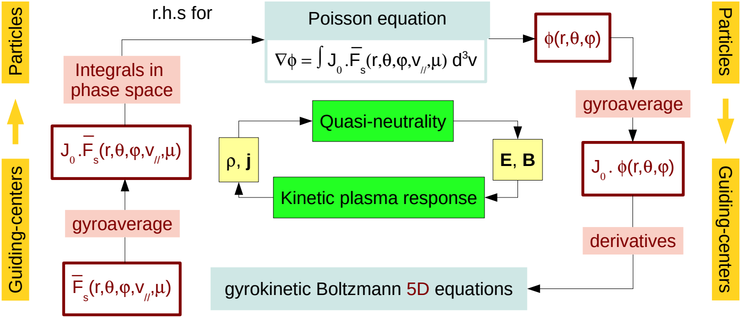
Gyrokinetic codes require state-of-the-art HPC

- Gyrokinetic codes require state-of-the-art HPC techniques and must run efficiently on several thousands processors.
  - non-linear 5D simulations
  - multi-scale problem in space and time
    - time: $\Delta t \approx \gamma^{-1} \sim 10^{-6}s \rightarrow t_{\mathrm{simul}} \approx$ few $\tau_E \sim 10s$
    - space: $\rho_i \rightarrow$ machine size a $\quad \rho_* \equiv \frac{\rho_i}{a} \ll 1 \quad (\rho_*^{\mathrm{ITER}} \approx 10^{-3})$

➠ GK codes are extremely CPU time consuming

- GK code development is an highly international competitive activity
  - US: $\sim$ 8 codes - EU: 5 codes - Japan: 2 codes
- European collaboration ⇒ Eurofusion project (2014 + 2015-2018?)
  - Validation and verification of european GK codes
    - GYSELA (France) - GENE (Germany) - ORB5 (Switzerland)

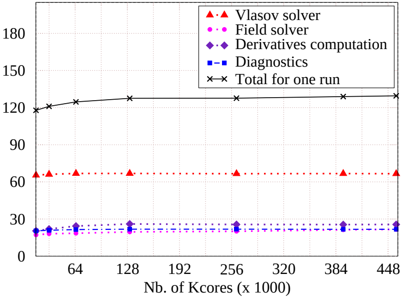
- Gyrokinetic complexity due to the fact the Poisson is solved with the charge density of particles and the Vlasov equation describe the guiding-center evolution.



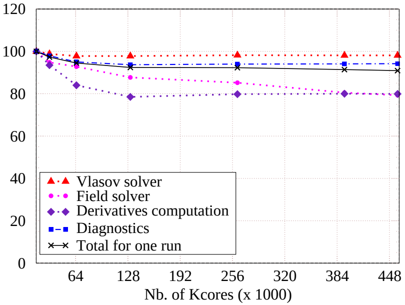
WEAK scaling: (on JUQUEEN - Juelich)
Relative efficiency of 91% on 458 752 cores

- Hybrid MPI/OpenMP parallelism
- Relative efficiency of 91% on 458 752 cores (performed on the totality of JUQUEEN/Blue Gene machine (Juelich))

Execution time, one Gysela (Weak Scaling - Juqueen)



Relative efficiency, one run (Weak scaling - Juqueen)



- Poisson solver ~ 15% of the total time
- Efficiency of Poisson solver ~ 41% ➠ Work still under progress.

- Solving the 3D quasi-neutrality equation is equivalent to:

  Find $\phi(r,\theta,\varphi)$ such that:

$$\underbrace{-\frac{1}{n_{e_0}}\sum_s Z_s \nabla_\perp \cdot \left(\frac{n_{s,\mathrm{eq}}}{B\Omega_s}\nabla_\perp\phi\right)}_{\text{polarization term due to } \neq \text{ between guiding-centers and particles}} + \underbrace{\frac{e}{T_{e,\mathrm{eq}}}\left(\phi - \left\langle\phi\right\rangle_{\mathrm{FS}}\right)}_{\text{adiabatic electrons}} = \frac{1}{n_{e_0}}\sum_s Z_s \int J_0 \cdot \left(\bar{F}_s - \bar{F}_{s,\mathrm{eq}}\right) d^3v$$

- Numerical methods:
  - Fourier projection in periodic directions θ and φ
  - Finite differences in radial direction

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- <u>Difficulties:</u>

  ☹ R.H.S = integral over the velocity space
  $\Rightarrow$ *Parallel communications* $++$

Numerical methods for 3D quasi-neutrality equation solving

- Solving the 3D quasi-neutrality equation is equivalent to:

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polarization term due to $\neq$ between guiding-centers and particles

adiabatic electrons

- Difficulties:
  - ☹ R.H.S = integral over the velocity space

    $\Rightarrow$ *Parallel communications* $++$
  - ☹ $\langle\phi\rangle_{\mathrm{FS}} = \int\int \phi \mathcal{J}_x \mathrm{d}\theta \mathrm{d}\varphi / \int\int \mathcal{J}_x \mathrm{d}\theta \mathrm{d}\varphi$ flux surface average of ϕ

