PhD subject : Model order reduction with mesh adaptation.

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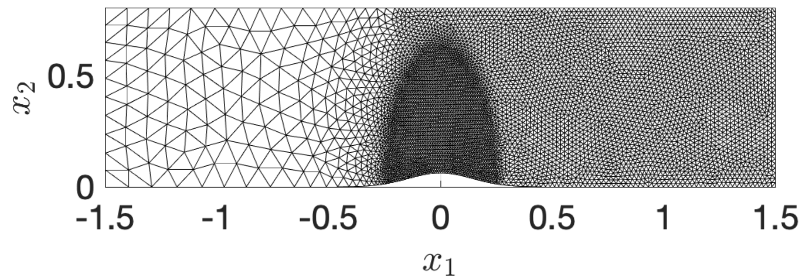
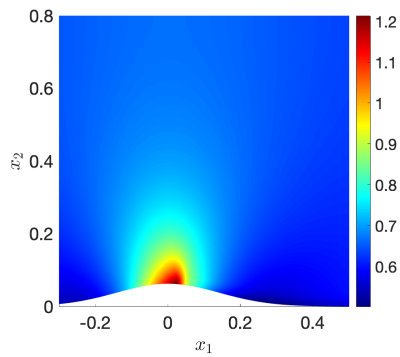
A wealth of applications in science and engineering involve the solution to computational fluid dynamics (CFD) problems for many different system configurations. For this class of problems, it is important to reduce the *marginal* cost associated with a given simulation over a range of parameters. Model order reduction (MOR) techniques rely on an **offline/online decomposition to reduce marginal costs**. During the offline phase, we rely on high-fidelity (`hf`) simulations to generate a reduced-order model (ROM) to estimate the solution over a range of parameters. During the online or deployment phase, given a new value of the parameter, we query the ROM to estimate the solution field. **We particularly focus on the development of automated procedures** for both training and deployment stages: autonomy refers to the ability to complete the analysis with minimal user intervention.

Approximation to advection-dominated PDEs poses several fundamental challenges to state-of-the-art model order reduction methods. First, despite the recent advances in high-performance computing and numerical analysis, `hf` numerical approximation of these problems requires extensive computational resources: as a result, reduced-order approximations are built using a moderate number of solutions (*snapshots*). Second, it is well known (e.g. [4]) that linear-approximation-based methods are completely inadequate to deal with parameter-dependent discontinuities: this motivates the development of **nonlinear approximation methods**. Third, MOR techniques rely on the projection of the equation onto a unique shared `hf` discretization and thus rely on the assumption that the underlying `hf` discretization is accurate *for all* parameters in a prescribed range [6]: for problems with parameter-dependent shocks, this requires accurate adaptive mesh refinement (AMR) over a broad portion of the spatial domain and is often unfeasible (Fig. 1).

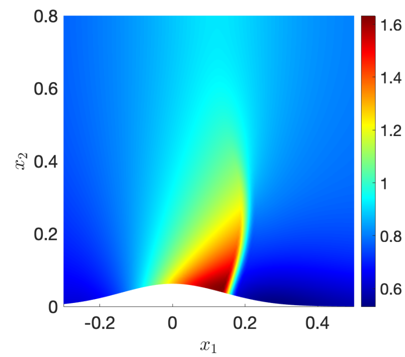
Mesh adaptation aims at improving the accuracy of numerical simulations while reducing the computational cost through automatic optimisation of the mesh resolution during the computation. More precisely, in anisotropic mesh adaptation, the mesh elements sizes and orientations are modified in order to minimize a certain numerical error model, to guarantee an optimal mesh size for a desired accuracy (Fig. 2). A non-linear process is considered that ensures convergence of the mesh/solution pair to the optimum with respect to the error model considered [3]. Anisotropic mesh adaptation is also able to handle unsteady problems, with complex geometries undergoing large displacements [1].



(a)

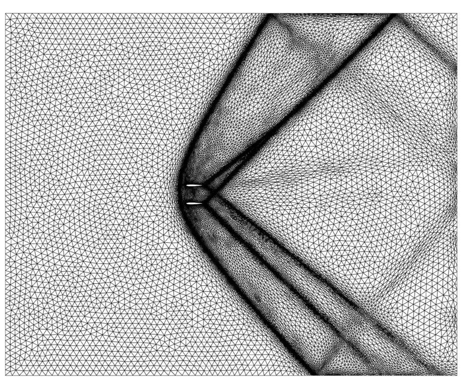


(b) $\text{Mach}_\infty = 0.62$

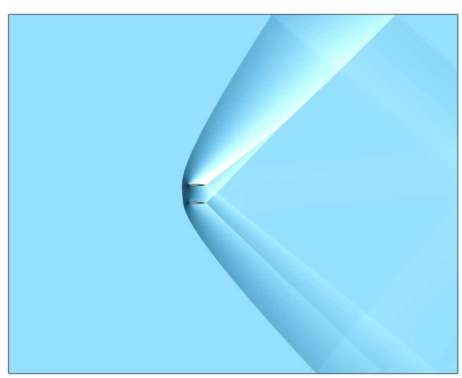


(c) $\text{Mach}_\infty = 0.74$

Figure 1: Transonic inviscid flow past a bump for several configurations. (a) common mesh. (b)-(c) snapshots of the Mach number close to the bump for two values of the free-stream Mach, Mach_∞. The mesh needs to be refined over a vast portion of the domain to capture the features of the flow.



(a)



(b)

Figure 2: Adaptive transonic inviscid flow past two NACA airfoils for one configuration: mesh (a) and density field (b). The mesh is refined tightly around the flow discontinuities.

The aim of the PhD project is to study a **novel integrated model reduction mesh adaptation approach for nonlinear advection-dominated systems of partial differential equations (PDEs)**, notably aerodynamic and hydraulic flows. Relevant solutions to these problems are characterized by parameter-dependent *shocks* and *contact* *discontinuities.*

To this aim, **registration-based model reduction will be combined with parametric mesh adaptation. Registration-based model reduction** [5] relies on a parametric mapping Φ to identify and then track relevant features of the solution field and ultimately improve performance of linear compression methods such as proper orthogonal decomposition (POD, [2]). **Parametric mesh adaptation** refers to the task of determining an accurate mesh for a range of system configurations. Provided that Φ is effective to track moving features of the solution, mesh adaptation should lead to considerably more parsimonious discretizations compared to uniform refinement, for any target accuracy.

The PhD student is expected to develop, analyze and then implement the combined model-reduction mesh-adaptation procedure for nonlinear advection-dominated PDEs. Automated procedures will be developed for both training and deployment stages: rigorous *a priori* and *a posteriori* analyses will inform the construction of the ROM and the mesh at training stage; *a posteriori* error indicators will be developed to certify the accuracy of the online predictions.

References

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