    $\Rightarrow$ *Pb in Fourier due to coupling between* θ *and* φ

Quasi-neutrality: Treatment of $\langle \phi \rangle_{\mathrm{FS}}$

- Quasi-neutrality → Poisson equation form:

$$\mathcal{L}\phi(r,\theta,\varphi) + \alpha(r)\left(\phi(r,\theta,\varphi) - \langle\, \phi \,\rangle_{\mathrm{FS}}(r)\right) = \rho(r,\theta,\varphi) \qquad (1)$$

with $\mathcal{L} = \frac{1}{n_{e_0}} \sum_s Z_s \nabla_\perp \cdot \left(\frac{n_{s,\mathrm{eq}}}{B\Omega_s} \nabla_\perp \phi \right)$ and $\langle\cdot\rangle_{\mathrm{FS}} = \int\int \cdot\, \mathcal{J}_{\mathrm{x}} \mathrm{d}\theta \mathrm{d}\varphi \Big/ \int\int \mathcal{J}_{\mathrm{x}} \mathrm{d}\theta \mathrm{d}\varphi$

- Compute $\rho(r,\theta,\varphi)$ and $\langle\rho\rangle_{\theta,\varphi}(r)$ with $\langle\, \cdot \,\rangle_{\theta,\varphi}(r) = \int\int \cdot\, \mathrm{d}\theta \mathrm{d}\varphi / L_\theta L_\varphi$
- Solve for all φ, solve with Fourier Projection in θ and Finite differences in r

$$(\mathcal{L} + \alpha(r))\ \tilde{\Phi} = \rho - \langle\, \rho \,\rangle_{\theta,\varphi} \quad \text{with} \quad \tilde{\Phi} = \phi - \langle\, \phi \,\rangle_{\theta,\varphi} \qquad (2)$$

- Compute $\langle\, \tilde{\Phi} \,\rangle_{\mathrm{FS}}$
- Solve the 1D radial system with Finite differences of second order

$$\mathcal{L}\langle\, \phi \,\rangle_{\theta,\varphi} + \alpha(r)\left(\langle\, \phi \,\rangle_{\theta,\varphi} - \langle\, \phi \,\rangle_{\mathrm{FS}}\right) = \langle\, \rho \,\rangle_{\theta,\varphi} \qquad (3)$$

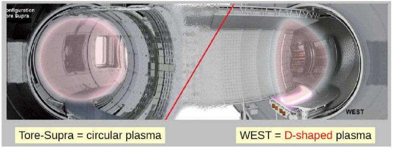
- Finally, ϕ is reconstructed as :

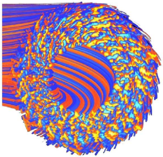
$$\phi = \underbrace{\tilde{\Phi}}_{\text{solution of (2)}} - \underbrace{\langle\, \tilde{\Phi} \,\rangle_{\mathrm{FS}}}_{\text{compute from } \tilde{\Phi}} + \underbrace{\langle\, \phi \,\rangle_{\mathrm{FS}}}_{\text{solution of (3)}}$$

More realistic magnetic configurations for GYSELA(1/2)

Collaboration IPP-Garching (L. Mendoza – PhD (joint supervision with IRFM) + A. Ratnani + E. Sonnendrucker), Univ Nancy (A. Back – Post-Doc) + INRIA Strasbourg (SELALIB team)

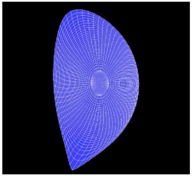
- Aim: More realistic magnetic configurations for GYSELA





Avoid hole

+ D-Shape



- Development of an hybrid method based on a coupling between semi-Lagrangian scheme and ISOgeometric approach.
  - Generalized Poisson solver under tests in SELALIB
  - Hybrid Vlasov-Poisson solver for simplified 4D drift-kinetic code using B-Splines under tests in SELALIB

CEMRACS 2014 project

More realistic magnetic configurations for GYSELA(2/2)

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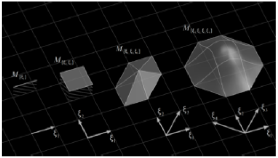
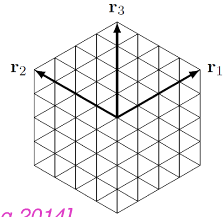


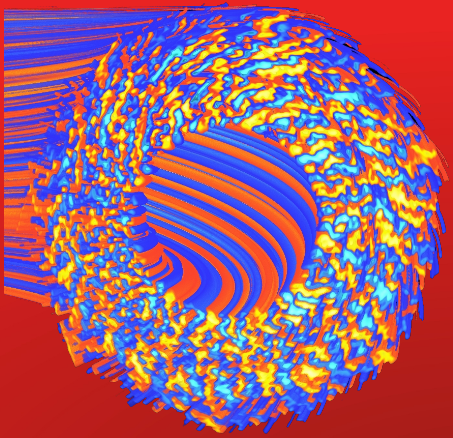
- A new mapping with Box-Splines
  - Generalization of B-splines
  - No singular points
  - No need of multiple patches for the core
  - Twelve-fold symmetry
    → more efficient programming
  - Regularity of the mesh
    → easy to find feet of characteristic for semi-Lagrangian scheme
- Test of feasibility of diocotron instability in progress in SELALIB



[L. Mendoza, Numkin-Garching 2014]

Collaborations:

- ANR GYPSI (2010-2014)
  ↪ Strasbourg, Nancy, Marseille
- ANR Nufuse G8@exascale (2012-2016)
  ↪ France, Germany, Japan, US, UK
- ADT INRIA Selalib (2011-2015)
  ↪ Strasbourg, Bordeaux
- Action C2S@Exa - IPL INRIA (march 2013-2017)
  ↪ Nice, Bordeaux
- New project following AEN INRIA Fusion (evaluation in progress)
  ↪ Strasbourg, Lyon, Nice
- Collaborations with IPP Garching (Germany) since 2012
- Collaborations with “Maison de la Simulation”- Saclay (Paris) since 2012



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